教育部"高等理工教育教学改革与实践"项目成果
国家工科物理教学基地双语教学教材

A Guide to the Study of University Physics

大学物理学学习指导

唐英 吴烨 主编

英文版

北京邮电大学出版社
www.buptpress.com

教育部“高等理工教育教学改革与实践”项目成果
国家工科物理教学基地双语教学教材

A Guide to the Study of University Physics

大学物理学学习指导

主编　唐英　吴烨

北京邮电大学出版社
www.buptpress.com

内 容 简 介

本书是作者根据多年来开展双语教学所积累的经验和我国工科大学物理双语教学实践需要编著的，目的是为学生提供一本便于使用的大学物理学英文版学习指导书，以取得良好的双语教学效果。全书共30章，其内容编排顺序与北京邮电大学出版社2007年2月出版的《大学物理学》（英文版）上、下册完全相同，便于读者将本学习指导书与相应教材配套使用。书中对工科大学物理学各部分内容的重点和难点进行了详细解说，精选了国内外教材中的一些典型例题，收集了大量练习题，设计了多套全真模拟试题，既便于教师开展大学物理双语教学，又便于学生巩固学习效果。

本书作为高等工科院校及理科非物理专业的大学物理双语教学课程的课外参考书，有利于帮助学生更好地理解双语教学的基本内容和学习重点、难点，对从事大学物理教学的教师和物理学专业的学生也具有较大的参考价值。本书内容全面、系统，包括了大学物理教学的主要内容及教学的主要构成元素：原理、例题、习题、解答、模拟考题等，因此既可以作为学习指导书与原教材配套使用，也可以作为一本大学物理双语教学的简明教程独立使用，以满足不同层次、不同学时的大学物理学双语教学的要求。

图书在版编目(CIP)数据

大学物理学学习指导；英文/唐英，吴烨主编．—北京：北京邮电大学出版社，2008

ISBN 978－7－5635－1844－9

Ⅰ．大…　Ⅱ．①唐…②吴…　Ⅲ．物理学—高等学校—教学参考资料—英文Ⅳ．O4

中国版本图书馆 CIP 数据核字(2008)第121355号

书　　名	大学物理学学习指导
主　　编	唐　英　　吴　烨
责任编辑	沙一飞　唐咸荣
出版发行	北京邮电大学出版社
社　　址	北京市海淀区西土城路10号（100876）
电话传真	010－62282185（发行部）　010－62283578（传真）
电子信箱	ctrd@buptpress.com
经　　销	各地新华书店
印　　刷	北京忠信诚胶印厂
开　　本	850mm×1 168mm　1/16
印　　张	17.75
字　　数	473千字
版　　次	2008年11月第1版　2008年11月第1次印刷

ISBN 978－7－5635－1844－9　　定价：32.00元

前　　言

为适应经济全球化和知识经济时代的要求，培养具有较强国际竞争力和国际学术交流能力的高素质创新人才，教育部启动了“高等学校教学质量及教学改革工程”和“国家级精品课程建设工程”。2005 年 11 月，作者承担了湖南省教育厅普通高等学校教学改革重点研究项目“基于工科物理双语教学平台的创新思维及创新能力培养体系研究”；2005 年 12 月，作者又承担了教育部高等理工教育教学改革与实践项目“国家工科物理教学基地大学物理双语教学课程建设”，本书即为上述项目的研究成果之一。

作者在多年开展大学物理双语教学实践的基础上，吸收国内外优秀的大学物理教材的成功之处，结合我国工科物理双语教学实践的需要，于 2007 年编写并由北京邮电大学出版社出版了大学物理学英文版教材 *University Physics*（上、下册）。该教材在多所高校大学物理的双语教学中使用，取得了良好的教学效果，得到了任课教师、学生及同行专家的充分肯定。

在教材使用过程中，很多教师和学生都提出，为了更好地开展双语教学，取得良好的教学效果，急需一本与该教材配套使用的英文版大学物理学习指导书。为此，作者按照大学物理学课程教学的三级要求“掌握、理解、了解”精心编著了本书。

本书共 30 章，其内容编排顺序与北京邮电大学出版社 2007 年 2 月出版的《大学物理学》（英文版）上、下册完全相同，便于读者将本学习指导书与相应教材配套使用。每一章包括 Core Material and Study Guide（内容提要及难点指导）；Problem-Solving Tactics（解题技巧）；Typical Samples（典型例题）；Supplementary Problems（练习题）以及 Answers to Supplementary Problems（练习题答案）等内容，对工科大学物理学各部分内容的重点和难点给予了详细解说；同时，本书还精选了国内外教材中的一些典型例题，并在例题的讲解中给出了一题多解，以培养学生的分析能力和创新思维；收集和编写了大量练习题，设计了多套全真模拟试题，并给出了每一道习题的答案及模拟试题的详细解答，既便于教师开展双语教学，也便于学生检查学习效果，使该书真正具备“指导学习”的功能。

本书注重理论联系实际，根据各章内容的特点编写了 Physics in Practice（物理学的应用），力图通过介绍现代技术中的物理学原理来激发学生的学习兴趣，启迪学生的创新思维，例如在光学部分介绍了 Using Interference to Read CDs and DVDs（光的干涉在光盘读取中的应用），Use of Diffraction Grating in CD Tracking（衍射光栅在光盘读取中的应用），Liquid Crystal Displays（液晶显示）等现代技术的光学原理。将现代高新技术与大学物理双语教学相结合，这既是对原教材教学内容的延伸，更是通过双语教学培养学生创新思维、增强学生创新能力的一种新的尝试。

本书由唐英、吴烨主编。其中第 1 至第 14 章由唐英编写，第 15 至第 30 章由吴烨编写。全书由彭小奇负责统稿和定稿。

由于作者水平有限，加之时间仓促，疏漏和不妥之处在所难免，恳请读者批评指正。

编　者

2008 年 8 月

CONTENTS

Chapter 1

Kinematics

1-1 Core Material and Study Guide

1. Position

One general way of locating a particle is with a position vector. As shown in Fig. 1-1, the location of a particle relative to the origin of a coordinate system is given by a position vector $\boldsymbol{r}$, which in unit-vector notation is

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k},$$

where $x\boldsymbol{i}$, $y\boldsymbol{j}$ and $z\boldsymbol{k}$ are the vector components of $\boldsymbol{r}$, and the coefficients x, y, and z are its scalar components.

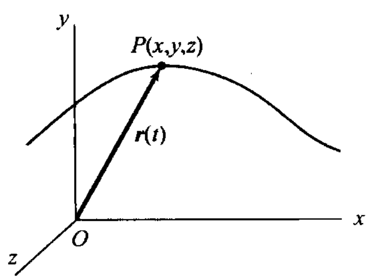

Fig. 1-1 The position vector of a particle.

The magnitude of position vector is defined as the distance between the location P of the particle and the origin, that is

$$r = |\boldsymbol{r}| = \sqrt{x^2 + y^2 + z^2}.$$

The dimensions of length is [L], with SI units of meters (m). The direction of the position vector can be represented by the direction cosines

$$\cos\alpha = \frac{x}{r}, \cos\beta = \frac{y}{r}, \cos\gamma = \frac{z}{r},$$

where α, β and γ are the angles between the position vector $\boldsymbol{r}$ and the positive x-, y- and z-axes respectively. They are called direction angles.

2. Displacement

As a particle moves, in Fig. 1-2, its position vector changes from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$, then the particle's displacement is

$$\Delta\boldsymbol{r} = \boldsymbol{r}_2 - \boldsymbol{r}_1.$$

We can rewrite this displacement as

$$\begin{aligned}\Delta\boldsymbol{r} &= (x_2\boldsymbol{i} + y_2\boldsymbol{j} + z_2\boldsymbol{k}) - (x_1\boldsymbol{i} + y_1\boldsymbol{j} + z_1\boldsymbol{k}) \\ &= \Delta x\boldsymbol{i} + \Delta y\boldsymbol{j} + \Delta z\boldsymbol{k},\end{aligned}$$

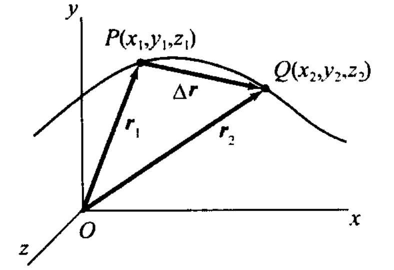

Fig. 1-2 The displacement vector of a particle.

where coordinates (x_1, y_1, z_1) correspond to position vector $\boldsymbol{r}_1$ and coordinates (x_2, y_2, z_2) correspond to the position vector $\boldsymbol{r}_2$.

We need to make a distinction between the distance an object has traveled, and its displacement, which is defined as the change in position of the object. That is, displacement is how far the object is from its starting point.

3. Average Velocity and (Instantaneous) Velocity

If a particle moves through a displacement $\Delta \boldsymbol{r}$ in a time interval Δt, then its average velocity $\boldsymbol{v}_{avg}$ is

$$\boldsymbol{v}_{avg} = \frac{\Delta \boldsymbol{r}}{\Delta t}.$$

As $\Delta t \to 0$, $\boldsymbol{v}_{avg}$ reaches a limit called the velocity or instantaneous velocity:

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{r}}{\Delta t} = \frac{d\boldsymbol{r}}{dt},$$

which can be rewritten in unit-vector notation as

$$\boldsymbol{v} = v_x \boldsymbol{i} + v_y \boldsymbol{j} + v_z \boldsymbol{k},$$

where $v_x = dx/dt$, $v_y = dy/dt$, $v_z = dz/dt$. The direction of the velocity is always directed along the tangent to the particle's path at the particle's position.

The terms velocity and speed are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with its units. Velocity, on the other hand, is used to signify both the magnitude (numerical value) of how fast an object is moving and the direction in which it is moving.

There is a second difference between speed and velocity: namely, the average velocity is defined in terms of displacement, rather than total distance traveled.

Average speed and average velocity have the same magnitude when the motion is all in one direction. In other cases, they may differ.

4. Average Acceleration and (Instantaneous) Acceleration

If a particle's velocity changes from $\boldsymbol{v}_1$ to $\boldsymbol{v}_2$ in time interval Δt, its average acceleration during Δt is

$$\boldsymbol{a}_{avg} = \frac{\boldsymbol{v}_2 - \boldsymbol{v}_1}{\Delta t} = \frac{\Delta \boldsymbol{v}}{\Delta t}.$$

The instantaneous acceleration (or simply acceleration) is the limit of $\Delta \boldsymbol{v} / \Delta t$ as time interval Δt goes to zero.

$$\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{d\boldsymbol{v}}{dt}.$$

In unit-vector notation,

$$\boldsymbol{a} = a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k},$$

where $a_x = dv_x/dt$, $a_y = dv_y/dt$, $a_z = dv_z/dt$.

Note carefully that acceleration tells us how fast the velocity changes, whereas velocity tells how fast the position changes.

5. Projectile Motion

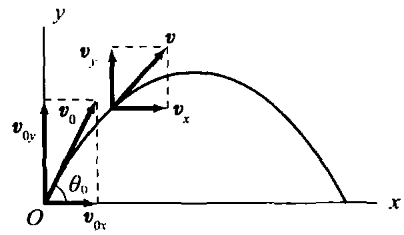

Fig. 1-3 Projectile motion.

A particle moves in a vertical plane with some initial velocity $\boldsymbol{v}_0$ but its acceleration is always the free-fall acceleration $\boldsymbol{g}$ (in the absence of air resistance), which is downward. The projectile is launched with an initial velocity $\boldsymbol{v}_0$ and an angle θ_0, the particle's equation of motion and velocity are

$$x = (v_0\cos\theta_0)t,$$

$$y = (v_0\sin\theta_0)t - \frac{1}{2}gt^2,$$

$$v_x = v_0\cos\theta_0,$$

$$v_y = v_0\sin\theta_0 - gt.$$

The equation of the path of a particle in projectile motion is given by

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2.$$

We can see that projectile motion is parabolic.

The horizontal range of the projectile is the horizontal distance the projectile has traveled when it returns to its initial (launch) height.

$$R = \frac{v_0^2\sin2\theta_0}{g}.$$

We can see that the maximum range, for a given initial velocity $\boldsymbol{v}_0$ is obtained when the sine takes on its maximum value of 1.0, which occurs for $2\theta_0 = 90°$.

6. Circular Motion

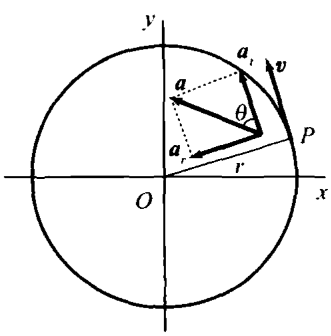

Fig. 1-4 The acceleration for a circular motion.

In a circular motion, we use tangential acceleration to describe the change in magnitude of the velocity of the moving particle,

$$a_t = \lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = \frac{dv}{dt}.$$

That is the rate of change of the speed.

And we introduce centripetal acceleration (radial acceleration) to describe the change in direction of the velocity of the moving particle,

$$a_r = \frac{v^2}{r}$$

or

$$a_r = r\omega^2.$$

The magnitude of the total acceleration is given by

$$a = \sqrt{a_r^2 + a_t^2}.$$

As shown in Fig. 1-4, the direction of the total acceleration is expressed by $\tan\theta = a_r/a_t$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{v}$.

7. Uniform Circular Motion

In uniform circular motion, the speed v and the angular speed ω are constant, and the period is given by

$$T = \frac{2\pi}{\omega}.$$

Because tangential acceleration $a_t = 0$, total acceleration $a = a_r = \frac{v^2}{r}$.

An object moving in a circle of radius r with constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is v^2/r. It is not surprising that this acceleration depends on v and r. For the greater the speed v, the faster the velocity changes direction; and the larger the radius, the less rapidly the velocity changes direction.

The acceleration vector points toward the center of the circle. But the velocity vector always points in the direction of motion, which is tangent to the circle. Thus the velocity and acceleration vectors are perpendicular to each other at every point in the path for uniform circular motion.

8. Relative Motion

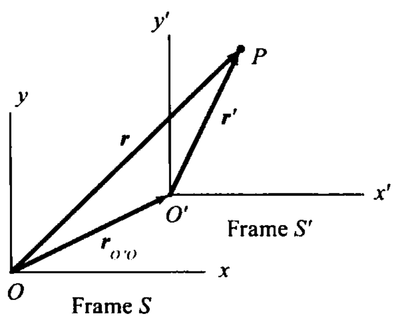

Fig. 1-5 Relative motion.

When two frames of reference S and S' are moving relative to each other at constant velocity $\boldsymbol{u}$, as in Fig. 1-5, the velocities of a particle P as measured from the two frames have relation

$$\boldsymbol{v} = \boldsymbol{v}' + \boldsymbol{u},$$

in which $\boldsymbol{v}$, and $\boldsymbol{v}'$ are the velocities of particle relative to S and S' respectively.

Both observers measure the same acceleration:

$$\boldsymbol{a} = \boldsymbol{a}'.$$

It is often useful to remember that for any two objects or reference frames, A and B, the velocity of A relative to B has the same magnitude, but opposite direction, as the velocity of B relative to A:

$$\boldsymbol{v}_{BA} = -\boldsymbol{v}_{AB}.$$

1-2 Problem-Solving Tactics

1. Read and reread the whole problem carefully before trying to solve it.

2. Draw a diagram or picture of this situation, with coordinate axes wherever applicable. (You can choose to place the origin of coordinates and the axes wherever you like, so as to make your calculations easier.)

3. Write down what quantities are "known" or "given", and then what you want to know.

4. Consider which equations (and/or definitions) relate the quantities involved. Before using equations, be sure their range of validity includes your problem. Solve the equations for the desired unknown.

5. A very important aspect of doing problems is keeping track of units. An equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has been made. This can serve as a check on your solution.

1-3 Typical Samples

Sample 1-1 The position of a particle is given in a certain coordinate system by the vector.

$$\boldsymbol{r}(t) = 4\cos(\pi t/T)\boldsymbol{i} - 4\sin(\pi t/T)\boldsymbol{j}.$$

(a) Find the displacement vector at time $t = T/3$, $t = T/2$, and $t = 2T$. In each case compute the distance to the origin.

SOLUTION: By substituting these values of time, we can directly calculate the displacements corresponding to each value of time respectively,

$$\boldsymbol{r} = 2\boldsymbol{i} - 2\sqrt{3}\boldsymbol{j}, \text{ for } t = \frac{T}{3}. \qquad \text{(Answer)}$$

$$\boldsymbol{r} = -4\boldsymbol{j}, \text{ for } t = \frac{T}{2}. \qquad \text{(Answer)}$$

$$\boldsymbol{r} = 4\boldsymbol{i}, \text{ for } t = 2T. \qquad \text{(Answer)}$$

(b) What is the angle that the position vector makes with the $+x$-axis for arbitrary?

SOLUTION: For this two-dimentional motion, the position vector can be expressed as

$$\boldsymbol{r}(t) = x\boldsymbol{i} + y\boldsymbol{j}.$$

And the angle that the position vector makes with the $+x$-axis is defined as

$$\tan\theta = \frac{y}{x},$$

where $x = 4\cos(\pi t/T)$,

and $y = -4\sin(\pi t/T)$.

Therefore

$$\tan\theta = -\frac{4\sin(\pi t/T)}{4\cos(\pi t/T)} = -\tan\left(\frac{\pi t}{T}\right),$$

$$\theta = -\frac{\pi t}{T}. \qquad \text{(Answer)}$$

Sample 1-2 A particle moves with the velocity $\boldsymbol{v} = (4 + 2t^2)\boldsymbol{i} + (-10 - 4t)\boldsymbol{j}$.

(a) What are the particle's position and velocity after 3 s, assuming that it starts at origin?

SOLUTION: From the definition of velocity,

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt}.$$

Multiplied by dt for both sides of this equation, we obtain

$$d\boldsymbol{r} = \boldsymbol{v}\, dt.$$

Taking the definite integral for two sides of this relation corresponding to the motion's initial and final conditions,

$$\int_0^r d\boldsymbol{r} = \int_0^t \boldsymbol{v} dt.$$

Replacing $\boldsymbol{v}$ by $(4 + 2t^2)\boldsymbol{i} + (-10 - 4t)\boldsymbol{j}$, and by some calculations, we have

$$\boldsymbol{r} = \int_0^t [(4 + 2t^2)\boldsymbol{i} + (-10 - 4t)\boldsymbol{j}]dt$$

$$= \left(4t + \frac{2}{3}t^3\right)\boldsymbol{i} + (-10t - 2t^2)\boldsymbol{j}.$$

Substituting $t = 3$ s into the position vector

$$\boldsymbol{r} = \left(4t + \frac{2}{3}t^3\right)\boldsymbol{i} + (-10t - 2t^2)\boldsymbol{j},$$

we have

$$\boldsymbol{r} = (12 + 18)\boldsymbol{i} + (-30 - 18)\boldsymbol{j} = 30\boldsymbol{i} - 48\boldsymbol{j}. \qquad \text{(Answer)}$$

Alike, substituting $t = 3$ s into the velocity vector

$$\boldsymbol{v} = (4 + 2t^2)\boldsymbol{i} + (-10 - 4t)\boldsymbol{j},$$

obtains,

$$\boldsymbol{v} = (4 + 18)\boldsymbol{i} + (-10 - 12)\boldsymbol{j} = 22\boldsymbol{i} - 22\boldsymbol{j}. \qquad \text{(Answer)}$$

(b) What is the particle's moving direction with respect to the x-axis at time $t = 3$ s?

SOLUTION: Because the particle's moving direction depends on its velocity, and the velocity for the two-dimentional motion can be expressed as

$$\boldsymbol{v} = v_x \boldsymbol{i} + v_y \boldsymbol{j},$$

here $$v_x = 22, v_y = -22.$$

Therefore the particle's moving direction with respect to the x-axis at time $t = 3$ s is given by

$$\tan\theta = \tan\frac{v_y}{v_x} = \tan\frac{-22}{22} = -1,$$

$$\theta = -45°. \qquad \text{(Answer)}$$

Sample 1-3 Figure 1-6 shows a massless pulley mounted on a fixed axel on the bank of a lake, which has a vertical distance h above the surface of water. A boat is at rest in the lake, which is fastened by a cord that is wrapped around the rim of the pulley. Suppose that a person is pulling the cord at a constant speed of v_0. Calculate the velocity and the acceleration of the boat when the horizontal distance between the boat and bank is s.

SOLUTION: Set up a coordinate system as shown in Fig. 1-6. At $t = 0$, the boat locates at x_0, the length of the cord between the boat and the pulley is l_0; At time t, the boat locates at x, the length of the cord between the boat and the pulley is l. Then

$$l = l_0 - v_0 t.$$

The motion of equation for the boat is given by

$$x = \sqrt{l^2 - h^2} = \sqrt{(l_0 - v_0 t)^2 - h^2}.$$

The velocity of the boat at time t is

$$\boldsymbol{v} = \frac{\mathrm{d}x}{\mathrm{d}t}\boldsymbol{i} = -\frac{v_0\sqrt{x^2 + h^2}}{x}\boldsymbol{i} = -\frac{v_0}{\cos\theta}\boldsymbol{i}.$$

The acceleration of the boat at time t is

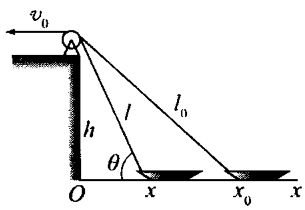

Fig. 1-6 Sample 1-3.

$$\boldsymbol{a} = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}\boldsymbol{i} = -\frac{h^2 v_0^2}{x^3}\boldsymbol{i}.$$

For $x = s$, the velocity and the acceleration of the boat are

$$\boldsymbol{v} = -\frac{v_0\sqrt{s^2 + h^2}}{s}\boldsymbol{i}, \qquad \text{(Answer)}$$

and $$\boldsymbol{a} = -\frac{h^2 v_0^2}{s^3}\boldsymbol{i}. \qquad \text{(Answer)}$$

It shows that the acceleration increases with s decreasing as the boat approaches the bank of the lake.

Sample 1-4 In Fig. 1-7, a river with width of d, which has an zero current speed at the shore and v_0 in the middle way of the river. Suppose the speed of current varies linearly with the distance between the shore and the middle way of the river. A person rows a boat at a constant speed of u perpendicularly to the current of the river toward the center of the river. Find (a) the motion of equation of the boat and (b) the path equation of the boat.

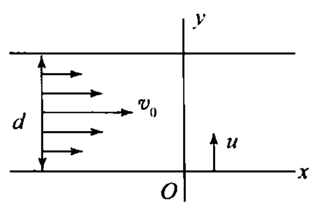

Fig. 1-7 Sample 1-4.

SOLUTION: (a) Take the bank as the reference frame and set up the coordinate system as shown in Fig. 1-7. The components of the velocity of the boat are given by

$$v_x = ky,$$
$$v_y = u,$$

where k is a positive constant.

For $y = \frac{d}{2}$, $v_x = v_0$, we can get

$$k = \frac{v_x}{y} = \frac{2v_0}{d}.$$

Therefore $v_x = \frac{2v_0}{d}y$.

From $v_y = u$, we have

$$\frac{dy}{dt} = u,$$

then $$y = \int_0^t u\,dt = ut.$$

Substituting y with ut in equation $v_x = \frac{2v_0}{d}y$, obtains

$$v_x = \frac{2v_0}{d}ut.$$

Therefore

$$x = \int_0^t \frac{2v_0 ut}{d}dt = \frac{v_0 u}{d}t^2.$$

Then the motion of equation of the boat is given by

$$\boldsymbol{r} = \frac{v_0 u}{d}t^2\boldsymbol{i} + ut\boldsymbol{j}.$$ (Answer)

SOLUTION: (b) Eliminating t in equations

$$x = \frac{v_0 u}{d}t^2 \quad \text{and} \quad y = ut,$$

we can get the path equation of the boat

$$x = \frac{v_0}{ud}y^2,$$ (Answer)

which shows that the boat moves at constant acceleration in the xy-plane; its path is a parabola in the range of $0 \leqslant y \leqslant \frac{d}{2}$.

Sample 1-5 The Moon's nearly circular orbit about the Earth has a radius of about 3.84×10^8 m and a period T of 27.3 days. Determine the acceleration of the Moon toward the Earth.

SOLUTION: In orbit around the Earth, the Moon travels a distance $2\pi r$, where $r = 3.84 \times 10^8$ m is the radius of its circular path.

The speed of the Moon in its orbit about the Earth is

$$v = \frac{2\pi r}{T}.$$

The period T in seconds is

$$T = 27.3 \times 24 \times 3600 = 2.46 \times 10^6 \text{ s}.$$

Therefore the centripetal acceleration of the Moon toward the Earth is given by

$$a_r = \frac{v^2}{r} = \frac{(2\pi r)^2}{T^2 r} = 2.72 \times 10^{-3} \text{ m/s}^2.$$

(Answer)

We can write this in terms of $g = 9.80$ m/s^2 (the acceleration of gravity at the Earth's surface) as

$$a_r = \frac{2.72 \times 10^{-3} g}{9.80} = 2.78 \times 10^{-4} g.$$

Sample 1-6 A particle is fired horizontally at speed of v_0. Calculate the particle's tangential acceleration, radial acceleration, and the radius of curvature at time t. Ignore the resistance of air.

SOLUTION: As shown in Fig. 1-8, suppose the velocity of the particle at time t is $\boldsymbol{v}$, then

$$v_x = v_0, v_y = -gt,$$
$$v^2 = v_0^2 + g^2t^2.$$

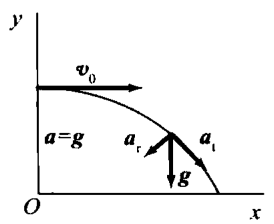

Fig. 1-8 Sample 1-6.

The speed of the particle is given by

$$v = \sqrt{v_0^2 + g^2t^2}.$$

From the definition of tangential acceleration, we have

$$a_t = \frac{dv}{dt} = \frac{g^2 t}{\sqrt{v_0^2 + g^2 t^2}}. \qquad \text{(Answer)}$$

The total acceleration is $\boldsymbol{g}$, and $\boldsymbol{a}_t$ and $\boldsymbol{a}_r$ are perpendicular to each other, then

$$g^2 = a_t^2 + a_r^2.$$

Therefore $a_r = \sqrt{g^2 - a_t^2} = \frac{g v_0}{\sqrt{v_0^2 + g^2 t^2}}$. (Answer)

Suppose the radius of curvature is ρ, then from the relation $a_r = \frac{v^2}{\rho}$, we have

$$\rho = \frac{v^2}{a_r} = \frac{(v_0^2 + g^2 t^2)^{3/2}}{g v_0}. \qquad \text{(Answer)}$$

Supplementary Problems

1. If a body moves at a constant speed v along a closed path, its average speed in comparison to v is (a) greater; (b) less; (c) the same; (d) sometimes greater, sometimes less; (e) none of these.

2. On a distance – time graph, the slope at any point is (a) the distance traveled; (b) the time elapsed; (c) the average speed; (d) the instantaneous speed; (e) none of these.

3. A body moving with an acceleration having a constant magnitude must experience a change in (a) velocity; (b) speed; (c) average speed; (d) acceleration; (e) none of these.

4. The speed of a body traveling in a straight line with a constant positive acceleration increases linearly with (a) distance; (b) time; (c) displacement; (d) distance squared; (e) none of these.

5. Given the position vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$, write the expression for the unit vector $\hat{\boldsymbol{r}}$.

6. A remote-controlled vehicle climbs a winding road whereupon its path-length, measured from the starting point ($l = 0$ at $t = 0$), is given by the equation $l(t) = 10.0 + 5.0t^2$. Write an expression for the scalar value of its tangential acceleration as a function of time.

7. The equation of motion of a particle moving in three dimensions within an experimental test setup is $l(t) = 2.0t^3 - 5.0t^2 + 3.0t + 0.5$. Write an expression for the scalar value of the tangential acceleration and evaluate it numerically at $t = 3.0$ s.

8. A small bright lamp hangs from the celling at a height of h_l meters above the floor of a large hall. A man (who is h_m meters tall) walking at a constant velocity $\boldsymbol{v}$ across the hall passes directly under the lamp at $t = 0$. Write an expression for the speed at which the very end of its shadow subsequently sweeps across the floor in front of him as he walks. What happens to the speed as h_m gets smaller.

9. A football is kicked at ground level with a speed of 18.0 m/s at an angle 32.0° to the horizontal. How much later does it hit the ground?

10. A projectile is fired with an initial speed of 51.2 m/s at an angle of 44.5° above the horizontal on a long flat firing range. Determine (a) the maximum height reached by the projectile, (b) the total time in the air, (c) the total horizontal distance covered (that is, the range), and (d) the velocity of the projectile 1.50 s after firing.

11. Suppose the space shuttle is in orbit 400 km from the Earth's surface, and circles the Earth about once every 90 minutes. Find the centripetal acceleration of the space shuttle in its orbit. Express your answer in terms of g, the gravitational acceleration at the Earth's surface.

12. The position of a particle moving in the xy plane is given by $\boldsymbol{r} = 2.0\cos 3.0t\boldsymbol{i} + 2.0\sin 3.0t\boldsymbol{j}$, where $\boldsymbol{r}$ is in meters and t is in seconds. (a) Show that this represents circular motion of radius 2.0 m centered at the origin. (b) Determine the velocity and acceleration vectors as functions of time. (c) Determine the speed and magnitude of the acceleration. (d) Show that $a = v^2/r$.

(e) Show that the acceleration vector always points toward the center of the circle.

13. Two cars approach a street corner at right angles to each other. Car 1 travels at 30 km/h and car 2 at 50 km/h. What is the velocity of car 1 as seen by car 2? What is the velocity of car 2 relative to car 1?

14. An ocean current has a northward velocity given by (4.80 km/h)$\boldsymbol{j}$. A boat capable of traveling at a top speed with respect to the water of 18.0 km/h must cross the current always traveling due east (i. e., in the direction of $\boldsymbol{i}$). What must be the boat's maximum velocity with respect to the water? At what speed is the boat approaching the shore?