

volume 105

lecture notes in pure and applied mathematics



# geometry and topology

manifolds, varieties, and knots

edited by  
Clint McCrory  
Theodore Shifrin

 **CRC Press**  
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Clint McCrory  
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## Preface

This volume is the proceedings of the 1985 Georgia topology conference, which was held at the University of Georgia in Athens from August 5 to August 16, 1985. This conference followed the tradition of the Georgia topology conferences of 1961, 1969, and 1977, in bringing together many of the leading research mathematicians working in topology and allied fields.

Topics of discussion ranged from traditional areas of topology, such as knot theory and the topology of manifolds, to areas such as differential and algebraic geometry. Geometry continues to provide a rich source of both topological examples and tools with which to attack fundamental topological problems. The recent work of Simon Donaldson on four-manifolds is a striking example of the power of geometry, and is also a reminder of the essential unity of mathematics. An application of these ideas is given in the article by Friedman and Morgan.

Knot theory was well represented at the conference. An exciting trend in knot theory is its increasing application to chemistry, a provocative example of which is described in DeWitt Sumners' survey article on DNA research. The tendency of applications to engender new theoretical questions is illustrated by Keith Wolcott's work on the knotting of graphs.

Other topics discussed in this volume are three-manifolds, group actions, and algebraic varieties. The article by Akbulut and King is an exposition of their important work on the topological characterization of real algebraic varieties. A beautiful application of classical geometry to topology is described by Davis in his article on convex cell complexes.

Helpful advice in planning the conference was received from R. H. Bing, R. C. Kirby, J. Morgan, D. Sullivan, and W. Thurston. We are grateful, in addition, to J. C. Cantrell and J. G. Hollingsworth for their patient advice and unwavering assistance in the conference.

Invited addresses were presented by S. Akbulut, J. Cannon, S. Cappell, M. Davis, S. Donaldson, R. Edwards, F. T. Farrell, D. Gabai, W. Goldman, J. Harer, A. Hatcher, W-C. Hsiang, S. Kerckhoff, W. Meeks, W. Pardon, F. Quinn, P. Scott, P. Shalen, S. Weinberger, and J. West.

Financial support for the conference was provided by the National Science Foundation, the University of Georgia Research Foundation, the Franklin College of Arts and Sciences of the University of Georgia, and the Department of Mathematics of the University of Georgia. The papers submitted for this volume were referred by conference participants. The referees have done an excellent job of improving the exposition of the mathematics presented here. Mrs. Dianne Byrd typed the proceedings, and we wish to thank her warmly for her patient and painstaking job. We also thank the staff of Marcel Dekker for their assistance in producing this book.

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