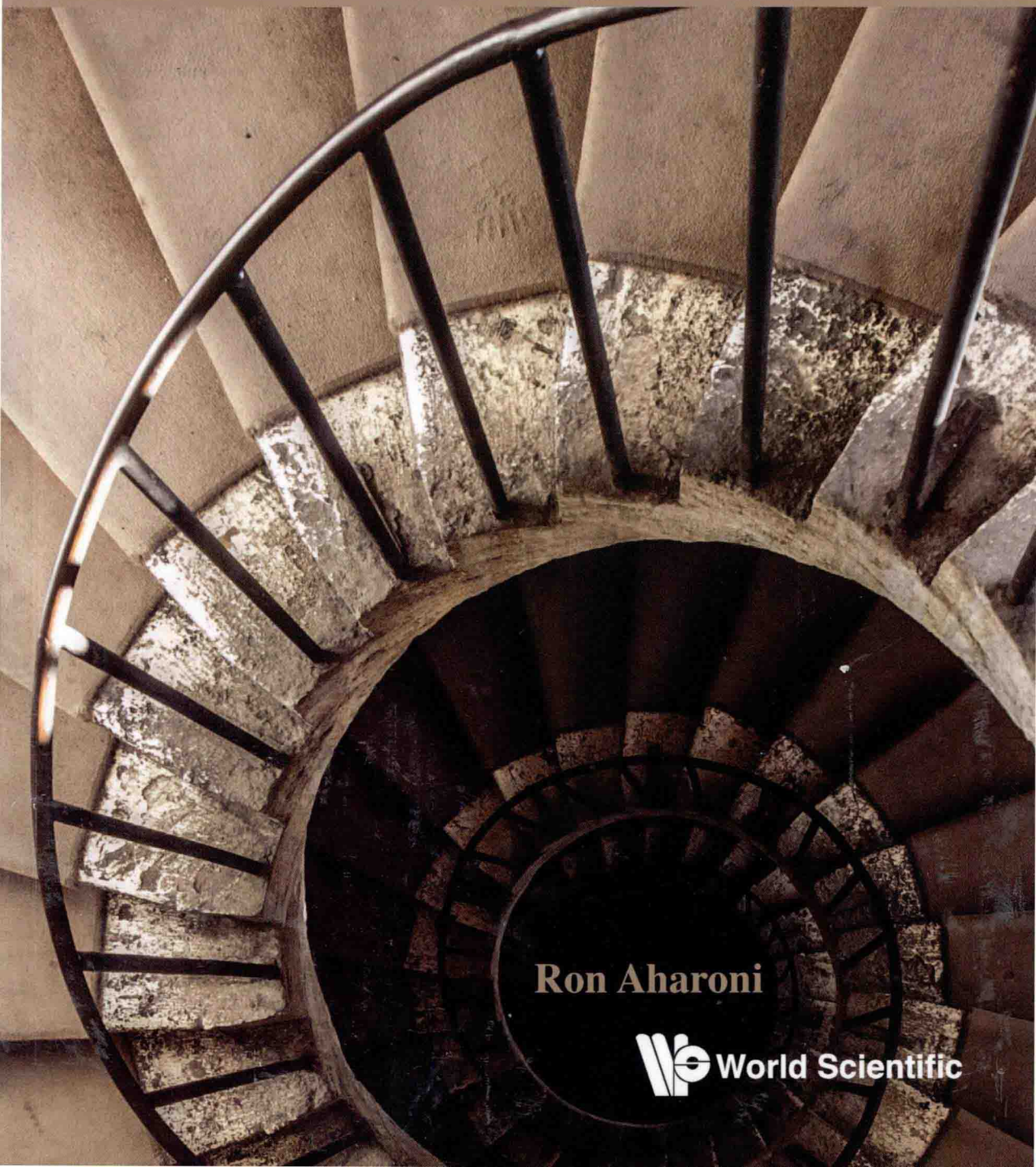
# Circularity

## The Common Secret to Paradoxes, Scientific Revolutions and Humor



Ron Aharoni

World Scientific

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## A Common Secret to Paradoxes, Scientific Revolutions and Humor

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**Ron Aharoni**
Technion, Israel

World Scientific

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# Circularity

A Common Secret to Paradoxes,
Scientific Revolutions and Humor

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# Cats and Tails

*Before I start speaking, let me say a few words.*

(An opening to a speech)

My little daughter returned one day from a visit to the dentist, and asked me: do you know how to make the anesthetic shot painless? You give an anesthetic shot before. Of course, to make this one painless, another shot is needed.

Amusing? Indeed. But strangely, this mode of thought also has serious sides. It is called "circularity", or "self-reference", and it is the protagonist of our story. The anesthetic shot is an example of a circularly defined task, namely one whose performance depends on its having been carried out before. Another example is the advice given to somebody who has fallen into a deep pit: go, fetch a ladder, and climb out. Such a situation is called, for good reason, a "vicious circle". The viciousness is experienced, for example, by a visitor to the United States who wishes to obtain a credit card. He has first to show a history of paying debts, of which the almost only instance is having paid the bills of a credit card.

A cat chasing its tail, someone who needs his glasses to find his lost glasses, a fresh job applicant rejected because of lack of work experience — all these are attempting to perform circular tasks. Most will fail. Lifting yourself from a pond, together with your horse, by pulling at your hair, is a feat that only Baron Munchausen knows how to perform. Archimedes formulated this idea in "Give me a lever and a place to stand and I will move the earth." He was not merely

boasting of the wonderful pulleys he invented, but also wanted to explain that there is no leverage point from which you can lift yourself.

Circular tasks of another type are those that get in their own way. A photographer's command "Everybody, smile!" usually has the opposite effect. Polls published before elections can overturn the results and invalidate their own predictions. In Joseph Heller's *Catch 22* the pilots may evade bombing missions if they can prove insanity, but Item 22 in the army code says that anybody trying to evade bombing missions is sane. "Don't listen to this advice" is difficult advice to follow.

Circularity can be a pest. This happens when it outwits us, and passes circularly defined concepts as valid. For example, if you assume that the expression "The number hereby defined plus 1" points at a real number, you get an absurdity — a number equal to itself plus 1. "The father of Napoleon Bonaparte" points at a specific person; "The father of the person defined in this sentence" does not, and if you assume it does, you obtain the impossible situation of a person being his own father.

In these instances the circularity is transparent, and nobody will fall for it. But sometimes it succeeds in disguising well enough to fool us, and then absurdities arise. This is the dark side of circularity, its subversive and unruly face, to which the first part of the book is devoted. We shall meet in it a baby-snatching crocodile and agreements that cannot be fulfilled; proofs for the existence of God (or of the Loch Ness monster), and two famous philosophical paradoxes originating in circularity — the Determinism–Free Will conundrum and the Mind–Body problem.

Another, illuminated side, of circularity emerges when it is overt instead of acting from behind the scenes. It sometimes happens that nature itself poses circularly defined tasks, and in this case it is important to realize this. When a task is impossible, the impossibility had better be recognized. The second part of the book is devoted to this facet of circularity. We shall learn about three mathematical discoveries born from recognition of circular phenomena. One is a theorem of Georg Cantor from 1878, stating that there is no largest set in the world. For every set there is a bigger one. Then we shall meet the

greatest ever victory of circularity — Kurt Gödel's Incompleteness Theorem, that (roughly) states that not everything that is true can be formally proved. And then we shall learn about a conclusion of Alan Turing's from 1936, that there is no omniscient machine. In modern day terminology — there is no computer program that can solve all problems. If such a program existed, it could pose a problem that it itself could not solve — just as an almighty God could create a rock too heavy for Him to lift.

The two sides of circularity are not separate. The rebellious side and the useful side are intertwined and influence each other. Paradoxes are sometimes inspiration for mathematical proofs (as was the case in the proof of Gödel's theorem) and mathematical proofs are sometimes parodied by paradoxes. The dark side is a negative image of the illuminated one, and negative images can teach something about the original. Moreover, they have their own charm.

For those who are ready to go deeper, I added a few chapters in *Part VII For the Experienced Hikers.* In those chapters some technical points will be elucidated, and some more complex mathematical insights will be presented.

Finally, no justice can be done to circularity without mentioning its charming side — its capacity to amuse. It is an unending source of jokes, and a sure recipe for humorous effect.

> I always thought I was indecisive. But now I am not so sure.
>
> The pig goes to God and complains: "Everything bad is ascribed to me: gluttony, filthiness, laziness. Why me?" God scratches His head, and says: "Indeed, piggishness."
>
> How does the dictionary define "circularity"? — see definition for "circularity".

In the last part of the book I will tell about circularity in humor, and try to solve the riddle of what makes circularity funny.

# Acknowledgments

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# Contents

*Cats and Tails* v
*Acknowledgments* ix

**The Dark Side — Paradoxes** 1

**Part I: Magic** 3

An Elusive Crook 5
A Nation Enamored with Paradoxes 8
Zeno and Tortoises 10
Are there Sand Heaps in the World? 13
Crocodiles and Lawyers 15
Epimenides and the Mishtakes 17
A Useful Paradox 20
Smullyan's Island 22
Proving the Existence of the Loch Ness Monster 24

**Part II: Free Will** 27

Newcomb Overturns the Rules of Nature 29
Written in the Stars 32
Nothing to It — Or Perhaps There is Something? 35
The Idle Argument 37
Why Can We Not Change the Past? 40
And Yet It Moves? 43
Cat and Tail: Choosing the Choice 45
*The Matrix* 47

All is Foreseen (From the Outside) and Permission is Granted (From the Inside) 49

**Part III: The Mind–Body Problem** **51**

A Meeting of Non-Meeters 53
The Philosophical Discontent 55
A Therapist of Thought 57
Ryle Visits a University 60
From the Inside or From the Outside? 62
Direct Knowledge 64
Socrates and Lewis Carroll on Direct Knowledge 66
How do People Learn about Their Mental Events? 68
The Cat Sits, Therefore the Cat Is 70
The Red King's Dream 72

**The Illuminated Side — Scientific Breakthroughs** **75**

**Part IV: Large Infinities and Still Larger Ones** **77**

You Shall See the Land from Afar 79
What is "Counting" 82
A Surprise 84
Inequality Between Sets 87
Cantor Plays with Mr. Potato Head 90
The Paradoxes of Set Theory 94

**Part V: Gödel's Incompleteness Theorem** **97**

A Revolution in a Small Town 99
Copernican Revolutions 101
The Cement of Mathematical Thought 104
Meanwhile, Across the Channel ... 107
A Very Ambitious and Very Wrong Program 109
Gödel's Paradox 112
From Paradox to Proof 114
Surprise Tests 117

The Theorem of the Century? 119
Why it was Imperative to Mummify Lenin 122

**Part VI: Turing Invents the Computer 125**

From Olympus Down to Earth 127
The Halting Problem 129
The Imitation Game 131
Searle's Chinese Room 134
Personality and Genius 136
Can a Machine Know that it is a Machine? 137
Three Families, One Secret 139
The Jail of Personality 147

**Part VII: For the Experienced Hikers 151**

The Diagonal Method 153
Conclusion: The Real Numbers are Not Countable 155
A Methodical but Not Very Efficient Detective 156
Why Gödel's Paradox Doesn't Generate a Mathematical Contradiction 158
Why the Incompleteness Theorem Doesn't Contradict the Completeness Theorem 160

Bibliography 163

# The Dark Side — Paradoxes

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