CONTEMPORARY MATHEMATICS

629

Riemann and Klein Surfaces, Automorphisms, Symmetries and Moduli Spaces

Conference in Honor of Emilio Bujalance
Riemann and Klein Surfaces, Symmetries
and Moduli Spaces
June 24–28, 2013
Linköping University, Linköping, Sweden

Milagros Izquierdo
S. Allen Broughton
Antonio F. Costa
Rubí E. Rodríguez
Editors

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This volume contains the proceedings of the conference on Riemann and Klein Surfaces, Symmetries and Moduli Spaces, in honor of Emilio Bujalance, held from June 24–28, 2013, at Linköping University.

The conference and this volume are devoted to the mathematics that Emilio Bujalance has worked with in the following areas, all with a computational flavor: Riemann and Klein surfaces, automorphisms of real and complex surfaces, group actions on surfaces and topological properties of moduli spaces of complex curves and Abelian varieties.

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Riemann and Klein Surfaces, Automorphisms, Symmetries and Moduli Spaces

To our master and colleague Prof. Bujalance
and specially to our great friend Emilio who united us all in mathematics

Preface

Group actions on Riemann surfaces, Klein surfaces, and related structures such as hyperbolic manifolds, complex surfaces, and Teichmüller spaces constitute an important nexus for group theory, combinatorics, geometry, topology, Galois theory and complex analysis. Compact Riemann surfaces are complex projective curves and have been studied for well over one hundred years. These surfaces have spawned a rich, interconnected theory in algebraic geometry, complex analysis, hyperbolic geometry, topology, and combinatorial/geometric group theory using tools such as uniformizing Fuchsian groups, Jacobian varieties, and moduli spaces. The use of combinatorial/geometric group theory for studying Riemann surfaces goes all the way back to Klein, Hurwitz and others; has continued through topological methods such as mapping class groups, Teichmüller and moduli spaces; and is a key part of the more recent developments in cryptographic applications, the Grothendieck theory of dessins d'enfants, and the use of powerful computer algebra systems such as Magma and GAP.

This volume is based on the conference, Riemann and Klein Surfaces, Symmetries, and Moduli Spaces, that took place at Linköping University 24–28 June 2013, in honor of Professor Emilio Bujalance. The conference and this volume are devoted to the mathematics that Professor Bujalance and his many colleagues have worked on, with special focus on Riemann and Klein surfaces, i.e., complex curves, automorphisms of surfaces (real and complex), group actions on surfaces, Grothendieck theory of dessins d'enfants, topological properties of moduli spaces of complex curves, and Abelian varieties. The conference had a computational/combinatorial flavour; the presence of a group action—geometric, combinatorial, or topological—was felt in almost every paper presented.

There is considerable current activity in these fields. As noted, these topics range from classical problems of classifying Riemann and Klein surfaces and their automorphism groups to the discovery of the topological structure of moduli spaces and the Grothendieck theory of dessins d'enfants. This volume should be a very useful source for anyone working in the area since the conference covered such a wide spectrum of topics. To guide mathematicians new to the area, several articles of an expository nature have been included. The volume will also serve as a tool in communicating the interplay among the different areas.

S. Allen Broughton, Antonio F. Costa, Milagros Izquierdo and Rubí E. Rodríguez

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On Jacobians of curves with superelliptic components

L. Beshaj, T. Shaska, and C. Shor

Dedicated to Professor Emilio Bujalance on the occasion of his 60th birthday

ABSTRACT. We construct a family of non-hyperelliptic curves whose Jacobians decomposes into a product of superelliptic Jacobians. Moreover, we investigate the decomposition of Jacobians of superelliptic curves based on their automorphisms. For a curve given by the equation $y^n = f(x^m)$, we provide a necessary and sufficient condition in terms of m and n for the the Jacobian of the curve to decompose.

1. Introduction

Let $\mathcal{X}$ be a genus $g \geq 2$ smooth, irreducible projective curve, defined over an algebraically closed field k and $\pi_i : \mathcal{X} \to \mathcal{X}_i$, $1 \leq i \leq s$ be coverings to genus g_i curves $\mathcal{X}_i$, respectively. Denote by $\mathrm{Jac}(\mathcal{X})$ and $\mathrm{Jac}(\mathcal{X}_i)$ the Jacobians of these curves respectively. A classical question is to determine when the Jacobian $\mathrm{Jac}(\mathcal{X})$ is isogenous to a product of $\mathrm{Jac}(\mathcal{X}_i)$. The problem has been studied in the XIX century by Legendre, Jacobi, Hermite, Klein, Kovalevskaya, Hecke, et al. In this context, these curves were of interest due to the fact that split Jacobians give relations between abelian and elliptic integrals. Most of these efforts were focused on the case when the coverings $\pi_i : \mathcal{X} \to \mathcal{X}_i$ are Galois coverings.

The case when genus $g(\mathcal{X}_i) = 1$ for some i and $\pi_i : \mathcal{X} \to \mathcal{X}_i$ not necessarily Galois is especially interesting due to its implications in number theory and relations to Lang's conjecture; see [**7**, **8**] and [**10**]. When $g = 2$ such coverings have different ramification structures when $\deg \pi_i$ is odd or even; see [**4**] for a summary of results in genus two. In both cases the moduli space of such covers with fixed ramification structure can be embedded as a subvariety of $\mathcal{M}_2$.

The Jacobian $\mathrm{Jac}(\mathcal{X})$ is said to (completely) split if it is isogenous to the product of elliptic curves. Ekedahl and Serre [**6**] posed the following questions: i) Is it true that for every integer $g > 1$ there exists a curve of genus g with split Jacobian? ii) Is there a bound on the genus of a curve with split Jacobian? Towards answering these questions they used modular curves, and coverings of curves of genus 2 and 3. The maximal genus of their examples was 1297. Many other authors have considered these questions as well, and in recent work, Shaska [**15**, **16**] and Yamauchi [**20**], have produced somewhat unusual families of curves with decomposable Jacobians.

In this paper we will focus on the cases when the coverings $\pi_i : \mathcal{X} \to \mathcal{X}_i$ are cyclic Galois coverings. Hence, these coverings induce automorphisms on $\mathcal{X}$. By

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focusing on such curves we have a more organized way to study the decompositions of such Jacobians starting from the automorphism group $\mathrm{Aut}(\mathcal{X})$ of $\mathcal{X}$. While we can determine the full list of groups that occur as full automorphism groups of genus $g \geq 2$, the corresponding equations of curves for each group are not known; see [**3**]. There is, however, a nice family of curves when such decomposition can be fully explored and the factors of such Jacobians can be fully determined. This is the class of superelliptic curves, namely the curves which can be written as affine varieties with equation $y^n = f(x)$ for some $n \geq 2$ and discriminant $\Delta(f, x) \neq 0$. For such curves we know precisely the structure of the automorphism group G, the signature, the equations of the curves, and their invariants; see [**5**]. Hence, it is possible to determine explicitly the components of such Jacobians and in some cases the moduli space of such curves in the moduli space $\mathcal{M}_g$. A complete description of curves for which such decompositions is based on their automorphisms is intended in [**3**].

The second goal of the paper is to extend the family of curves introduced by Yamauchi in [**20**]. The family of curves introduced in [**20**] are non-hyperelliptic curves with arbitrary large genus such that the Jacobian has hyperelliptic components. We attempt to extend this family to curves whose Jacobians have superelliptic components. Our proof is based on the automorphisms of curves. We show that such decomposition based on the automorphisms of the curve induces some arithmetic condition among the orders of such automorphisms. This condition implies that the family $F_{m,n}$ constructed in [**20**] agrees with our family of curves only for the values $F_{m,1}$ and $F_{m,2}$.

This paper is organized as follows. In section 2 we briefly define the Jacobian of a curve and give two classical results of Accola; see [**1**, **2**]. In these results it is described how one can start from a partition of a group of automorphisms of the curve $\mathcal{X}$ and get a decomposition of the $\mathrm{Jac}(\mathcal{X})$. Further in this section we describe results of Kani and Rosen were the previous results were generalized; see [**9**]. We use such results in our decomposition of Jacobians of the superelliptic curves for the family of curves described in section 4.

In section 3 we study the decomposition of Jacobians of superelliptic curves. A superelliptic curve $\mathcal{X}$ is a curve with equation $y^n = f(x)$ defined over a field k, where $(\mathrm{char}\ k, n) = 1$ and k is algebraically closed. Such curves have the *superelliptic* automorphism $\tau : (x, y) \to (x, \zeta_n y)$, where ζ_n is an n-th primitive root of unity and τ is central in $G := \mathrm{Aut}(\mathcal{X})$. Let σ be another automorphism of $\mathcal{X}$ such that its projection $\bar{\sigma} \in \overline{G} := G/\langle\tau\rangle$ has order m. Then σ and $\sigma\tau$ fix two subfields of the function field $k(\mathcal{X})$ and therefore there are two quotient curves $\mathcal{X}_1 := \mathcal{X}/\langle\sigma\rangle$ and $\mathcal{X}_2 := \mathcal{X}/\langle\sigma\tau\rangle$. We determine equations of $\mathcal{X}_1$ and $\mathcal{X}_2$ and a necessary and sufficient condition in terms of m and n such that $\mathrm{Jac}(\mathcal{X})$ is isogenous to $\mathrm{Jac}(\mathcal{X}_1) \times \mathrm{Jac}(\mathcal{X}_2)$.

In section 4 we generalize a construction of Yamauchi of a family $F_{m,n}$ of non-hyperelliptic curves with decomposable Jacobians, where all components are hyperelliptic Jacobians. Instead we construct a family $\mathcal{X}_{r,s}$ of curves whose Jacobians are superelliptic Jacobians. We prove that the automorphism group of these *component curves* are cyclic or dihedral groups. Moreover, we find a necessary and sufficient condition in terms of r and s for this decomposition to occur. Our decomposition is based solely on the decomposition induced by the automorphisms of the curves.

Notation: Throughout this paper by g we denote an integer ≥ 2 and k denotes an algebraically closed field. Unless otherwise noted, by a *curve* we always mean the isomorphism class of an algebraic curve defined over k. The automorphism group of a curve always means the full automorphism group of the curve over k. We denote the cyclic group of order n by C_n and the dihedral group of order $2n$ by D_{2n}. V_4 denotes the Klein 4-group and ζ_n denotes an n-th primitive root of unity.

2. Preliminaries

Let $\mathcal{X}$ be a genus $g \geq 2$ algebraic curve defined over $\mathbb{C}$. We choose a symplectic homology basis for $\mathcal{X}$, say $\{A_1, \ldots, A_g, B_1, \ldots, B_g\}$, such that the intersection products $A_i \cdot A_j = B_i \cdot B_j = 0$ and $A_i \cdot B_j = \delta_{ij}$, where δ_{ij} is the Kronecker delta. We choose a basis $\{w_i\}$ for the space of holomorphic 1-forms such that $\int_{A_i} w_j = \delta_{ij}$. The matrix $\Omega = \left[\int_{B_i} w_j\right]$ is the *Riemann matrix* of $\mathcal{X}$ and the matrix $[I \,|\Omega]$ is called the *period matrix*. The columns of the matrix $[I \,|\Omega]$ form a lattice L in $\mathbb{C}^g$. The complex torus $\mathbb{C}^g/L$ is called the Jacobian of $\mathcal{X}$ is denoted by $\mathrm{Jac}(\mathcal{X})$.

Let $\mathcal{H}_g$ be the *Siegel upper-half space*. Then $\Omega \in \mathcal{H}_g$ and there is an injection

$$\mathcal{M}_g \hookrightarrow \mathcal{H}_g / Sp_{2g}(\mathbb{Z}) =: \mathcal{A}_g$$

where $Sp_{2g}(\mathbb{Z})$ is the *symplectic group*. A non-constant morphism $f : A \to B$ between two Abelian varieties which is surjective and of finite kernel is called an **isogeny**. An Abelian variety is called *decomposable* if it is isogenous to a product of Abelian varieties, it is *simple* if it has no non-trivial Abelian subvarieties. An Abelian variety is called **completely decomposable** or **completely split** if it is isogenous to a product of elliptic curves.

A map of algebraic curves $f : \mathcal{X} \to \mathcal{Y}$ is called a **maximal covering** if it does not factor over a nontrivial isogeny. A map $f : \mathcal{X} \to \mathcal{Y}$ induces maps between their Jacobians $f^* : \mathrm{Jac}(\mathcal{Y}) \to \mathrm{Jac}(\mathcal{X})$ and $f_* : \mathrm{Jac}(\mathcal{X}) \to \mathrm{Jac}(\mathcal{Y})$. When f is maximal then f^* is injective and $\ker(f_*)$ is connected, see [**14**, p. 158] for details. Hence, $\mathrm{Jac}(\mathcal{X}) \cong \mathrm{Jac}(\mathcal{Y}) \times A$, where A is some Abelian variety. Thus, coverings $f : \mathcal{X} \to \mathcal{Y}$ give factors of the Jacobian $\mathrm{Jac}(\mathcal{X})$. Such methods have been explored for genus 2 curves by Shaska et al. in [**11**, **17**, **18**]. They are the only examples that we know when explicit computations have been performed and the corresponding locus has been computed for non-Galois coverings.

If the covering $f : \mathcal{X} \to \mathcal{Y}$ is a Galois covering then its monodromy group is isomorphic to a subgroup H of the automorphism group $G = \mathrm{Aut}(\mathcal{X})$. Hence a common procedure to produce decompositions of Jacobians is to explore the automorphism group of the curve.

Fix an integer $g \geq 2$ and a finite group G. Let $C_1, \ldots, C_r$ be conjugacy classes $\neq \{1\}$ of G. Let $\mathbf{C} = (C_1, \ldots, C_r)$, be an unordered tuple, repetitions are allowed. We allow r to be zero, in which case $\mathbf{C}$ is empty.

Consider pairs $(\mathcal{X}, \mu)$, where $\mathcal{X}$ is a curve and $\mu : G \to \mathrm{Aut}(\mathcal{X})$ is an injective homomorphism. Mostly we will suppress μ and just say $\mathcal{X}$ is a curve with G-action, or a G-curve, for short. Two G-curves $\mathcal{X}$ and $\mathcal{X}'$ are called equivalent if there is a G-equivariant isomorphism $\mathcal{X} \to \mathcal{X}'$.

We say a G-curve $\mathcal{X}$ is **of ramification type** $(g, G, \mathbf{C})$ if g is the genus of $\mathcal{X}$ and the points of the quotient $\mathcal{X}/G$ that are ramified in the cover $\mathcal{X} \to \mathcal{X}/G$ can be labelled as $p_1, \ldots, p_r$ such that C_i is the conjugacy class in G of distinguished inertia group generators over p_i (for $i = 1, \ldots, r$).

Let $\mathcal{X}$ be a G-curve and $H < G$. Then there is a covering $\mathcal{X} \to \mathcal{X}/H$. Let the genus of $\mathcal{X}/H$ be denoted by g_H. How is g_H determined in terms of g and G? Consider the following problem: let $H_1, \dots, H_r$ be subgroups of G and $g_1, \dots, g_r$ the genera of the $\mathcal{X}/H_1, \dots, \mathcal{X}/H_r$ respectively. Is there any arithmetic relation between $g, g_1, \dots, g_r$?

Accola proved the following results which provide a method for decomposing Jacobians, see [**1**].

THEOREM 2.1 ([**1**]). *Let $\mathcal{X}_g$ be a G-curve, $H_i < G$, g_i the genus of $\mathcal{X}_g/H_i$ for $i = 1, \dots s$, and $g_0 = g(\mathcal{X}_g/G)$. Assume that $H_i \cap H_j = \{e\}$ for all $i \neq j$. Then,*

$$g_0 \, |G| = g - sg + \sum_{i=1}^{s} |H_i| \, g_i.$$

Moreover, we have the following relation among the genera; see [**2**].

THEOREM 2.2 ([**2**]). *Let $\mathcal{X}_g$ be a G-curve, and H_i, for $i = 1, \dots s$, subgroups of G such that $G = \cup_i^s H_i$. Denote by g_i the genus of $\mathcal{X}_g/H_i$ and by $H_{ij\dots k} = H_i \cap H_j \cap \cdots \cap H_k$.*

Then,

$$g_0 \, |G| = \sum_{i=1}^{s} |H_i| \cdot g_i - \sum |H_{ij}| \cdot g_{ij} + \sum |H_{ijk}| \cdot g_{ijk} - \cdots - (-1)^s \sum |H_{12\dots s}| \cdot g_{12..s}.$$

See [**2**] for details.

2.1. Decomposing the Jacobian by group partitions. Let $\mathcal{X}$ be a genus g algebraic curve with automorphism group $G := \mathrm{Aut}(\mathcal{X})$. Let $H \leq G$ such that $H = H_1 \cup \cdots \cup H_t$ where the subgroups $H_i \leq H$ satisfy $H_i \cap H_j = \{1\}$ for all $i \neq j$. Then,

$$\mathrm{Jac}^{t-1}(\mathcal{X}) \times \mathrm{Jac}^{|H|}(\mathcal{X}/H) \cong \mathrm{Jac}^{|H_1|}(\mathcal{X}/H_1) \times \cdots \mathrm{Jac}^{|H_t|}(\mathcal{X}/H_t).$$

The group H satisfying these conditions is called a group with partition. Elementary Abelian p-groups, the projective linear groups $PSL_2(q)$, Frobenius groups, dihedral groups are all groups with partition.

Let $H_1, \dots, H_t \leq G$ be subgroups with $H_i \cdot H_j = H_j \cdot H_i$ for all $i, j \leq t$, and let g_{ij} denote the genus of the quotient curve $\mathcal{X}/(H_i \cdot H_j)$. Then, for $n_1, \dots, n_t \in \mathbb{Z}$ the conditions $\sum n_i n_j g_{ij} = 0$, $\sum_{j=1}^{t} n_j g_{ij} = 0$, imply the isogeny relation

$$\prod_{n_i > 0} \mathrm{Jac}^{n_i}(\mathcal{X}/H_i) \cong \prod_{n_j < 0} \mathrm{Jac}^{n_j}(\mathcal{X}/H_j). \tag{2.1}$$

In particular, if $g_{ij} = 0$ for $2 \leq i < j \leq t$ and if $g = g_{\mathcal{X}/H_2} + \cdots + g_{\mathcal{X}/H_t}$, then

$$\mathrm{Jac}(\mathcal{X}) \cong \mathrm{Jac}(\mathcal{X}/H_2) \times \cdots \times \mathrm{Jac}(\mathcal{X}/H_t). \tag{2.2}$$

The proof of the above statements can be found in [**9**].

3. Jacobians of superelliptic curves

A curve $\mathcal{X}$ is called superelliptic if there exist an element $\tau \in G := \mathrm{Aut}(\mathcal{X})$ such that τ is central and $g(\mathcal{X}/\langle\tau\rangle) = 0$. Let $\mathcal{X}_g$ have affine equation given by some polynomial in terms of x and y and denote by $K = k(x, y)$ the function field of $\mathcal{X}_g$. Let $H = \langle\tau\rangle$ be a cyclic subgroup of G such that $|H| = n$ and $H \triangleleft G$, where $n \geq 2$. Moreover, we assume that the quotient curve $\mathcal{X}_g/H$ has genus zero. The

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