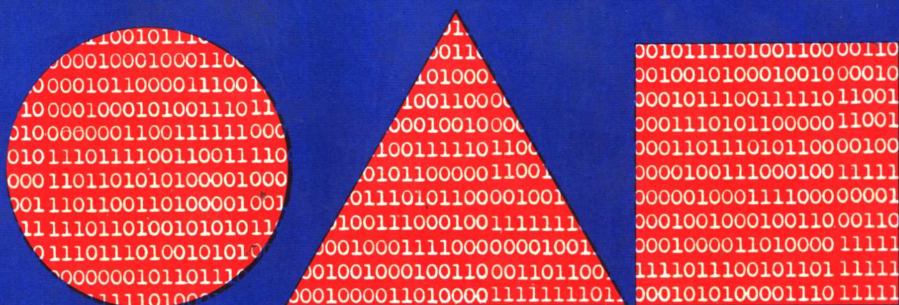
MODELING AND IDENTIFICATION OF DYNAMIC SYSTEMS

N. K. Sinha and
B. Kuszta



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Preface

Although during the last twenty years a great deal of research has been done on the subject of system identification, and several very good books have been written on the subject, it has been felt that several topics of great importance to practicing engineers as well as control theorists have not appeared in a systematic form in a book. This book is an attempt to fill this gap.

Two basic types of models are discussed in this book: (i) models for control applications, and (ii) models of many physical, biological and socioeconomic phenomena. Both groups of models are studied by means of differential or difference equations. In the first group, although it may be possible to obtain a model by detailed analysis using the basic laws of physics and chemistry, it is usually preferable to determine the model from the observations of the input and the output. In the second group, it is not even possible to perform an analysis due to lack of complete knowledge; in such cases we may obtain stochastic models which are often very useful for understanding the phenomenon.

Even the systems belonging to the first group are generally of the distributed-parameter and nonlinear type. Although it may be possible to obtain suitable models using nonlinear partial differential equations, these are, generally, of little value for practical applications. On the other hand, a suitable linear lumped-parameter model, can be quite useful within certain limitation of operating ranges which can be estimated by proper examination. Hence, for such systems, it is necessary not only to be able to determine a suitable linear lumped-parameter model, but also to fully understand the conditions under which it is valid. We try to answer both of these basic questions.

The popular approach of investigating nonlinear systems by means of the Taylor series expansion of nonlinear operators (for example, Volterra series) or the describing function method will not be included in this book because of its limited practical application. Our approach is to determine suitable linear models, along with the regions over which they are valid.

The organization of the text will now be discussed. In Chapters 2 to 6, linear models are presented, including multivariable systems as well as stochastic models. In Chapter 7, the problem of identification of closed-loop linear systems is discussed. This is followed, in Chapter 8, by techniques for obtaining low-order models of high-order systems because of their usefulness in the understanding of such systems, as well as for the preliminary design of controllers. In Chapter 9, we consider the problem of combined state and parameter estimation because of its importance in the adaptive control of complex processes. Identification of distributed-parameter systems is discussed in Chapter 10. In Chapter 11 we consider nonlinear lumped-parameter systems, with emphasis on the determination of regions of bifurcation. Chapter 12 presents the design of optimal input signals for system identification with special consideration given to practical limitations. In Chapter 13, we describe methods for determining the order and the structure of linear models. Diagnostic tests for linear as well as nonlinear models are presented in Chapter 14. Concluding remarks along with a list of unsolved problems in the area are given in Chapter 15. A number of appendices are included to describe some of the theoretical background, as required.

It may be pointed out that the most of the material in Chapters 5 (multivariable systems), 8 (model reduction techniques), 9 (combined state and parameter estimation), 10 (distributed-parameter systems), 11 (nonlinear systems), and 12 (design of optimal inputs) appears for the first time in the form of a book in English in the control literature.

This book will be of value to both practicing engineers as well as students of control theory. The material has been arranged in such an order that it can be followed without difficulty by a person who has taken a first course on control theory and has the usual mathematical background in transform calculus and the theory of state equations. Most of this material has been used as a graduate course on system identification at McMaster University and at Tianjin University in China. The authors are grateful to the former students for finding several typographical errors.

This work owes a lot to the efforts of many former students. The authors are indebted to Drs. A. Sen, J. D. Hickin and H. El-Sherief for permission to reproduce some portions of their Ph.D. theses. The support of the research by the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. Discussions with many colleagues were very helpful, and in particular, the authors would like to thank Dr. B. Beliczynski of the Technical University of Warsaw, and Drs. J. F. McGregor and J. D. Wright of McMaster University. The authors are very grateful for the encouragement received from the late Professor N. S. Rajbman of the Institute of Control Sciences, Moscow.

Thanks are due to Pat Dillon, Nancy Sine and Amy Stott of the Word Processing Centre, McMaster University, for their very cheerful and untiring efforts in typing the manuscript and making innumerable corrections. Finally, this work would have proved impossible without the support and understanding of our wives, Meena Sinha and Krystyna Kuszta.

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B. KUSZTA

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1 Introduction

The problem of system modeling and identification has attracted considerable attention during the past twenty years mostly because of a large number of applications in diverse fields like chemical processes, biomedical systems, socio-economic systems, transportation, ecology, electric power systems, hydrology, aeronautics, and astronautics. In each of these cases, a model consists basically of mathematical equations which can be used for understanding the behavior of the system, and wherever possible, for prediction and control.

Two basic types of modeling problems arise. In the first type one can associate with each physical phenomenon, a small number of measurable causes (inputs) and a small number of measurable effects (outputs). The outputs and the inputs can generally be related through a set of mathematical equations, in most cases nonlinear partial differential equations. The determination of these equations is the problem of modeling in such cases. These can be obtained either by writing a set of equilibrium equations based on mass and energy balance and other physical laws, or one may use the "black-box" approach which consists of determining the equations from the past records of the inputs and the outputs. Modeling problems of this type appear quite often in engineering practice. Some typical problems are modeling of (i) a stirred-tank chemical reactor, (ii) a multimachine electrical power system, (iii) a synchronous-orbit communications satellite, and (iv) the control mechanism of a nuclear power reactor. In each of these examples one can easily identify certain input and output quantities, and then obtain the mathematical model relating them. Some of these will be discussed in the book in the later chapters.

Another type of modeling problem arises in those situations where although we can identify a certain quantity as a definite measureable output or effect, the causes are not so well defined. Some typical examples are (1) the annual population of the United States, (2) the annual rainfall in a certain country, (3) the average annual flow in a river, and (4) the daily value of a certain stock in the stock market. In all these cases, we have available a sequence of outputs, which will be called a time series, but the inputs or causes are numerous and not quite known in addition to often being unobservable. Nevertheless, it is important to develop a model in order that one may have some understanding of the process which may be used for planning. The models in such cases are called stochastic models, due to a certain amount of uncertainty which is unavoidable.

In this book we shall be studying both of these types of modeling problems. The first will be referred to as the problem of system identification, whereas the second will be called the problem of stochastic modeling. It must be clearly understood that the two problems are related closely. Moreover in both cases, we must be able to choose the best from a set of rival models. This requires development of methods for testing such models as well as suitable criteria for deciding on the optimum.

In system identification, we are concerned with the determination of system models from records of system operation. The problem can be represented diagrammatically as below

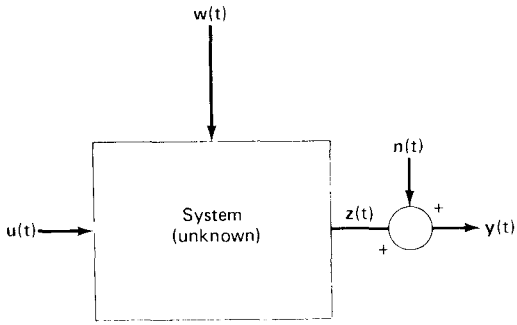

Figure 1.1

where

$\underset{\sim}{u}(t)$ is the known input vector of dimension m
$\underset{\sim}{z}(t)$ is the output vector of dimension p
$\underset{\sim}{w}(t)$ is the input disturbance vector
$\underset{\sim}{n}(t)$ is the observation noise vector
$\underset{\sim}{y}(t)$ is the measured output vector of dimension p

Thus, the problem of system identification is the determination of the system model from records of u(t) and y(t).

At this point it is important to distinguish between the system and its model. A system is defined as "a collection of objects arranged in an ordered form, which is, in some sense, purpose or goal directed." What constitutes a 'system' depends upon the viewpoint of the analyst or designer. For instance, an electronic amplifier consisting of a large number of components may be regarded as a system by the electronic engineer. On the other hand, the same amplifier may be one of the many parts of a "feedback control system." Furthermore, this feedback control system may be a part of a chemical process (or system) containing many loops of this type. Finally, we may have a plant containing many such units.

A model may be defined as "a representation of the essential aspects of a system which presents knowledge of that system in a usable form." A model, to be useful, must not be so complicated that it cannot be understood and thereby be unsuitable for predicting the behavior of the system; at the same time it must not be trivial to the extent that predictions of the behavior of the system based on the model are grossly inaccurate.

A fundamental problem in system identification is the choice of the nature of the model which should be used for the system. The model may be one of the following types:

(a) linear time-invariant (lumped-parameter)–ordinary linear differential equations
(b) linear time-varying (lumped parameter)–ordinary linear differential equations
(c) linear but with distributed parameters–partial differential equations
(d) nonlinear–nonlinear differential equations

Although, in practice, most systems are nonlinear with distributed parameters, linear models for such systems are often used because of their simplicity. In a large number of cases, "incremental," or "piecewise" linear models can be conveniently used for approximate understanding of the system. In using such models, one must be careful and should have an idea of the limits of their validity. Nevertheless, a great deal of work has been done on obtaining linear models for systems; so much that often by system identification one understands the determination of the parameters of "suitable" linear model for the system.

Some of the problems in system identification are:

(a) determining the order of the linear model
(b) selection of a suitable criterion for determining the "accuracy" of the model
(c) designing an input signal which will maximize the accuracy of the estimates of the parameters of the model.

Although most systems are of the "continuous-time" type, the application of the digital computer for identification makes it desirable to use "discrete-time" models. Often the determination of the parameters of a discrete-time model is more straightforward. Furthermore, provided that the sampling interval satisfies certain conditions, the determination of the continuous-time model from the discrete-time model is fairly straightforward.

Many applications require "on-line" identification instead of "off-line." An identification method is said to be of the "off-line" type when one collects a large amount of input and output data for the system which may be stored in a computer or recorded in some manner. These data are then processed in a batch to estimate the parameters of the model and obtain the best fit according to a prescribed cost function. In off-line identification, one may often select the type of input most suitable. Also, there is a greater flexibility in selecting computational methods without any restriction on computing time. As a result the accuracy of the estimates can be made fairly high, approaching the Cramer-Rao bound.

In a number of control applications, especially adaptive control, it is necessary to identify the system in a fairly short time. An identification scheme is said to be of the "on-line" type if it satisfies the following conditions:

(a) it does not require a special input
(b) all the data need not be stored
(c) a recursive algorithm is used for adjusting the estimates of the parameter after each sampling instant
(d) the amount of computation required for "model adjustment" is a fraction of the sampling period.

It may be added that, in general, on-line methods will not lead to as accurate models as possible with off-line methods which can use a much larger amount of data. But in many practical situations one cannot afford to wait for the time required to collect all the data. As a matter of fact, it will be recognized that life is the art of reaching sufficient conclusions from insufficient data. Some typical examples of situations where one must make an important decision on the basis of insufficient information are: (i) getting married, (ii) accepting a job, (iii) hiring a new employee, and (iv) investing in the stock market.

An important application of on-line identification is the development of the self-tuning regulator, proposed recently by Professor K. Aström and his colleagues.

A large variety of methods have been applied to system identification, both off-line and on-line. The methods can be classified in many ways; one scheme for classification is given below.

I. Classical Methods: (mostly off-line)
  (a) Frequency Reponse Identification
  (b) Impulse response identification by deconvolution
  (c) Step response identification
  (d) Identification from correlation functions

II. Equation-error Approach: (batch-processing)
  (a) Least-squares
  (b) Generalized least squares
  (c) Maximum likelihood
  (d) Minimum variance
  (e) Gradient Methods

III. Model Adjustment Techniques:
  (a) Least-squares (recursive)
  (b) Generalized least squares (recursive)
  (c) Instrumental variables
  (d) Bootstrap
  (e) Maximum likelihood (recursive)
  (f) Correlation (recursive)
  (g) Stochastic approximation

In Chapter 2 we shall be discussing the classical methods for system identification, which have been known for more than fifteen years.

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2
Classical Methods of System Identification

We shall now consider a number of methods of system identification that are called classical in the sense that they have been around for a longer time than the so-called modern methods. The first such method is based on the frequency response of the system.

2.1 FREQUENCY RESPONSE METHOD

The *frequency response method* for identification of linear systems is based upon the familiar Bode diagrams of frequency response.

In this method, sine-wave inputs are applied to the system and the steady-state output is observed; both the magnitude ratio and the phase shift between the output and input are measured. These measurements are made over the entire range of frequencies of interest. If the transfer function of the system (Figure 2.1) is G(s), then the frequency response is obtained by replacing s by $j\omega$, i.e.,

$$G(j\omega) = M(\omega) \cdot e^{j\phi(\omega)} = \frac{Y(j\omega)}{X(j\omega)} \tag{2.01}$$

where M is the ratio of the magnitudes, and ϕ is the phase shift between the output and the input.

The plot of $M(\omega)$ in decibels against ω (log scale), as well as the plot of $\phi(\omega)$ against ω (log scale), can then be used for estimating the various break-frequencies (poles and zeros) of the transfer function.

In practical application of this method, one must be able to generate the sine-wave inputs of various frequencies, and also be able to measure the magnitude ratios and phase-shifts accurately at these frequencies. The method is applicable

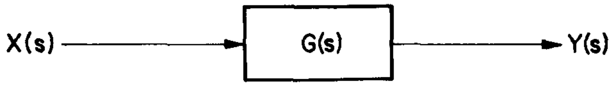

Figure 2.1

only to linear stationary processes and assumes off-line identification. Moreover, the method is applicable only to stable systems, since the frequency-response of an unstable system cannot be measured in practice.

The estimation of the transfer function is based on approximating the magnitude response curves with straight lines of slopes 6n db/octave, where n is an integer. These give the break frequencies and hence the transfer function, which is then verified from the phase shift curve. Although the case of real poles or zeros is quite straightforward, the case of complex poles requires estimating the damping ratios as well. The following figures illustrate the variation of M and ϕ with the damping ratio.

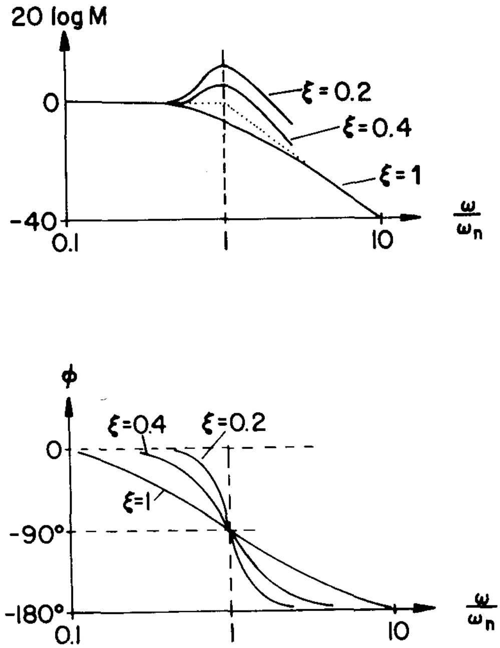

Figure 2.2. Frequency response curves for a second-order system given by

$$G(s) = \frac{1}{1 + 2\xi s/\omega_n + (s/\omega_n)^2}$$

EXAMPLE 2.1.1

Consider the frequency response data obtained for a d.c. servomotor/servo-amplifier combination in an undergraduate control laboratory, as shown below

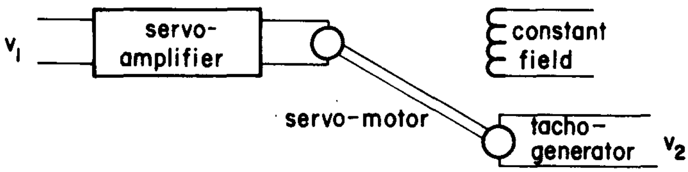

Figure 2.3. Identification of a servomotor/servoamplifier combination.

Frequency (Hz)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$20 \log \lvert v_2/v_1 \rvert$	7.1	7.0	6.7	6.4	6.0	5.6	5.1	4.6	4.1
$\measuredangle v_2/v_1$	-6.35	-12.6	-18.5	-24.1	-29.2	-33.8	-38.0	-41.8	-45.1

1.0	1.2	1.5	2.	2.5	3.	4.	5.	7.	10.
3.7	2.7	1.4	-0.6	-2.3	-3.7	-6.0	-7.9	-10.8	-13.8
-48.1	-53.3	-59.1	-65.9	-70.3	-73.4	-77.4	-79.8	-82.7	-84.9

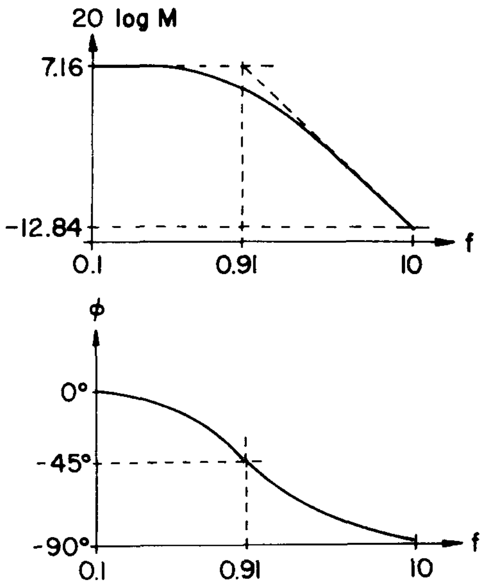

Figure 2.4

The frequency response plots are shown in Figure 2.4. By drawing straight-line asymptotes to the magnitude ratio curve, the break frequency is estimated as 0.9 Hz. This is confirmed by the phase-shift plot giving phase shift equal to 45° at this frequency. Estimating the d.c. gain as 7.2 decibels the transfer function is estimated as

$$G(s) = \frac{10^{7.2/20}}{1 + s/(2\pi \times 0.9)} = \frac{2.28}{1 + s/1.8\pi} = \frac{12.89}{s + 5.65} \tag{2.02}$$

In this case it is rather straightforward to obtain the transfer function because the model was of the first order. For a system of higher order, it is not so easy to estimate the transfer function accurately from the frequency response plots; specially if the poles and zeros are not far apart. The following example will illustrate the difficulty.

EXAMPLE 2.1.2

Consider the transfer function

$$G(s) = \frac{200(s+2)}{(s+4)(s^2 + 10s + 100)} \tag{2.03}$$

This represents the overall transfer function of a position control servo with a lead compensator. Samples of the frequency response are given in the following table

ω	0.1	0.2	0.3	0.4	0.5	0.7	1.0
$20 \log \lvert G \rvert$	0.32	1.10	2.04	2.93	3.72	4.96	6.15
$\sphericalangle G$ (deg)	4.90	7.43	7.01	4.33	0.16	-10.77	-31.25
	1.5	2.0	2.5	3	4	5	7
	5.95	2.89	-0.83	-4.15	-9.45	-13.50	-19.52
	-72.22	-106.13	-126.02	-137.64	-150.21	-156.88	-163.92
	10	15	20	30	40	50	100
	-25.81	-32.91	-37.92	-44.98	-49.98	-53.86	-65.9
	-168.51	-172.66	-174.51	-176.35	-177.26	-177.81	-178.9

The frequency response curves are shown in Figure 2.5.