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A First Course in Probability (6th Ed)

概率基础

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出版说明

21世纪的竞争是人才的竞争，是全球性人才培养机制的较量。如何培养面向现代化、面向世界、面向未来的高素质的人才成为我国人才培养的当务之急。为此，教育部发出通知，倡导在全国普通高等学校中使用原版外国教材，进行双语教学，培养适应经济全球化的人才。

为了响应教育部的号召，促进统计教材的改革，培养既懂统计专业知识又具备较高英语语言能力的统计人才，全国统计教材编审委员会在国家统计局领导的大力支持下，组织引进了这套“外国优秀统计学教材”。

为了做好“外国优秀统计学教材”引进工作，全国统计教材编审委员会将其列入了“十五”规划，并成立了由海内外统计学家组成的专家委员会。在对国外统计学教材的使用情况进行了充分了解，对国内高等院校使用外国统计学教材的需求情况进行了仔细分析，并对从各种渠道推荐来的统计教材进行了认真审定的基础上，制定了引进教材书目。在确定引进教材书目的过程中，我们得到了国内外有关专家、有关院校和外国出版公司及其北京办事处的支持和帮助，在此致谢。中国人民大学统计学系的吴喜之教授不仅推荐了大量的优秀候选书目，而且校译了影印教材的翻译目录，为这套教材的及早出版作了大量的工作，我们表示衷心的谢意。

这套引进教材多数是国外再版多次、反响良好，又比较适合国内情况、易于教学的统计教材。我们希望这套引进教材的出版对促进我国统计教材的改革和高校统计学专业双语教学的发展能够起到重要的推动作用。

全国统计教材编审委员会

2002 年 8 月 28 日

Preface

"We see that the theory of probability is at bottom only common sense reduced to calculation; it makes us appreciate with exactitude what reasonable minds feel by a sort of instinct, often without being able to account for it.... It is remarkable that this science, which originated in the consideration of games of chance, should have become the most important object of human knowledge.... The most important questions of life are, for the most part, really only problems of probability." So said the famous French mathematician and astronomer (the "Newton of France") Pierre Simon, Marquis de Laplace. Although many people might feel that the famous marquis, who was also one of the great contributors to the development of probability, might have exaggerated somewhat, it is nevertheless true that probability theory has become a tool of fundamental importance to nearly all scientists, engineers, medical practitioners, jurists, and industrialists. In fact, the enlightened individual had learned to ask not "Is it so?" but rather "What is the probability that it is so?"

This book is intended as an elementary introduction to the theory of probability for students in mathematics, statistics, engineering, and the sciences (including computer science, the social sciences and management science) who possess the prerequisite knowledge of elementary calculus. It attempts to present not only the mathematics of probability theory, but also, through numerous examples, the many diverse possible applications of this subject.

In Chapter 1 we present the basic principles of combinatorial analysis, which are most useful in computing probabilities.

In Chapter 2 we consider the axioms of probability theory and show how they can be applied to compute various probabilities of interest.

Chapter 3 deals with the extremely important subjects of conditional probability and independence of events. By a series of examples we illustrate how conditional probabilities come into play not only when some partial information is available, but also as a tool to enable us to compute probabilities more easily, even when no partial information is present. This extremely important technique of obtaining probabilities by "conditioning" reappears in Chapter 7, where we use it to obtain expectations.

In Chapters 4, 5, and 6 we introduce the concept of random variables. Discrete random variables are dealt with in Chapter 4, continuous random variables in Chapter 5, and jointly distributed random variables in Chapter 6. The important concepts of the expected value and the variance of a random variable are introduced in Chapters 4 and 5: These quantities are then determined for many of the common types of random variables.

Additional properties of the expected value are considered in Chapter 7. Many examples illustrating the usefulness of the result that the expected value of a sum of random variables is equal to the sum of their expected values are presented. Sections on conditional expectation, including its use in prediction, and moment generating functions are contained in this chapter. In addition, the final section introduces the multivariate normal distribution and presents a simple proof concerning the joint distribution of the sample mean and sample variance of a sample from a normal distribution.

In Chapter 8 we present the major theoretical results of probability theory. In particular, we prove the strong law of large numbers and the central limit theorem. Our proof of the strong law is a relatively simple one which assumes that the random variables have a finite fourth moment, and our proof of the central limit theorem assumes Levy's continuity theorem. Also in this chapter we present such probability inequalities as Markov's inequality, Chebyshev's inequality, and Chernoff bounds. The final section of Chapter 8 gives a bound on the error involved when a probability concerning a sum of independent Bernoulli random variables is approximated by the corresponding probability for a Poisson random variable having the same expected value.

Chapter 9 presents some additional topics, such as Markov chains, the Poisson process, and an introduction to information and coding theory, and Chapter 10 considers simulation.

The sixth edition continues the evolution and fine tuning of the text. There are many new exercises and examples. Among the latter are examples on utility (Example 4c of Chapter 4), on normal approximations (Example 4i of Chapter 5), on applying the lognormal distribution to finance (Example 3d of Chapter 6), and on coupon collecting with general collection probabilities (Example 2v of Chapter 7). There are also new optional subsections in Chapter 7 dealing with the probabilistic method (Subsection 7.2.1), and with the maximum-minimums identity (Subsection 7.2.2).

As in the previous edition, three sets of exercises are given at the end of each chapter. They are designated as **Problems, Theoretical Exercises**, and **Self-Test Problems and Exercises**. This last set of exercises, for which complete solutions appear in Appendix B, is designed to help students test their comprehension and study for exams.

All materials included on the Probability Models diskette from previous editions can now be downloaded from the Ross companion website at http://www.prenhall.com/Ross. Using the website students will be able to perform calculations and simulations quickly and easily in six key areas:

- Three of the modules derive probabilities for, respectively, binomial, Poisson, and normal random variables.
- Another module illustrates the central limit theorem. It considers random variables that take on one of the values $0, 1, 2, 3, 4$ and allows the user to enter the probabilities for these values along with a number n. The module then plots the probability mass function of the sum of n independent random variables of this type. By increasing n one can "see" the mass function converge to the shape of a normal density function.
- The other two modules illustrate the strong law of large numbers. Again the user enters probabilities for the five possible values of the random variable along with an integer n. The program then uses random numbers to simulate n random variables having the prescribed distribution. The modules graph the number of times each outcome occurs along with the average of all outcomes. The modules differ in how they graph the results of the trials.

We would like to thank the following reviewers whose helpful comments and suggestions contributed to recent editions of this book: Anastasia Ivanova, University of North Carolina; Richard Bass, University of Connecticut; Ed Wheeler, University of Tennessee; Jean Cadet, State University of New York at SUNY, Stony Brook; Jim Propp, University of Wisconsin; Mike Hardy, Massachusetts Institute of Technology; Anant Godbole, Michigan Technical University; Zakkula Govindarajulu, University of Kentucky; Richard Groeneveld. Iowa State University; Bernard Harris, University of Wisconsin; Stephen Herschkorn, Rutgers University; Robert Keener, University of Michigan; Thomas Liggett, University of California, Los Angeles; Bill McCormick, University of Georgia; and Kathryn Prewitt, Arizona State University. Special thanks go to Hossein Hamedani, Marquette University, and Ben Perles for their hard work in accuracy checking this manuscript.

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DEDICATION

For Rebecca

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CHAPTER 1

Combinatorial Analysis

1.1 INTRODUCTION

Here is a typical problem of interest involving probability. A communication system is to consist of n seemingly identical antennas that are to be lined up in a linear order. The resulting system will then be able to receive all incoming signals—and will be called *functional*—as long as no two consecutive antennas are defective. If it turns out that exactly m of the n antennas are defective, what is the probability that the resulting system will be functional? For instance, in the special case where $n = 4$ and $m = 2$ there are 6 possible system configurations—namely,

$$\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{array}$$

where 1 means that the antenna is working and 0 that it is defective. As the resulting system will be functional in the first 3 arrangements and not functional in the remaining 3, it seems reasonable to take $\frac{3}{6} = \frac{1}{2}$ as the desired probability. In the case of general n and m, we could compute the probability that the system is functional in a similar fashion. That is, we could count the number of configurations that result in the system being functional and then divide by the total number of all possible configurations.

From the preceding we see that it would be useful to have an effective method for counting the number of ways that things can occur. In fact, many problems in probability theory can be solved simply by counting the number of different ways that a certain event can occur. The mathematical theory of counting is formally known as *combinatorial analysis*.