THE CONSTRUCTION OF MORTALITY AND SICKNESS TABLES

by

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Fifth Edition

The principles on which mortality investigations are based have not changed since this book was first written, but recent investigations show many points of changed emphasis; these are discussed in the revised last chapter, and the problems facing a compiler to-day are indicated. Much new tabular data is included in a variety of figures for recent mortality and sickness investigations. The book is of great value to any general student of life assurance who wishes to come to grips with the fundamentals, and it is considered essential reading for the Chartered Insurance Institute examinations. The approach throughout is simple, formulae being excluded entirely so that any reader should find little difficulty in this treatment of a fascinating and important subject.

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A PRIMER OF STATISTICS

by Sir William Elderton, k.b.e.
and Ethel M. Elderton, d.sc.

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PREFACE TO FIFTH EDITION

THIS book was originally written jointly, but later editions were amended by Sir William Elderton alone after the untimely death of R. C. Fippard at Gallipoli in June 1915. At the request of Sir William I have undertaken the fifth edition for which the major changes are those made in Chapter IX where data from new tables have been introduced.

In the preface to the first edition the authors wrote that they had attempted to describe with as littlc technicality as possible the usual methods of constructing mortality and similar tables. Since that time much work has been done leading to changed emphasis in some respects ; but the basic principles they expounded are still essential for a clear understanding of the subject.

E. B. O. S.

CONTENTS

THE CONSTRUCTION OF MORTALITY AND SICKNESS TABLES

CHAPTER I

INTRODUCTORY

IT is well known that the practical conclusions of an actuary are based on mortality, sickness, and other tables, which are prepared from the statistics given in the census returns or collected by life assurance offices and friendly societies.

The study of the methods by which these tables are constructed is therefore of great importance, and it would naturally be anticipated that the fundamental character of the subject would make it attractive; but, as a matter of fact, there is no other part of actuarial work that appears to the average student so troublesome and uninteresting. This may be because he learns about several tables of mortality which are never used, some of which never have been used, and many of which were constructed by more or less unsuitable methods, and he is left with the erroneous impression that the whole subject is a mass of bewildering detail. He may even go so far as to think that he would employ his time more profitably by evolving methods to suit special circum-

stances, than in learning the way other people have solved, or failed to solve, similar problems. And here he is right. Thinking out such solutions is the best way to understand the subject; and when it has been mastered the history is easy enough and less uninteresting.

The real difficulty for most people is that the amount of detail obscures not only the importance of the subject but even the problem that has to be solved. This is, of course, fatal; when we are trying to solve any problem we must be clear about its nature, and if, as is nearly always the case in statistical work, an approximate result is all that can be hoped for, we must try to see where the approximation falls short of accuracy.

Let us begin then by saying that the object of our investigation is to find the rate of mortality at any age from data obtained from censuses and death registers of the general population, or from the books of insurance offices, where the "rate of mortality" at a specified age may be defined as the ratio of the number of persons in a particular population dying within one year from the attainment of that age, to the number who were under observation for one year from the attainment of that age, or until death if occurring within the year. If, for example, there are 10,000 people aged 30 exactly, each of whom is kept in sight until he attains age 31 or dies (if death occurs before that age), and if it is found that there are 104 deaths among the 10,000, then the rate of mortality, or chance of dying in a year at age 30, is ·0104.

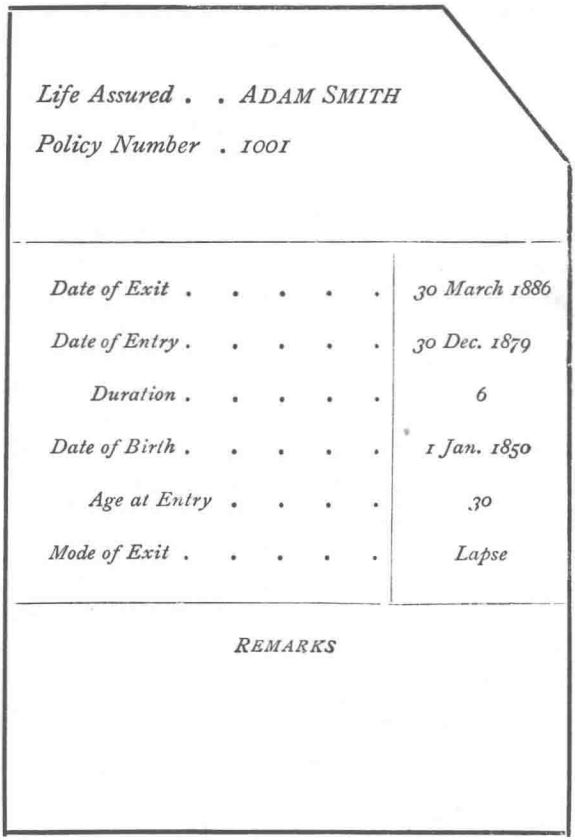
Now let us see how this definition can help us to appreciate some of the difficulties of our problem and the ways these difficulties can be met. In the first place, we want a large number of people (in our numerical illustration, 10,000) who are exactly 30 years of age. We cannot expect to trace many people who were all born on the same day, but we might trace 10,000 people all of whom attained age 30 in the same year (or some other period) and observe them from their thirtieth to their thirty-first birthdays, or we might observe for a year all those who are between $29\frac{1}{2}$ and $30\frac{1}{2}$ at a particular moment. In the second place, we must know the number of deaths among the people observed; and in the third place, our definition suggests that we must observe each case for a year. Here we are faced with the difficulty that people emigrate or, if we are dealing with particulars from the registers of an assurance office, some policies lapse or are surrendered and the lives are consequently lost sight of. If there were 10,200 people all aged 30, of whom 400 withdrew (by emigration or their policies lapsing) at the end of exactly half a year, we ought to consider the 400 people withdrawing as equivalent to 200 under observation throughout the year, because we do not know whether they died in the second half of the year or not; in other words, within our experience these 400 who withdrew only had half the chance of dying in the year that was given to the rest of the lives under observation. The consequence is that the 10,200 people in the circumstances described are equivalent to 10,000

people observed for the whole year. If the 400 left at various times during the year a similar argument will hold, and we shall see later how these points are dealt with in practice.

As we have already indicated, we can use either the data obtained from the general population, or the particulars that can be found in the registers of a life assurance company, to reach the rates of mortality at various ages; but more accurate results can be obtained from the latter, owing to the more detailed information available. We shall therefore deal with the life office methods first, and refer afterwards to the methods to be adopted in the case of the general population.

The course that is usually followed in order to obtain rates of mortality, is first to decide exactly how many years of the assurance company's experience are to be taken into account and whether any cases are to be excluded. It might, for instance, be thought well to exclude those who effected policies many years ago or those who were charged an extra premium, while for some purposes the separation of with-profit policies from without-profit policies might be deemed advisable. Males and females should be dealt with separately whenever possible. If it is decided to neglect certain cases, great care must be taken to see that their exclusion will not vitiate the results of the investigation. Thus deaths arising from some specified cause, such as cancer, could not be excluded, because each life has undergone the risk of dying from this disease, and if we exclude those persons whose deaths have actually taken place from

this cause the general rate of mortality would be under-estimated. There would, however, be no fallacy



Life Assured . . ADAM SMITH

Policy Number . 1001

Date of Exit	*30 March 1886*
Date of Entry	*30 Dec. 1879*
Duration	*6*
Date of Birth	*1 Jan. 1850*
Age at Entry	*30*
Mode of Exit	*Lapse*

REMARKS

in working only on people resident in a certain district or following a certain occupation.

Having decided on these points, the usual practice

is to fill up a card (see specimen) for every case to be included, giving the date of birth, date of entry, the date of withdrawal and the mode of withdrawal, *i.e.* death, lapse, surrender, or if the life was still living at the close of the observations.

The above specimen card is suitable, the corner is cut off for convenience in sorting, and the particulars are arranged so that the calculations can be made readily.[1]

As the easiest way to appreciate what these particulars give and how they can be used, is by taking one or two examples, Table I has been prepared so that the first four lines give the particulars furnished by the office, and the later lines the calculated ages and durations to be used in the subsequent work.

The particulars in the first four lines are easy to follow, and can be left to explain themselves while we consider the rest of the table.

It will be seen in the first place that the nearest age at entry has been calculated in each case. This is used as a convenient and accurate approximation to the exact age; it means that we group together all persons on whose lives policies have been granted between the ages $29\frac{1}{2}$ and $30\frac{1}{2}$, and assume the exact age at entry to have been 30.

The next two lines of the table show the method of calculating the time during which a policy is in

[1] If the experience extends to a very large number of cases, mechanical means of sorting would probably be used. Cards are printed with a series of columns in which holes can be punched, and these holes enable the sorting machine to group and enumerate the cards.

force in various circumstances. Taking the first example, we see that the duration from the date of entry to the day when the policy lapsed was six years and three months. The life assured comes under our

TABLE I

	EXAMPLE 1.	EXAMPLE 2.	EXAMPLE 3.	EXAMPLE 4.	EXAMPLE 5.
Date of birth .	Jan. 1, 1850	June 23, 1851	Dec. 15, 1860	March 10, 1845	Sept. 15, 1850
Date of entry .	Dec. 30, 1879	Jan. 1, 1880	Jan. 5, 1880	June 10, 1880	June 18, 1880
Date of withdrawal	March 30, 1886	Aug. 4, 1900	Dec. 10, 1895	Oct. 15, 1907	...
Mode of withdrawal	Lapse	Surrender	Death	Death	Existing on June 18, 1911
Nearest age at entry	30	29	19	35	30
Duration . .	$6\frac{3}{12}$	$20\frac{7}{12}$	$15\frac{11}{12}$	$27\frac{4}{12}$	31
Duration used in calculations	6	21	15	27	31
Age at exit used in calculations	36	50	34	62	61

observation for that time, and no longer; he might, within our experience, have died during that period, but as a matter of fact he did not do so; or in more technical language, he was exposed to risk of death for six years and three months. In the same way, the life assured in the second case was exposed to risk of death for twenty years and seven months. If we go back to our definition of the rate of mortality, we see that these cases do not fit in conveniently, because we ought to observe every case until the end of the year. The first six years in the first example fit well enough, but the life assured had a chance of dying in only three months of the seventh. We ought, strictly speaking, to count him for one quarter of that year, and the assured in the second example for twenty years and seven-twelfths of the twenty-first year. In practice it is convenient to avoid fractional durations, so we take the nearest duration in a similar way to that followed in connection with the age at entry. We assume, therefore, that Example 1 was exposed to risk for six years, and Example 2 for twenty-one years. We balance the understatement of some cases with the overstatement of others.

The next two examples relate to deaths, and have been dealt with differently from those that have just been considered. We have not taken the nearest duration but the number of complete years lived, and neglected the odd months altogether. To understand this we must go back again to our definition of the rate of mortality. In finding the rate of mortality at age 30, for instance, we want to obtain the ratio of the deaths between ages 30 and 31 (which in

practice we call the deaths at age 30) to the corresponding number living at age 30, so that if a person dies at any time between 30 and 31 we must assume he was exposed to risk for the whole of that year of age. Use of completed years for deaths seems to contradict this, but it will emerge in the next chapter that the method is convenient in practice. If we adopted any other method, such as the nearest duration, we should reach some other function than the rate of mortality as we have defined it.

The one remaining example relates to a policy which is still in existence when the observations end. The usual practice is to assume that the experience ends not on a fixed date, but on the anniversaries in a particular calendar year (*e.g.* 1911) of the dates when the policies were effected. This fits in well with our definition of the rate of mortality, because it enables us to observe these cases for an exact number of years.

Those who are studying the subject of the construction of mortality tables for the first time frequently have considerable difficulty in following the meaning of the term "those existing at the close of observations," and seem to find it hard to see why they have to be taken into account. It must be borne in mind that the experience covers a fixed period of years. Each policy is observed from the date of entry until its cessation by surrender, lapse, or death, or until its anniversary in the final year of the period. There will necessarily be several of the latter cases, some effected one year before the final year, some two years, and so on. These are termed

the "Existing at the close of Observations," and since the policy-holders might have died during the time covered by the experience, they must be included for each year of their existence up to the date when the observations end.

This may be made clearer if we consider four policies effected ten years ago. Let us assume that in one case the life assured died; in another the policy was allowed to lapse; a third was surrendered; and the remaining one is still being maintained. This last one would be termed "Existing." It must be taken into account for each of the ten years, because the life assured might have died during that period, even though he did not do so. If it was excluded altogether we should exaggerate the mortality, as can easily be seen by considering the extreme case of ten persons observed for ten years, of whom only one died and none withdrew. If we did not take the nine existing people into account for the whole of the ten years, we should assume that a person was certain to die within that period, whereas as a matter of fact only one out of ten had done so.

Having thus provided ourselves with the necessary data, we have to consider how it can best be manipulated, but before doing so it will be useful to recapitulate what has been said in the following way, which shows the procedure to be adopted:—

1. Decide the limits of the experience.
2. Write a card for each case, giving—(*a*) date of birth; (*b*) date of entry; (*c*) date of exit; (*d*) cause of exit, or if existing.