GENERAL TOPOLOGY AND MODERN ANALYSIS

edited by

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ACADEMIC PRESS
A Subsidiary of Harcourt Brace Jovanovich, Publishers
New York London Toronto Sydney San Francisco 1981

Academic Press Rapid Manuscript Reproduction

Proceedings of a Conference on General Topology and Modern Analysis Held at the University of California, Riverside, May 28-31, 1980, in honor of F. Burton Jones.

ACADEMIC PRESS, INC.
111 Fifth Avenue, New York, New York 10003

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data
Main entry under title:

General topology and modern analysis.

"Proceedings of the Conference on General Topology and Modern Analysis held in May 1980 at the University of California, Riverside, in honor of the retirement of Professor F. Burton Jones"--Pref.
Bibliography: p.
1. Topology--Congresses. 2.Mathematical analysis--Congresses. 3. Jones, F. Burton. I. McAuley, L. F. (Louis F.), Date. II. Rao, M. M. (Malempati Madhusudana), Date. III. Jones, F. Burton. IV. Conference on General Topology and Modern Analysis (1980: University of California, Riverside)
QA611.A1G454 514 81-2249
ISBN 0-12-481820-X

PRINTED IN THE UNITED STATES OF AMERICA

81 82 83 84 9 8 7 6 5 4 3 2 1

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PREFACE

This volume contains the proceedings of the Conference on General Topology and Modern Analysis held in May 1980 at the University of California, Riverside, in honor of the retirement of Professor F. Burton Jones. The variety of topics covered included set theory as well as some applications, and reflected Professor Jones' wide-ranging interest in mathematics.

Among Professor Jones' many contributions to topology, perhaps his idea of aposyndesis and his creation of its theory have generated the most research activity. So, we have made a special effort to present the current status of work in this area and have devoted one of the three major sections of the book to this topic. Each of these sections starts with a survey article, followed by other articles in alphabetical order by author. Professor E. E. Grace, who has provided able assistance for the special section, has prepared a nearly exhaustive annotated bibliography of aposyndesis that is included at the end of the volume. We warmly appreciate his work. For delivering the inaugural address of the conference, Professor R. H. Bing, a fellow creator of ''circles of pseudoarcs'' along with Professor Jones, also deserves applause.

The idea of honoring Burton Jones with such a broadly based conference originated with his colleagues, former students, and many friends. The organizing committee consisted of J. de Pillis, L. F. McAuley, M. M. Rao, P. Roy, and A. R. Stralka, with Professors McAuley and Rao as cochairmen and later as coeditors of the Proceedings. The responses to our invitations were quite enthusiastic; the sessions were well attended with a good geographic representation. Many people even paid their own way to attend the conference.

The seventieth birthday of Professor F. Burton Jones was celebrated on November 22, 1980. As a token of respect and affection for his mathematical and other contributions, as well as his humane qualities, we present this volume to him, and wish him many years of fruitful mathematical activity.

ACKNOWLEDGMENTS

The necessary financial assistance for the conference was graciously provided by Dr. W. Mack Dugger, Dean of the College of Natural and Agricultural Sciences, and Dr. Michael D. Reagan, Vice Chancellor, both of the University of California, Riverside. We are grateful to them for this support.

All members of the organizing committee worked in various ways to make the conference a success. Much of the local work became easier due to the interests of some of the faculty and staff at the University of California, Riverside. Special thanks go to John de Pillis, acting chairman of the Mathematics Department, and Florence Kelly, the administrative assistant of that department, who acted as the conference secretary. Her efficient and enthusiastic work made the meetings a pleasure to attend, and consequently, the subsequent work for the Proceedings became lighter.

We extend thanks and appreciation to the session chairpersons:

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All papers were retyped with care and diligence by Joyce Kepler and Patricia Baxter. In addition to the proofreading done by the authors, Dave Holmes proofread all the papers. Also, several graduate students helped with the transportation of guests arriving at and departing from Riverside. We are grateful to all these people for their enthusiastic assistance.

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SECTION I

INAUGURAL LECTURE

METRIZATION PROBLEMS*

R. H. Bing

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I. USING EXAMPLES

Burton Jones is the first mathematician with whom I collaborated after receiving my Ph.D. Although R. D. Anderson, E. E. Moise, C. E. Burgess, Mary Ellen Estill (Rudin), Eldon Dyer, Billy Jo Ball were at Texas at this time, they were students and R. L. Moore insisted that his students develop independent work habits. Hence, after receiving my Ph.D., I did not discuss research with these students. However, Jones had received the Ph.D. several years earlier and was just returning from Cambridge where he had done work on underwater sound related to war work. I did not feel restrained in discussing mathematical research with him.

An unsolved problem of considerable interest to me was the question Jones had asked in 1937 [J_1]---is a normal Moore space metrizable? To understand how we worked at unsolved problems, it is necessary to know our modus operandi. Our first approach in attacking a problem was to look for a counterexample. If no one of our vast store of examples worked, we would try modifying known examples to discover a counterexample. It was my gut-reaction (and still is) that there is a real counterexample to the normal Moore space conjecture but it may be more complicated than anything we have examined. I soon learned that Jones had examples in his repetoire that were missing from mine.

*Work on this paper was supported by NSF Grant MCS-790-4709.

ISBN 0-12-481820-X

II. JONES' TIN-CAN-SPACE T

One of these amazing examples is Jones' tin-can-space, which I call T . Jones found the example in the 1930's, showed it to me in the 1940's and wrote up a version of it in the 1960's [J_2]. Points of T are tin cans placed on horizontal shelves. There were ω_1 of these horizontal shelves placed above each other in a best well ordered fashion (each shelf had at most a countable number of other shelves below it). There were only a countable number of tin cans on each level so T had $\aleph_0 \times \aleph_1 = \aleph_1$ points (tin cans).

The bottom level (shelf) L_0 had only one tin can and the can was assigned a size of 0 . The next level L_1 had a countable number of disjoint tin cans, and each of these was above the can on the bottom row. These cans were assigned sizes $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$. There were $\aleph_0$ tin cans on the next level L_2 above each can on L_1 and they were assigned sizes $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$, etc. Inductively cans are also placed on levels $L_2, L_3, \ldots$ and assigned sizes.

Figure 1 shows cans on levels L_0, L_1, L_2 with certain sizes indicated. It was too crowded to show all cans on higher levels, but we note that for levels $L_1, L_2, \ldots, L_\alpha, \ldots$ ($\alpha < \omega_0$), the following properties hold.

1. There are only a countable number of cans on L_i ($0 < i < \omega_0$).
2. If L_α and L_β are levels with L_α below L_β , n is a positive integer, and p is a can on L_α , then there is a stack of cans such that the stack is based on p , has its top in L_β , and the sum of the sizes of the cans in the stack above p (size of p is ignored) is $1/2^n$.

Only a countable number of cans are put on the shelf L_{ω_0} and this causes some difficulty. Stacks of cans lead up to a Cantor set of positions for cans at the L_{ω_0} level, but we ignore most of them and use only a countable number of them---being guided by Properties 1 and 2 of the

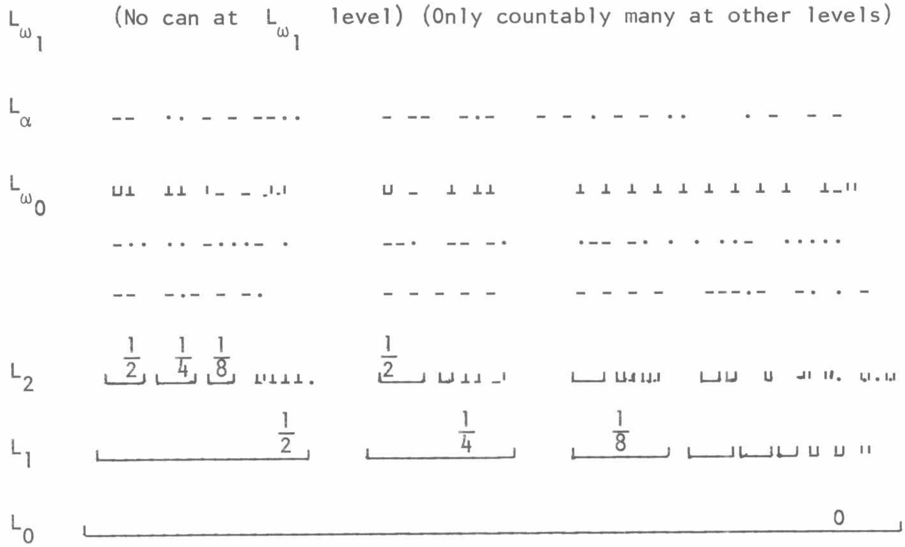

Fig. 1

previous paragraph. This is done as follows. There are only a countable number of cans in the union of all levels below L_{ω_0}. For each such can p on a lower level and each positive integer n, we pick a stack of cans based on p and reaching up to the L_{ω_0} level, so that the sum of the sizes of the cans in the stack above p is $1/2^n$. We then put a can p on the L_{ω_0} level at the top of the stack. We do this for each lower can and each n, but even so---put only a countable number of cans on the L_{ω_0} level. However, conditions 1 and 2 prevail even through level L_{ω_0}. Cans on level L_{ω_0} are assigned a size of 0.

The procedure is continued inductively to get cans on each level L_α with $\alpha < \omega_1$. Pages 116-117 of [J_2] gives details of a variation of the inductive procedure.

For a topology of T, let each can on a nonlimit level be isolated. If p is a can in some limit level, it is the top of some stack. The set of cans in such a stack is a neighborhood.

The space T is locally compact and locally separable. It contains an uncountable collection of mutually disjoint open sets and is not separable.

To show that T is a Moore space, we subdivided T into a countable number of mutually disjoint closed sets $A_{r_1}, A_{r_2}, \dots$ where r_i is a non-negative rational, and if $p \in A_{r_i}$, the sume of the sizes of cans in the maximal stack topped by p is r_i. To get a development $G_1, G_2, \dots$, we would let an element $g \in G$ be one of the neighborhoods previously described with the additional restriction that if $p \in A_{r_1} \cup A_{r_2} \cup \cdots \cup A_{r_i}$, and $p \in g \in G_i$, this p is the top can of g.

We show that T is not metrizable by showing that if it were, it would be the union of a monotone increasing sequence of countable sets. With this objective in mind, we suppose T has a bounded metric with diameter less than 1 and for each $p \in T$ let

$$\varepsilon(p) = LUB\{x \mid x \text{ neighborhood of } p \text{ is countable}\} .$$

For subsets A, B, C of T let

$$f_1(A) = \bigcup_{a \in A} N(a, \varepsilon(a)/2) ,$$

$f_2(B) = \{q$, some element of B lies at same or a higher level than $q\}$, and

$f_3(C) =$ union of $f_2(C)$ and next level.

If X is the one point set which is bottom of T, consider the monotone increasing sequence of countable sets

$$X, f_3 f_2 f_1(X), (f_3 f_2 f_1)^2(X), \dots .$$

Note that f_3 pushes us past isolated levels and $f_2 f_1$ makes short order of limit ones.

If T were normal, it would be an example showing that the normal Moore space conjecture is false. In his 1965 paper, Jones said he had not yet discovered whether T was normal or not. We discuss the normality of T