WALLACE D. HAYES

Gasdynamic Discontinuities

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GASDYNAMIC DISCONTINUITIES

BY WALLACE D. HAYES

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PREFACE

The favorable response of many engineers and scientists throughout the world to those volumes of the Princeton Series on High Speed Aerodynamics and Jet Propulsion that have already been published has been most gratifying to those of us who have labored to accomplish its completion. As must happen in gathering together a large number of separate contributions from many authors, the general editor's task is brightened occasionally by the receipt of a particularly outstanding manuscript. The receipt of such a manuscript for inclusion in the Princeton Series was always an event which, while extremely gratifying to the editors in one respect, was nevertheless, in certain particular cases, a cause of some concern. In the case of some outstanding manuscripts, namely those which seemed to form a complete and self-sufficient entity within themselves, it seemed a shame to restrict their distribution by their inclusion in one of the large and hence expensive volumes of the Princeton Series.

In the last year or so, both Princeton University Press, as publishers of the Princeton Series, and I, as General Editor, have received many enquiries from persons engaged in research and from professors at some of our leading universities concerning the possibility of making available at paperback prices certain portions of the original series. Among those who actively campaigned for a wider distribution of certain portions of the Princeton Series, special mention should be made of Professor Irving Glassman of Princeton University, who made a number of helpful suggestions concerning those portions of the Series which might be of use to students were the material available at a lower price.

In answer to this demand for a wider distribution of certain portions of the Princeton Series, and because it was felt desirable to introduce the Series to a wider audience, the present Princeton Aeronautical Paperbacks series has been launched. This series will make available in small paperbacked volumes those portions of the larger Princeton Series which it is felt will be most useful to both students and research engineers. It should be pointed out that these paperbacks constitute but a very small part of the original series, the first seven published volumes of which have averaged more than 750 pages per volume.

For the sake of economy, these small books have been prepared by direct reproduction of the text from the original Princeton Series, and no attempt has been made to provide introductory material or to eliminate cross references to other portions of the original volumes. It is hoped that these editorial omissions will be more than offset by the utility and quality of the individual contributions themselves.

Coleman duP. Donaldson, General Editor

PUBLISHER'S NOTE: Other articles from later volumes of the clothbound series, *High Speed Aerodynamics and Jet Propulsion*, may be issued in similar paperback form upon completion of the original series in 1961.

CONTENTS

SECTION D

THE BASIC THEORY OF GASDYNAMIC DISCONTINUITIES

WALLACE D. HAYES

It is the aim of this section to present a theoretical introduction to the subject of gasdynamic discontinuities, from as general a point of view as is practicable. A gasdynamic discontinuity is a surface in a fluid field across which various properties of the fluid appear from a macroscopic point of view to change discontinuously and across which there is some flow of the fluid. A gasdynamic discontinuity is thus distinguished from a contact discontinuity across which there is no flow of fluid and across which the pressure is continuous. Hydrodynamic discontinuities occur in many fields of fluid mechanics; in the field of gas dynamics or high speed aerodynamics they appear in essentially all flows of practical importance. Their existence was first recognized in the middle of the nineteenth century, though it was not until the work of Hugoniot (1889) [*1*, p. 80] that the presently accepted formulation of the discontinuity relations was established. For a discussion of the early history of the study of discontinuities in hydrodynamics the reader is referred to the book of Courant and Friedrichs [*2*].

The most important gasdynamic discontinuity is the shock wave, in which a gas or other material undergoes a sudden increase in pressure, density, temperature, and entropy. In an ordinary shock wave the gas behind the shock obeys the same equation of state as does the gas in front of the shock. Many other types of gasdynamic discontinuity do not have this property; perhaps the most important type is the combustion wave, in which a chemical reaction takes place in the discontinuity proper so that the materials on the two sides of the discontinuity obey quite different equations of state. Another important type which may appear in aerodynamic or wind tunnel problems is the condensation shock, in which the gas in front of the discontinuity contains a vapor in a supersaturated state which partially condenses in the discontinuity proper.

Important in very strong discontinuities or shocks are phenomena which heretofore have been of interest particularly to physicists. Among these effects may be cited ionization, dissociation, and relaxation. With large amounts of radiation the energy effects of the radiation may strongly affect the discontinuity, and with sufficiently high velocities relativistic effects might have to be taken into account.

It is true, of course, that these gasdynamic discontinuities are not discontinuities in the strict sense; a shock, combustion wave, or condensation shock has a finite thickness across which the physical properties change continuously. If this thickness is small compared with some appropriate macroscopic dimension of the flow field, such as the radius of curvature of a curved shock, the physical relationships may be obtained by an analysis which treats the discontinuity as strict. The assumption that the discontinuity thickness is small compared with a macroscopic dimension is a fundamental one for this section.

The term "structure" as applied to a gasdynamic discontinuity refers to the values of the physical properties of the fluid within the small but finite thickness of the discontinuity. If thermodynamic equilibrium in a substance is disturbed, a characteristic time must elapse before equilibrium can be approximately reestablished; this time times the velocity of the fluid defines a characteristic distance which is of the order of a molecular mean free path or greater. If the physical and chemical changes occurring in the discontinuity are sufficiently slow, so that the thickness of the discontinuity is large compared with this characteristic distance, the concept of thermodynamic quasi-equilibrium may be considered to apply. In this case the Navier-Stokes equations are applicable, as are the classical chemical kinetic laws for slow reactions. If the discontinuity is thin, with the physical and chemical changes occurring rapidly, the essential absence of thermodynamic equilibrium must be taken into account. This may involve abandoning the continuum concept and taking the point of view of kinetic theory. Certain molecular processes such as diffusion and nucleation must be taken into account in investigating the structure of discontinuities involving, for example, chemical reactions or condensation. (See Sec. F, G, and H.)

It is clear that the thorough study of gasdynamic discontinuities and their structures combines in an essential way the fields of hydrodynamics, physics, and chemistry, and that there is no lack of problems which deserve attention. At the present time there is a strong need for additional concepts which properly describe phenomena taking place in discontinuities and which will permit some simplification of the complicated laws governing the attendant phenomena, so that more suitable theoretical approaches to the problems may be made.

D,1. Basic Relations in a Normal Discontinuity. The basic laws governing normal discontinuities in hydrodynamics are now given and discussed. The investigation is made on the basis of a steady state process; the flow is considered to be one-dimensional and the discontinuity surface is assumed to be perpendicular to the direction of the flow. Although most discontinuities as they occur in nature do not satisfy these restrictions, they do not introduce any essential lack of generality. The motion of the discontinuity may be eliminated by having the observer move so as to always lie on the discontinuity. Unsteadiness and nonuniformity of the flow field may be eliminated by having the observer take a sufficiently microscopic point of view, with small space and time scales. Obliquity of the discontinuity may be eliminated by having the observer move along the discontinuity so that the tangential velocity components are zero.

In the establishment of the basic laws certain assumptions are necessary, on which the validity of the results depends. The principal basic assumption which must be made is that the fluid on each side of the discontinuity obeys a known equation of state[1] and has unique definable values of the velocity and of the various intensive and specific extensive thermodynamic variables. This assumption may pose difficulties in certain cases, such as in a mixture of solid and gaseous combustion products, or a case where there is a marked deviation from thermal equilibrium. Energy exchange through radiation is neglected, and the laws of Newtonian or nonrelativistic mechanics are assumed.

The basic laws of discontinuities are derived by use of the basic conservation principles of mechanics. Since these laws, or very similar ones, have already been derived in previous sections, no detailed derivation is given here. The standard notation is used, with p the pressure, ρ the density, u the velocity, h the specific enthalpy, and e the specific internal energy. For convenience the additional symbol v is used for the inverse of the density, i.e. for the specific volume.

Referring to Fig. D,1a, the principle of the conservation of mass gives the basic law

$$m = \rho_1 u_1 = \rho_2 u_2 = \frac{u_1}{v_1} = \frac{u_2}{v_2} \tag{1-1}$$

The quantity m thus defined is termed the mass flow. The principle of the conservation of momentum gives the law

$$p_0 = p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \tag{1-2}$$

Finally, the principle of the conservation of energy gives the law

$$h_0 = h_1 + \tfrac{1}{2}u_1^2 = h_2 + \tfrac{1}{2}u_2^2 \tag{1-3}$$

These three are termed the conservation laws.

[1] The term "equation of state" here and subsequently in this section is used in a sense encompassing all the usual thermodynamic variables, and not in the engineering sense specifying only the relationship connecting pressure, volume, and temperature. The specification of enthalpy as a function of entropy and pressure, for example, may be considered the equation of state in the sense used here, because such a relation gives complete thermodynamic information. Also, the term "an equation of state" is occasionally used to refer to any equation connecting state variables.

A convenient result, valid for all types of gasdynamic discontinuity, may be obtained immediately from the first two conservation laws (Eq. 1-1 and 1-2). It is

$$p_2 - p_1 = m(u_1 - u_2) = m^2(v_1 - v_2) \tag{1-4}$$

and shows that a decrease of the flow velocity across the discontinuity is associated with an increase in the pressure and an increase in the density. Conversely, an increase in the flow velocity is associated with a decrease in the pressure and a decrease in the density. This permits the immediate classification of all discontinuities into those of compression (deceleration) types and those of expansion (acceleration) types. If the three quantities equated in Eq. 1-4 are zero, there is no discontinuity. If they are small the

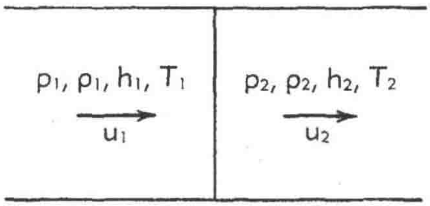

Fig. D,1a. Normal discontinuity.

discontinuity is termed weak. Alternate forms obtainable from Eq. 1-4 are

$$\frac{p_2 - p_1}{\rho_2 - \rho_1} = u_1 u_2 \tag{1-5}$$

$$(u_1 - u_2)^2 = (p_2 - p_1)(v_1 - v_2) \tag{1-6}$$

The energy function in Eq. 1-3 is the specific enthalpy, related to the specific internal energy by the relation

$$h = e + pv \tag{1-7}$$

An alternate expression for the energy equation may be obtained as

$$h_0 = e_1 - \tfrac{1}{2}u_1^2 + \frac{p_0 u_1}{m} = e_2 - \tfrac{1}{2}u_2^2 + \frac{p_0 u_2}{m} \tag{1-8}$$

From the conservation laws may be derived the so-called Hugoniot relation, which may be expressed in two alternate forms:

$$h_2 - h_1 = \tfrac{1}{2}(p_2 - p_1)(v_1 + v_2) \tag{1-9}$$

$$e_2 - e_1 = \tfrac{1}{2}(p_2 + p_1)(v_1 - v_2) \tag{1-10}$$

This relation is of great importance in considerations of detonations and deflagrations and has the particular property that the flow velocities do not appear, so that the equation is a purely thermodynamic one. This relation leads to the Hugoniot diagram, in which p_2 is plotted vs. v_2 for a given choice of p_1 and v_1. If the fluid on both sides of the discontinuity obeys the same equation of state, the point p_1, v_1 lies on the curve. The cases in which a reaction or change of state occurs, such that the point p_1, v_1 does not lie on the curve, may conveniently be divided into those for which the point lies below the curve, termed exothermic, and those for which the point lies above the curve, termed endothermic.[2] Only the exothermic case is generally encountered; this is the case for which the Hugoniot diagram is illustrated in Fig. D,1b. Other diagrams are useful in the study of discontinuity phenomena, though they do not have the property of the Hugoniot diagram of involving only thermodynamic variables.

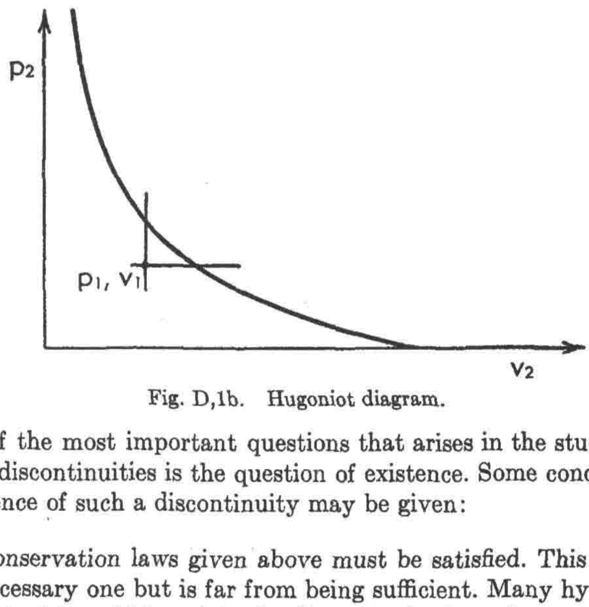

Fig. D,1b. Hugoniot diagram.

One of the most important questions that arises in the study of gasdynamic discontinuities is the question of existence. Some conditions for the existence of such a discontinuity may be given:

1. The conservation laws given above must be satisfied. This condition is a necessary one but is far from being sufficient. Many hypothetical discontinuities which satisfy the discontinuity laws do not exist.
2. The specific entropy of the material must increase. This condition is imposed to satisfy the second law of thermodynamics and is a necessary one. Together with condition 1 it is still not sufficient to ensure the existence of the discontinuity, but may be useful in eliminating

[2] This use of the words exothermic and endothermic is unconventional, and perhaps some such terms as expansive and contractive might be more appropriate. However, this usage would lead to such confusing entities as a compression expansive discontinuity, and the author prefers the terms exothermic and endothermic despite the lack of a clear-cut definitional connection with the release or absorption of heat. In general a reaction that gives off heat would give a discontinuity that is exothermic in the sense used here, and one that absorbs heat would give a discontinuity that is endothermic; hence the terminology does have some connection with conventional usage.

certain nonexistent discontinuities which would be permitted by condition 1.

3. The discontinuity must correspond in its structure to a physically realizable process. Here the term "physically" includes considerations of the chemistry and chemical kinetics of any reactions taking place. This condition is necessary and sufficient for the existence of the discontinuity in the small provided the discontinuity is internally stable. Since the appropriate physical and thermodynamic laws must be satisfied within the structure of the discontinuity, conditions 1 and 2 are automatically satisfied. The precise physical process must in most cases be approximated by a somewhat idealized process for purposes of theoretical treatment. For example, the Navier-Stokes equations are not strictly valid within a shock of finite strength and if they are used in an analysis the resultant picture of the physical process is an approximation. The pertinent point here is that the validity of an existence proof based on the demonstration of a physical process depends upon the accuracy with which the assumed process approximates the actual one.
4. The discontinuity must be internally stable. This means that if an equilibrium solution undergoes a disturbance allowed by the local hydrodynamic conditions the solution must return to the equilibrium one. The theory for such internal or local stability has not as yet been greatly developed. An attempt is given in Art. 4 below to outline such a theory and to present a few results.
5. The discontinuity must be stable in the large. A demonstration of the satisfaction of conditions 3 and 4 given above can at most prove the possible existence of a discontinuity in the small. A condition for the existence of a discontinuity in any particular case is that it must be stable with respect to possible changes in the configuration and hydrodynamic solution of the entire flow. The question of this type of stability lies outside the scope of this section.

Gasdynamic discontinuities must be classified according to whether or not the equations of state governing the material change as the material crosses the discontinuity. If the equations of state are the same for the material on both sides of the discontinuity it is termed a shock. A special case sometimes encountered is that in which the equation of state for a material has different forms in different thermodynamic regions because of different phases, so that the equations of state for the same material on the two sides of a discontinuity might be of different form. An example would be one in which a partially condensed gas changed through the discontinuity to a state with no condensed phase; such a discontinuity might be called an evaporation shock and would occur, for example, if a sufficiently strong shock were passed through a mist. However, in most