College Mathematics

Second Edition

Tan

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PWS-KENT
Publishing
Company
Boston

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Publishing Company

20 Park Plaza
Boston, Massachusetts 02116

PWS–KENT Publishing Company is a division of Wadsworth, Inc.

Library of Congress Cataloging-in-Publication Data

Tan, Soo Tang.
College mathematics / S.T. Tan.
p. cm.
Rev. ed. of: College mathematics for the managerial and social sciences.
Includes index.
ISBN 0-534-91791-7
1. Mathematics—1961– . 2. Management—Mathematics. 3. Social sciences—Mathematics.
I. Tan, Soo Tang. College mathematics for the managerial and social sciences. II. Title
QA37.2.T36 1988 87-19821
510—dc19 CIP

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Printed in the United States of America.
89 90 91 92 — 10 9 8 7 6 5 4 3 2

Sponsoring Editor *Tom Stone*
Production Coordinator/Designer *Elise Kaiser*
Production *Del Mar Associates*
Composition *Polyglot Pte. Ltd./Weimer Typesetting Company, Inc.*
Cover Printer *Phoenix Color Corp.*
Text Printer/Binder *The Maple-Vail Book Manufacturing Group*

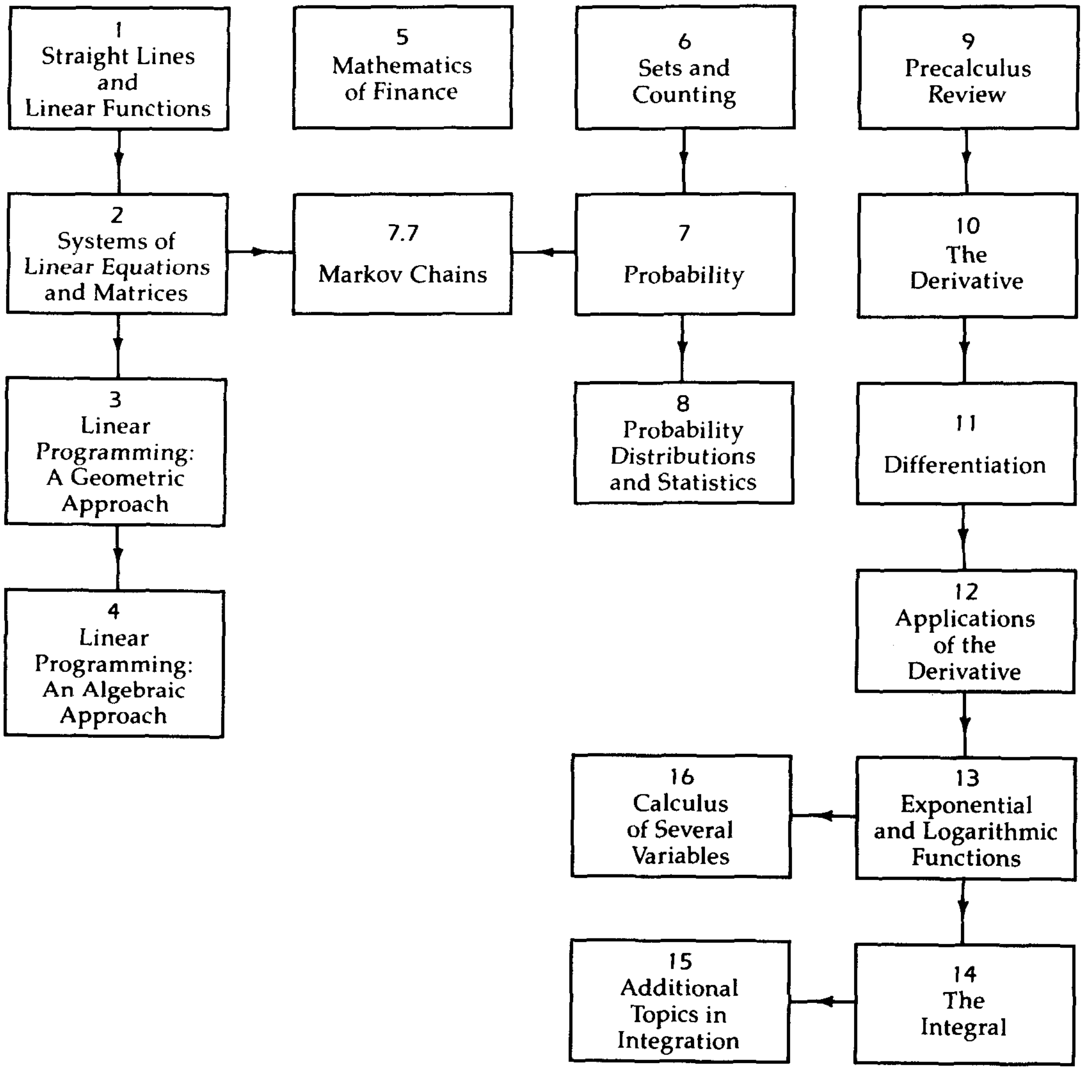
Cover art *"La Mer du Soir (g)"* by Kasai Masahiro; a silkscreen print used with the permission of the artist.

Preface

College Mathematics, Second Edition treats the standard topics in finite mathematics and calculus that are usually covered in a two-semester course for students in the managerial, life, and social sciences. The only prerequisite for understanding this book is a year of high school algebra. The objective of *College Mathematics* is two-fold: (1) to provide students with the background in the quantitative techniques necessary to better understand and appreciate their undergraduate courses and (2) to lay the foundation for more advanced courses, such as statistics and operations research. We hope that the careful balance of theory and applications accomplishes this.

Our approach is intuitive and we state the results informally. However, we took special care to ensure that this does not compromise the mathematical content and accuracy. The applications are drawn from many fields, and we made every effort to make them interesting, relevant, and up to date. Numerous examples and solved problems are used to amplify each new concept or result in order to facilitate students' comprehension of the new material. Each section is accompanied by an extensive set of exercises, which contains ample problems of a routine computational nature to help students master new techniques, followed by an extensive set of applications-oriented problems to test students' mastery of the topics. Each chapter of the text also ends with a set of review exercises. Answers to odd-numbered exercises appear in the back of the book.

Since the book contains more than enough material for the usual two-semester or three-quarter course, the instructor may be flexible in choosing the topics most suitable for his or her course. The following chart on chapter dependency is provided to help the instructor design a course that is most suitable for the intended audience.



Changes in the Second Edition

This edition contains an improved treatment of many of the topics in the previous edition, as well as a few additions and changes:

* Self-Check Exercises have been added at the end of each section. These exercises, with completely worked-out solutions appearing at the end of each exercise set, give students a chance to test themselves on their understanding of the material.
* Many more examples and exercises have been included. The examples, drawn from the fields of business, economics, social and behavioral sciences, life and physical sciences, and other fields of general interest, pro-

vide further illustrations of the concepts. The exercises are of varied degrees of difficulty ranging from rote to more challenging problems.

* Chapter 2 has been rewritten. A more traditional approach has been adopted with systems of linear equations appearing before matrices.
* Linear programming is covered in two chapters to allow for greater flexibility in planning one's syllabus.
* Sets and the algebra of sets have been moved forward to Chapter 6.
* Chapter 5, Mathematics of Finance, has been moved forward. Also, the section on arithmetic and geometric progression has been relegated to the end of the chapter and is now optional.
* A precalculus review has been added in Chapter 9. A special feature of this review is the inclusion of fully worked-out examples illustrating how algebraic techniques are used to simplify the more complicated expressions that occur later in the calculus portion of the text. These examples are indicated by the symbol Ⓡ. Many of the exercises in Chapter 9 have also been selected from the related calculus exercises.
* The introduction to limits has been rewritten and a real-world example has been included to motivate the concept.
* The chapter on the exponential and logarithmic functions is now covered after the student has mastered the differentiation of algebraic functions.
* Problems requiring the use of a calculator have been labeled [C]
* Added topics in the second edition include: (1) differentials, (2) applications to probability, (3) numerical integration, and (4) the method of least squares.

Acknowledgments

I wish to express my personal appreciation to each of the following reviewers whose many suggestions have helped make a much improved second edition: Wayne Andrepont, University of Southwestern Louisiana; Ralph Bravaco, Stonehill College; Lee Corbin, College of the Canyons; Robert Eicken, Illinois Central College; and Raymond Young, Embry-Riddle Aeronautical University. I also wish to thank Reverend Robert J. Kruse, Academic Dean of Stonehill College, for his enthusiastic support of this project. My thanks also go to the editorial and production staffs of PWS-KENT Publishing Company: Tom Stone and Elise Kaiser, for their assistance and cooperation in the development and production of this book. Finally, I wish to thank Nancy Sjoberg of Del Mar Associates and Andrea Olshevsky for doing an excellent job ensuring the accuracy and readability of this second edition.

S. T. Tan

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College Mathematics

Straight Lines and Linear Functions

Which road to take? Two towns are connected by two different routes: one runs along the coast and the other includes a stretch of mountain highway. The coastal route is faster but longer than the inland route. In Example 2, page 8, we will solve the problem of getting from one town to the other in the shortest time.

CHAPTER ONE

1.1 The Cartesian Coordinate System

- The Cartesian Coordinate System
- The Distance Formula
- Application

The Cartesian Coordinate System

The system of real numbers plays a fundamental role throughout this book. This system is made up of the set of real numbers together with the usual operations of addition, subtraction, multiplication, and division. We will assume that you are familiar with the rules governing these algebraic operations (see Appendix A).

It is convenient and fruitful to have a geometrical representation of the set of real numbers. Such a representation is called a **number line** and is constructed as follows: Arbitrarily select a point on a straight line to represent the number zero. This point is called the **origin.** If the line is horizontal, then a point at a convenient distance to the right of the origin is chosen to represent the number one. This determines the scale for the number line. Each positive real number x lies x units to the right of zero, and each negative real number $-x$ lies x units to the left of zero.

In this manner, a one-to-one correspondence is set up between the set of real numbers and the set of points on the number line, with all the positive numbers lying to the right of the origin and all the negative numbers lying to the left of the origin (see Figure 1.1).

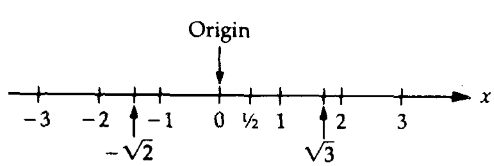

Figure 1.1 The real number line

A similar representation for points in a plane (a two-dimensional space) is realized through the **Cartesian coordinate system,** which is constructed as follows: Take two perpendicular lines, one of which is normally chosen to be horizontal. These lines intersect at a point O, called the **origin** (see Figure 1.2). The horizontal line is called the **axis of abscissas,** or more simply, the ***x*-axis.** The vertical line is called the **axis of ordinates,** or the ***y*-axis.** A number scale is set up along the x-axis with the positive numbers lying to the right of the origin and the negative numbers lying to the left of the origin. Similarly, a number scale is set up along the y-axis with the positive numbers lying above the origin and the negative numbers lying below the origin.

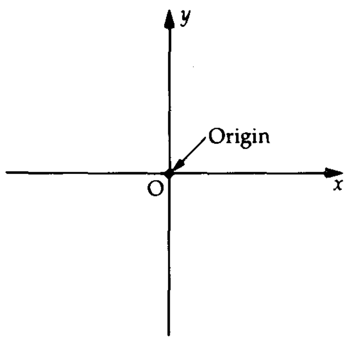

Figure 1.2 The Cartesian coordinate system

The number scales on the two axes need not be the same. Indeed, in many applications different quantities are represented by x and y. For example, x may represent the number of typewriters sold and y the total revenue resulting from the sales. In such cases it is often desirable to choose different number scales to represent the different quantities. Note, however, that the zeros of both number scales coincide at the origin of the two-dimensional coordinate system.

A point in the plane can now be represented uniquely in this coordinate system by an **ordered pair** of numbers, that is, a pair (x, y) where x is the first number and y the second. To see this, let P be any point in the plane (see Figure 1.3). Draw perpendiculars from P to the x-axis and y-axis, respectively. Then the number x is precisely the number corresponding to the point on the x-axis at which the perpendicular through P hits the x-axis. Similarly, y is the number

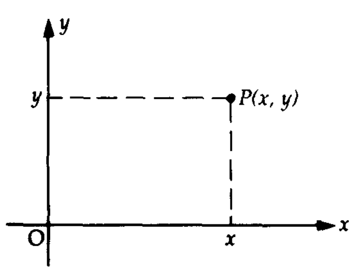

Figure 1.3