Precalculus

Larson / Hostetler

PRECALCULUS

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PREFACE

Success in college level mathematics courses for those interested in any one of a variety of disciplines such as computer science, engineering, management, statistics, or one of the natural sciences begins with a firm understanding of algebraic and trigonometric concepts. The goal of our textbook is to further the preparation of students, who have completed two years of high school algebra, in such important areas as graphical techniques, algebraic and transcendental functions, and analytic geometry. These are some of the fundamental elements used in the calculus and other mathematical endeavors that many students pursue.

The features of our book have been designed to create a comprehensive teaching instrument that employs effective pedagogical techniques.

• *Order of Topics.* Chapter 1 provides a quick review of the basic concepts of algebra. The algebra of polynomial and rational functions (Chapters 1 through 3) is brought to a logical conclusion with a discussion of the Fundamental Theorem of Algebra. Then, in Chapters 4 through 7 coverage is given to the transcendental functions and their applications. Chapters 8 through 10 include additional topics in algebra: systems of linear equations, matrices, sequences, and series. Analytic geometry is discussed in Chapters 11 and 12.

• *Algebra of Calculus.* Special emphasis has been given to the *algebra of calculus*. Many examples and exercises consist of algebra problems that arise in the study of calculus. These examples are clearly identified.

• *Examples.* The text contains over 550 examples, each carefully chosen to illustrate a particular concept or problem-solving technique. Each example is titled for quick reference and many examples include side comments (set in color) to justify or explain the steps in the solution.

• *Exercises.* Over 3800 exercises are included that are designed to build competence, skill, and understanding. Each exercise set is graded in difficulty to allow students to gain confidence as they progress. To help prepare students for calculus, we stress a graphical approach in many sections and have included numerous graphs in the exercises.

• *Graphics.* The ability to visualize a problem is a critical part of a student's ability to solve a problem. This text includes over 900 figures.

• *Applications.* Throughout the textual material we have included numerous word problems that give students concrete ideas about the usefulness of the topics included.

• *Calculators.* Although we do not require the use of calculators in any section, techniques for calculator use are provided at appropriate places throughout the text. In addition, calculators have allowed us to include many realistic applications that are often excluded from other texts because of lengthy or tedious computations. Exercises meant to be solved with the help of a calculator are clearly indicated.

• *Supplements.* An *Instructor's Guide* by Meredythe M. Burrows is available and it includes answers to the even-numbered exercises as well as sample tests for each chapter.

We would like to thank the many people who have helped us at various stages of this project. Their encouragement, criticisms, and suggestions have been invaluable to us. The following reviewers offered many excellent ideas: Ben P. Bockstage, Broward Community College; Daniel D. Bonar, Denison University; H. Eugene Hall, DeKalb Community College; William B. Jones, University of Colorado; Jimmie D. Lawson, Louisiana State University; Jerome L. Paul, University of Cincinnati; and Shirley C. Sorensen, University of Maryland.

The mathematicians listed below completed a survey conducted by D. C. Heath and Company in 1983 which helped outline our topical coverage: Stan Adamski, University of Toledo; Daniel D. Anderson, University of Iowa; James E. Arnold, University of Wisconsin; Prem N. Bajaj, Wichita State University; Imogene C. Beckemeyer, Southern Illinois University; Bruce Blake, Clemson University; Dale E. Boye, Schoolcraft College; Sarah W. Bradsher, Danville Community College; John H. Brevit, Western Kentucky University; Milo F. Bryn, South Dakota State University; Gary G. Carlson, Brigham Young University; Louis J. Chatterley, Brigham Young University; Mary Clarke, Cerritos College; Lee G. Corbin, College of the Canyons; Milton D. Cox, Miami University; Robert G. Cromie, St. Lawrence University; Bettyann Daley, University of Delaware; Clinton O. Davis, Brevard Community College; Karen R. Dougan, University of Florida; Richard B. Duncan, Tidewater Community College; Don Duttenhoeffer, Brevard Community College; Bruce R. Ebanks, Texas Technological University; Susan L. Ehlers, St. Louis Community College; Delvis A. Fernandez, Chabot College; Leslee Francis, Brigham Young University; August J. Garver, University of Missouri; Douglas W. Hall, Michigan State University; James E. Hall, University of Wisconsin; Nancy Harbour, Brevard Community College; Ferdinand Haring, North Dakota State University; Cecilia Holt, Calhoun Community College; James Howard, Ferris State College; Don Jefferies, Orange Coast College; William B. Jones, University of Colorado; Eugene F. Krause, University of Michigan; Richard Langlie, North Hennerin Community College; Jimmie D. Lawson, Louisiana State University; John Linnen, Ferris State College; Joseph T. Mathis, William Jewell College; Walter M. Potter, University of Wisconsin; Sandra M. Powers, College of Charleston; Nancy J. Poxon, California State University; George C. Ragland, St. Louis Community College; Ralph J. Redman, University of Southern Colorado; Emilio O. Roxin, University of Rhode Island; Charles I. Sherrill, University of Colorado; Donald R. Snow, Brigham Young University; William F. Stearns, University of Maine; Don L. Stevens, Eastern Oregon State College; Warren Strickland, Del Mar College; Billy J. Taylor, Gainesville Junior College; Henry Tjoelker, California State University; Richard G. Vinson, University of Southern Alabama; Glorya Welch, Cerritos College; Dennis Weltman, North Harris County College; Larry G. Williams, Schoolcraft College; and Rey Ysais, Cerritos College.

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On a personal level, we are grateful to our children for their interest and support during the three years the book was being written and produced, and to our wives, Deanna Larson and Eloise Hostetler, for their love, patience, and understanding.

If you have suggestions for improving this text, please feel free to write us. Over the past several years we have received many useful comments from both instructors and students and we value this very much.

Roland E. Larson
Robert P. Hostetler

THE LARSON AND HOSTETLER PRECALCULUS SERIES

To accommodate the different methods of teaching college algebra, trigonometry, and analytic geometry, we have prepared four volumes. These separate titles are described below.

COLLEGE ALGEBRA

A text designed for a one-term course covering standard topics such as algebraic functions, exponential and logarithmic functions, matrices, determinants, probability, sequences, and series.

TRIGONOMETRY

This text is used in a one-term course covering the trigonometric functions, exponential and logarithmic functions, and analytical geometry.

ALGEBRA AND TRIGONOMETRY

This title combines the content of the two texts mentioned above (with the exception of analytic geometry). It is comprehensive enough for two terms of courses or may be covered, with careful selection, in one term.

PRECALCULUS

With this book, students cover the algebraic and trigonometric functions, and analytic geometry in preparation for a course in calculus. This may be used in a one- or two-term course.

CONTENTS

1 REVIEW: REAL NUMBERS, THE PLANE, AND ALGEBRA

1.1 The Real Number System

In this text we will assume that you are familiar with basic algebra and will begin our study of precalculus with a look at the **real number system.** We use real numbers every day to describe quantities like age, miles per gallon, container size, population, and so on. To represent real numbers we use symbols such as

$$9, \quad -5, \quad \sqrt{2}, \quad \pi, \quad \tfrac{4}{3}, \quad 0.6666\ldots,$$
$$28.21, \quad 0, \quad \text{and} \quad \sqrt[3]{-32}$$

The set of real numbers is made up of the following five subsets:

Natural Numbers	$\{1, 2, 3, 4, \ldots\}$
Whole Numbers	$\{0, 1, 2, 3, \ldots\}$
Integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
Rational Numbers	{all numbers of the form p/q}* or {all terminating or repeating decimals}
Irrational Numbers	{all nonrepeating, nonterminating decimals}

The Real Number Line

The model we use to represent the real number system is called the **real number line.** It consists of a horizontal line with an arbitrary point (the

*Rational numbers can be expressed as the ratio of two integers; that is, they can be written in the form p/q, where p and q are integers with $q \neq 0$.

origin) labeled 0. Positive units are measured to the right from the origin and negative units are measured to the left, as shown in Figure 1.1.

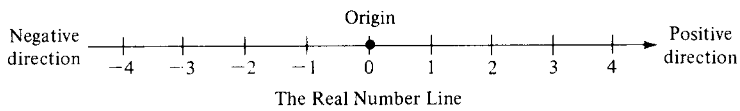


FIGURE 1.1

The importance of the real number line lies in the fact that *each point on the line corresponds to one and only one real number* and *each real number corresponds to one and only one point on the real line*. This type of relationship is called a **one-to-one correspondence.** (See Figure 1.2.)

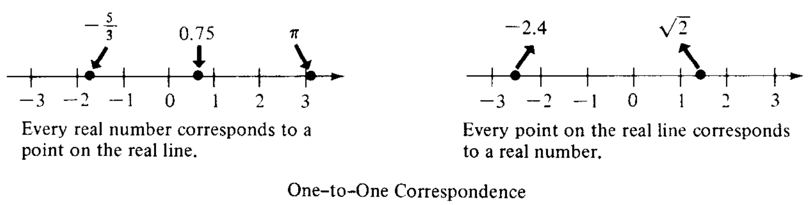


FIGURE 1.2

The number associated with a point on the real line is called the **coordinate** of the point. For example, in Figure 1.2, $-\frac{5}{3}$ is the coordinate of the left-most point, and $\sqrt{2}$ is the coordinate of the right-most point.

Ordering the Real Numbers

The real number line is useful in demonstrating the order property of real numbers. We say that the real number a **is less than** the real number b if a lies to the left of b on the real number line. Symbolically, we denote this relationship by

$a < b$ *(a is less than b)*

or equivalently

$b > a$ *(b is greater than a)*

Arithmetically, we have

$a < b$ if and only if $b - a > 0$ *($b - a$ is positive)*

$a > b$ if and only if $b - a < 0$ *($b - a$ is negative)*

Remark: In mathematics we use the phrase "if and only if" as a means of stating two impiications in one sentence. For instance, the statement $a < b$ if and only if $b - a > 0$ means

if $a < b$ then $b - a > 0$ *and* if $b - a > 0$ then $a < b$

The symbols $<$ and $>$ are referred to as **inequality signs.** They are sometimes combined with an equal sign as follows:

$a \le b$ *(a is less than or equal to b)*

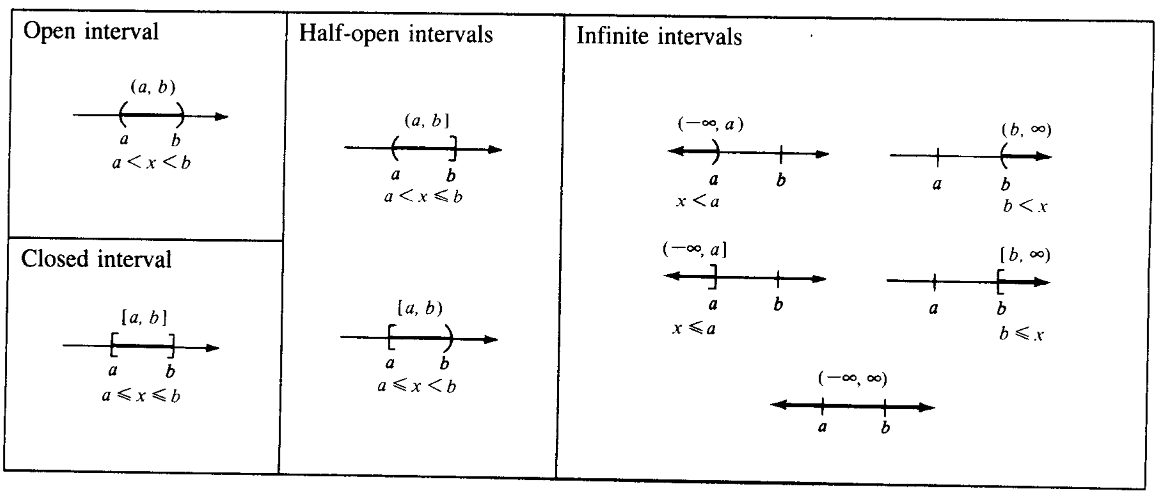
$b \ge a$ *(b is greater than or equal to a)*

Inequalities are useful in denoting subsets of the real numbers. For instance,

Inequality		**Subset of Reals**
$x \le 2$	denotes	All real numbers less than or equal to 2.
$-2 \le x < 3$	denotes	All real numbers between -2 and 3, including -2 (but not including 3).
$x > -5$	denotes	All real numbers greater than -5.

Subsets of real numbers are sometimes expressed in the **interval** forms shown in Table 1.1. (We use the symbols ∞ and $-\infty$ to denote positive and negative infinity.)

TABLE 1.1
Intervals on the Real Line



It should be clear from our discussion of order that for any two real numbers a and b *exactly one* of the following is true:

$$a = b, \quad a < b, \quad \text{or} \quad a > b$$

This is referred to as the **law of trichotomy.**

We have compared two numbers using the order relations $<$ and $>$. Two numbers can also be compared using their **absolute value.** By the absolute value of a number, we mean its *magnitude (its value disregarding* *its sign)*. We denote the absolute value of a by $|a|$. Thus, the absolute value of -5 is

$$|-5| = 5$$

DEFINITION OF ABSOLUTE VALUE

For any real number a:

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0 \end{cases}$$

The absolute value of a number can never be negative, for if a is negative ($a < 0$), then $-a$ is actually positive. For instance, let $a = -5$, then

$$|a| = |-5| = -a = -(-5) = 5$$

Absolute value is useful in finding the distance between two numbers on the real number line. To see how this is done, consider the numbers -3 and 4, shown in Figure 1.3.

The distance between −3 and 4 is 7.

FIGURE 1.3

To find the distance between these two points, we subtract *either* number from the other and then take the absolute value of the resulting difference. Thus, using absolute value, we have

$$\text{distance} = |4 - (-3)| = |4 + 3| = |7| = 7$$

or

$$\text{distance} = |-3 - 4| = |-7| = 7$$

DISTANCE BETWEEN TWO POINTS ON THE REAL LINE

For any real numbers a and b:

the distance between *a* and *b* is

$$\text{distance} = d(a, b) = |b - a| = |a - b|$$

EXAMPLE 1
Distance and Absolute Value

Use absolute value to denote each of the following.

(a) The distance between $\sqrt{7}$ and 4
(b) The distance between c and -2 is at least 7

(c) The distance between x and 2.3 is less than 1
(d) x is closer to 0 than to -4

Solution:

(a) $d(\sqrt{7}, 4) = |4 - \sqrt{7}| = 4 - \sqrt{7} \approx 4 - 2.646 \approx 1.354$
(b) Since $d(c, -2) = |c + 2|$, we have

$$|c + 2| \geq 7$$

(c) Since $d(x, 2.3) = |x - 2.3|$, we have

$$|x - 2.3| < 1$$

(d) Since $d(x, 0) = |x - 0| = |x|$ and $d(x, -4) = |x - (-4)| = |x + 4|$, we have

$$|x| < |x + 4|$$

Recall that *expressions* are the main building blocks of algebra. Some examples are

$$5t - 7, \qquad \frac{x}{x + 2}, \qquad \sqrt{x - 1}, \qquad \text{and} \qquad s^2 - 3s$$

where s, t, and x are *variables*. Unless specified otherwise, variables in this text will represent real numbers. On occasion it is necessary to restrict the numbers represented by a variable. For example, in the expression $x/(x + 2)$ we cannot use $x = -2$ since this would require division by zero, which we know is undefined. For the expression $\sqrt{x - 1}$, we restrict the numbers to $x \geq 1$ to avoid square roots of negative numbers, which yield nonreal results. The set of all *permissible* (or usable) values for a variable is called the **domain** of the variable.

Equations

An **equation** is a statement that two expressions are equal. Some examples of equations in one variable x are

$$3x - 5 = 7, \qquad x^2 - x - 6 = 0, \qquad x^2 - 9 = (x + 3)(x - 3)$$

$$\sqrt{2x} = 4, \qquad \frac{x}{x + 2} = \frac{2}{3}$$

To **solve** an equation means to find all values of the unknown (the variable) for which the equation is a true statement. Those values for which an equation is true are called **solutions** or **roots** of the equation. For instance, $x = 4$ is a solution of the equation $3x - 5 = 7$, because $3(4) - 5 = 7$ is a true statement. Similarly, $x = 8$ is a solution to $\sqrt{2x} = 4$, because $\sqrt{2(8)} = \sqrt{16} = 4$.

Equations that are true for *every* real number for which all terms of the equation are defined are called **identities.** For instance,

$$x^2 - 9 = (x + 3)(x - 3)$$

is an identity because every real number is a solution to this equation.

Most equations have values in their domains that are not solutions. For example, $x = 1$ is not a solution to $x^2 - 4 = 0$. We call such equations **conditional equations.**

The algebraic process of *solving an equation* usually generates a chain of intermediate equations, each with the same solution(s) as the original. Such equations are called **equivalent equations.**

GENERATING EQUIVALENT EQUATIONS

A given equation is transformed into an *equivalent equation* by:

1. Adding or subtracting the same quantity from both sides.
2. Multiplying or dividing both sides by the same nonzero* quantity.

*When multiplying or dividing by a *variable*, check to see that the resulting equation has the same solutions as the given one.

EXAMPLE 2
Solving an Equation

Solve the following equation for x:

$$6(x - 1) + 4 = 3(7x + 3)$$

Solution:
We have

$$\begin{aligned} 6(x - 1) + 4 &= 3(7x + 3) \\ 6x - 6 + 4 &= 21x + 9 \\ 6x - 2 &= 21x + 9 \\ -15x &= 11 \\ x &= -\tfrac{11}{15} \end{aligned}$$

These five equations are all equivalent, and we say they form the *steps* of the solution.

The solution method in Example 2 works well for equations that reduce to the *linear* form $ax = b$ (or $ax - b = 0$). *Quadratic* (second degree) equations in the standard form $ax^2 + bx + c = 0$ ($a \neq 0$) can be solved either by factoring or by the *quadratic formula*.

THE QUADRATIC FORMULA

The solutions of a quadratic equation in standard form

$$ax^2 + bx + c = 0, \qquad a \neq 0$$

are obtained from the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$