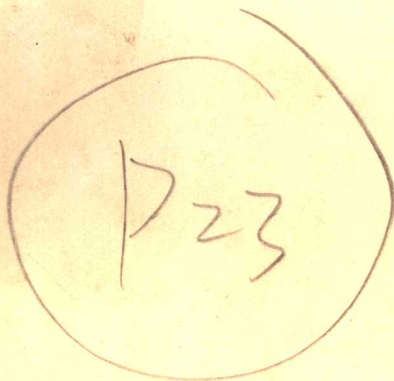# ELECTRICAL MACHINES AND DRIVES

WORKED EXAMPLES

SECOND EDITION

John Hindmarsh,

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CAL MACHINES AND DRIVES

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SECOND EDITION

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EDITOR'S PREFACE

THIS is a companion volume to the author's textbook *Electrical Machines and Their Applications*, a well-loved book now in its fourth edition. The success of that book is due to the unusual combination in the author of great teaching skill and long first-hand experience of the design, manufacture and application of electrical machines.

Although much has changed in electrical technology in the last few years, the role of rotating machines and transformers is as important as it ever was. It is by their means that electrical energy is generated and then reconverted to mechanical energy in homes and factories. Of course, these machines are continually being improved and modified, but their fundamental principles remain the same. Hence no electrical engineering course is complete without a study of the principles of electrical machines and drives.

But this is not an easy subject. The interaction of electrical and mechanical energy can be understood by students only when they have had practice at solving numerical problems. In his new book the author provides such problems, mostly with their solutions, graded in difficulty and interspersed with advice, all against a background of sound engineering practice. The reader who has worked through these examples will be able to face with confidence many problems about the behaviour of machines and the choice of drives. Not every electrical engineer needs to be a machine specialist, but he needs to be able to co-operate with such specialists and this book will enable him to do so.

I am very happy to commend the book both to students and to teachers, who will find it a great help in strengthening their lectures and practical classes.

University of Southampton   **P. HAMMOND**

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AUTHOR'S PREFACE TO THE SECOND EDITION

THE response to the first edition included a suggestion that the final section on simulation was somewhat limited in scope. This topic really requires a separate text to deal in depth with such a wide-ranging subject. Nevertheless, an attempt has been made in the present edition to augment the introductory treatment so that with sufficient interest, extensive simulations could be undertaken on the basis of the material given. Although this is really tending towards project or even long-term post-graduate work, requiring substantial computer-program development, its immediate importance is in the facility it provides to display computed performance, especially of power-electronic/machine circuits, which should give a better understanding of their special features. The author gratefully acknowledges permission to use the computational facilities at UMIST for this purpose.

There are other changes in the text to include reference for example to unbalanced operation, permanent-magnet machines and the universal motor. Also, the additional worked examples and tutorial examples should ensure that the rather wide topic of Electrical Drives is given sufficiently comprehensive coverage. Overall, the intention is to support the suggestion of teaching the subject by means of worked examples, after due preparation on the basic equations and with discussion of the problems and solutions.

Sale, Cheshire   J. H.
March 1985

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AUTHOR'S PREFACE TO THE
FIRST EDITION

DISCUSSION of the summary and objective of this book is deferred to the first section of Chapter 1 lest this preface, like many others, goes unread. However, there are some matters which need to be covered here, especially those which concern the author's debt to others. Every author likes to believe that he has created something original or shed new light on an old topic. But, inevitably, much of the work must have been derived from his experiences as a student, from his own teachers, from books read, from his own students and from his colleagues. This last is particularly true in the present case. A major influence has been the author's long association with Dr. N. N. Hancock, whose deep perception of the subject and ever-sympathetic ear when difficulties of understanding arose have been a source of constant sustenance. To Dr. A. C. Williamson, with his remarkable facility for cutting clean through the theoretical fog to grasp the essential nature of tricky machines problems, especially those associated with power-electronic circuits, I am deeply indebted. Much time was spent by him in kindly checking and correcting formative ideas for the material on constant-current and variable-frequency drives. Sections 4.3, 5.5, 7.3 and 7.4 are heavily reliant on his contributions. Dr. B. J. Chalmers' experience, with saturated a.c. machines particularly, was very important to the clarification of this section of the work. I am grateful, too, to Dr. M. Lockwood for his suggestions on simplifying simulation problems.

With regard to the examples themselves, an attempt has been made to cover thoroughly the basic machine types, but the subject is very wide. Very small and special machines have been omitted, quite apart from study of the economic and environmental factors which influence the decision in choosing an electrical drive, though some brief comments are made as appropriate. This selective treatment seemed to be the best way

of meeting the many requests in response to which this book was written. Many of the examples are taken, or modified, from examination papers set at the University of Manchester Institute of Science and Technology (UMIST), and permission to publish these is gratefully acknowledged. Other examples, especially those in the Appendix, are drawn from a variety of sources. The author cannot deny that some of them may have originated in form from other books read over the years since first meeting the topic as a student, and from discussions and contact with present and past colleagues.

Finally, the author would like to record his thanks to the Consulting Editor, Professor Percy Hammond, for his encouragement, for reading the text and making his usual perceptive comments and suggestions to get the balance right. To the Managing Editor, Mr. Jim Gilgunn-Jones, and his colleagues at Pergamon Press, who have been so patient in spite of delays and last-minute changes, I tender my grateful appreciation.

Sale, Cheshire   J. H.
August 1981

LIST OF SYMBOLS

THE following list comprises those symbols which are used fairly frequently throughout the text. Other symbols which are confined to certain sections of the book and those which are in general use are not included, e.g. the circuit symbols like R for resistance and the use of A, B and C for 3-phase quantities. Some symbols are used for more than one quantity as indicated in the list. With few exceptions, the symbols conform to those recommended by the British Standards Institution BS 1991.

> Instantaneous values are given small letters, e.g. e, i, for e.m.f. and current respectively.
>
> R.M.S. and steady d.c. values are given capital letters, e.g. E, I.
>
> Maximum values are written thus: $\hat{E}$, $\hat{I}$.
>
> **Bold face type** is used for phasor and vector quantities and for matrices, e.g. **E**, **I**. In general, the symbol E (e) is used for induced e.m.f.s due to mutual flux and the symbol V (v) is used for terminal voltages.

At   Ampere turns.
B Flux density, in teslas (T) (webers/metre^2).
d   Symbol for direct-axis quantities.
d Armature diameter, in metres.
e   Base of natural logarithms.
E_f Induced e.m.f. due to field m.m.f. F_f.
f Frequency, in hertz (Hz) (cycles per second).
F Magnetomotive force (m.m.f.) in ampere turns. Peak m.m.f. per pole per phase.
F'_a Effective d.c. armature-winding magnetising m.m.f. per pole.
F_a Peak armature-winding m.m.f. per pole.
F_f Peak field-winding m.m.f. per pole.

(Note that the suffices a and f are also used with the symbols for currents, fluxes and resistances of armature and field respectively.)

F_r Peak resultant m.m.f. per pole.

$I_{f.l.}$ Full-load current.

I_0 Current in magnetising branch.

I_m Reactive or magnetising component of I_0.

I_p Power component of I_0.

J Polar moment of inertia (rotational inertia), in kg m^2.

k Coefficient. A constant

k_f Generated volts per field ampere or per unit of m.m.f.

k_{fs} Saturated value of k_f.

k_ϕ Flux factor = generated volts per radian/sec or torque per ampere.

l Conductor length. Magnetic path length.

l (or l_1, l_2, etc.) Leakage inductance.

L General inductance symbol; e.g. L_{11} = self-inductance of coil 1; L_{12}, L_{13}, etc., for mutual inductances.

m   Number of phases.

M Alternative mutual-inductance symbol

n Rev/sec.

n_s Rev/sec synchronous $= f/p$.

N Number of turns. Rev/min.

N_s Rev/min synchronous $= 60f/p$.

p   Operator d/dt.

p Number of pole pairs.

$p.u.$ Suffix for *per-unit* quantities.

P Power.

$P_{control}$ Power at control terminals.

P_{elec} Power (total) at electrical terminals (P_e = per phase).

P_{gap} Air-gap power (total) (P_g = per phase).

P_m Mechanical power converted (per phase) (m.$P_m = \omega_m.T_e$).

P_{mech} Power at mechanical terminals ($\omega_m.T_{coupling} = P_{coupling}$).

q   Symbol for quadrature-axis quantities.

R_m Magnetising resistance, representing iron losses.

s Fractional slip $= (n_s - n)/n_s$.

S *Per-unit* relative motion $n/n_s \, (= 1 - s)$.

$T_{coupling}$ Torque at mechanical shaft coupling.

T_{loss} Sum of all machine internal loss torques.

T_e Torque developed electromagnetically, in newton metres.

T_m Torque arising mechanically $= T_e$ in steady state.

v Velocity, in metres per second.

V Voltage measured at the terminals of a circuit or machine.

x (or x_1, x_2, x_{a1}, etc.) Leakage reactance.

X General reactance symbol.

X_m Magnetising reactance.

X_{ms} Saturated value of X_m. X_{mu} Unsaturated value of X_m.

X_s Synchronous reactance $= X_m + x_{a1}$.

z_s Number of series-connected conductors per phase or per parallel path of a winding.

Z_s Synchronous impedance.

α General angle. Slot angle. Impedance angle $\tan^{-1} R/X$. Firing delay angle. Abbreviation for "proportional to".

δ Load angle. Chopper duty-cycle ratio

δ_{fa} (or δ_T) Torque angle.

η Efficiency.

Λ Magnetic permeance, webers/ampere-turn.

μ_0 Magnetic constant $= 4\pi/10^7$.

μ_r Relative permeability.

μ Absolute permeability $= B/H = \mu_0 \mu_r$.

φ Power-factor angle. N.B. This must be distinguished from the symbol for flux ϕ below.

ϕ Instantaneous value of flux. Flux per pole, in webers.

ϕ_m Mutual flux, in webers, due to resultant m.m.f.

Φ Flux time-phasor.

θ Shaft angular position in electrical radians, (θ_m mechanical angle). Temperature rise. General variable.

$\dot{\theta}$ Shaft speed in electrical rad/s.

τ Time constant.

ω Angular velocity of rotating time-phasors $= 2\pi f$ radians/sec.

ω_m Mechanical angular rotational velocity $= 2\pi n$ radians/sec.

ω_s Synchronous angular velocity $= 2\pi n_s = 2\pi f/p$ radians/sec.

Note: SI units (Systeme International d'Unites) are used in the text unless specifically stated otherwise, as in the case of *per-unit* quantities and the so-called Engineers' units; i.e. 1 hp $= 746$ watts; 1 lbf ft $= (746/550)$ Nm.

CONTENTS

N.B. Instead of an Index, the Example Numbers are given below with an indication of the topics covered.

CHAPTER 1

INTRODUCTION AND REVIEW OF BASIC THEORY

1.1 AIM OF THE BOOK

On entering the world of electrical machines, the student meets many conceptual difficulties not experienced for example in the early studies of digital systems, with their simple and precise 2-state operation. More assistance is required to permit the new-comer to gain confidence in dealing with non-linear, 3-dimensional, rotating electromagnetic devices. The purpose of this book is to provide this aid to understanding by showing how, with a limited number of equations derived from basic considerations of power flow and elementary circuit and electromagnetic theory, the electromechanical performance can be explained and predicted with reasonable accuracy.

Such an aim, which will permit the calculation of power-input/output characteristics almost close enough in engineering terms to those of the device itself, can be achieved by representing the machine as a simple electrical circuit—the equivalent-circuit model. This concept is explained in many books, for example in the author's companion volume *Electrical Machines and Their Applications*. Though more detailed theoretical treatment is given there, substantial portions of the present text may be regarded as suitable revision material. This expanded 2nd edition can, as a whole, be considered as a textbook on Electrical Drives, taught through worked examples, for a reader having some familiarity with basic machine theory.

Perhaps it is appropriate to point out that complete and exact analysis of machine performance is so complex as to be virtually impossible. The

additional accuracy achieved by attempts to approach such methods is primarily of interest to the specialist designer who must ensure that his product will meet the user's needs without breakdown and he must judge when the analytical complication is justified. For the user, and for the engineering student who is not yet a specialist, the simpler methods are adequate for general understanding and provide a lead-in if necessary for later specialisation.

There are many features of all machine types which are common, the obvious example being the mechanical shaft equations. But apart from these and the fundamental electromagnetic laws, the input/output relationships and modes of operation have many similarities. These are brought together where possible and also in this first chapter, some elementary mechanical, magnetic and circuit theory is discussed briefly, as a reminder of the basic knowledge required. Students should beware of underestimating the vital importance of this material, since experience shows that it is these very points, improperly understood, which hold back progress in coming to feel at ease with machines problems.

However familiar one may become with theory, as a student, the true test of an engineer is his ability to make things work. First steps to this goal of confidence are reached when a student is prepared to commit himself to selecting equations and inserting values in the algebraic expressions, producing answers to a specific problem. Hence the importance of practice with numerical examples. Understanding grows in proportion to one's ability to realise that the equations developed really can be used in a systematic fashion to solve such problems, since they describe the physical behaviour in mathematical terms. Appreciation of this last statement is the key to successful problem-solving.

The chapters are planned to sequence the examples at increasing levels of difficulty. Much theoretical support is given, in that the equations are discussed either at the beginning of each chapter, or as the need arises. Solution programmes indicate the kind of problems which can be formulated for the three basic types of rotating machine: d.c., induction, and synchronous. Readers are encouraged to adopt an ordered approach to the solution; for example it is a good idea to incorporate the question data on a diagram. One of the difficulties of machines problems often lies in the amount of data given. By putting the values on a simple diagram,

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assimilation is easier and it helps to avoid mistakes of interpretation, especially when working with 3-phase circuits. In following this recommended pattern, it is hoped that the text will help to remove the mystery with which some students feel the machines area is shrouded.

The emphasis is on machine terminal-characteristics, rather than on the internal electromagnetic design. In other words, the electrical-drives aspect is uppermost since this is the area in which most engineering students need to have some good knowledge. It is worth noting that about 60–70% of all electrical power is consumed by motors driving mechanical shafts and virtually all this power is produced by generators driven through mechanical shafts, so that the subject is of considerable importance to engineers. The problems and solutions are discussed where appropriate, to draw out the engineering implications. Electro-mechanical transients are not neglected and opportunity is also taken to consider the effects introduced by the impact of power-electronic control. In general, the usual methods of analysis are still reasonably effective in predicting machine performance. Full account of the influence of this important environment, in which harmonics proliferate, is somewhat beyond the scope of this book but some indication is given of the means used to deal with the machines problems which arise. Detailed study of machine/semi-conductor systems requires a knowledge of mathematical and computer simulation procedures. Chapter 8 considers this topic in sufficient depth to provide real understanding and a sound basis for further, specialised investigations. Finally, in Appendix C, some tutorial examples are given along with the answers. Some of the worked examples in the text have been taken from Appendix E of *Electrical Machines and Their Applications*, but many of these remain as further exercises for the determined student.

1.2 FOUNDATION THEORY

Excitation Calculations

Virtually all machines have iron in the magnetic circuit to enhance the flux value and/or to reduce the excitation requirements. The price to pay