# BRIEF CALCULUS WITH APPLICATIONS

SECOND EDITION

Larson/Hostetler

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SECOND EDITION

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### Preface

Brief Calculus with Applications, Second Edition is designed for use in a beginning calculus course for students in business, economics, management, and the social and life sciences. In writing this edition, we were guided by two primary objectives formed over many years of teaching calculus. For the student, our objective was to write in a precise and readable manner, with the basic concepts, techniques, and applications clearly defined and demonstrated. For the instructor, our objective was to create a comprehensive teaching instrument that employs proven pedagogical techniques, thus freeing the instructor to make the most efficient use of classroom time.

As full-time calculus instructors, we have found the following features to be valuable aids to the teaching and learning of calculus.

#### **FEATURES**

**Introductory Examples** Each section in Chapters 1–10 begins with a one-page motivational example designed to show the applicability of the material in the section. These examples are to be read for enjoyment and not for complete mastery, since many of the concepts and techniques involved are discussed in the material that follows.

**Prerequisites** Chapter 0 is designed as a quick review of the algebra needed to study calculus. An instructor may elect to skip Chapter 0 and begin the course with Chapter 1, and in such cases, Chapter 0 can serve as an algebraic reference for students.

**Examples** The text contains over 600 titled examples that have been chosen to illustrate specific concepts or problem-solving techniques.

**Exercises** Many new exercises were added to the Second Edition and the text now contains over 4,500 exercises. The answers to the odd-numbered exercises are given in the back of the text.

**Applications** There are over 500 applications taken from a variety of fields, with a special emphasis on applications in business and economics. An index of applications is given on the front inside covers of the text.

**Graphics** There are over 1,700 figures and graphs in the text. Of these, approximately 450 are in the actual exercise sets.

**Theorems and Definitions** Special care has been taken to state the theorems and definitions simply, without sacrificing accuracy. The theorems and definitions are set off by a color screen to emphasize their importance.

**Section Topics** Each section begins with a list of the major topics that are covered in the section.

**Remarks** The text contains special instructional notes in the form of "Remarks." These appear after definitions, theorems, or examples and are designed to give additional insight, help avoid common errors, or describe generalizations.

**Calculators** Special emphasis is given to the use of hand calculators in sections dealing with limits, numerical integration, Taylor polynomials, and Newton's Method. In addition, many of the exercise sets contain problems identified by the symbol as calculator exercises.

**Study Aids** For students who need algebraic help, the *Study and Solutions Guide* by Dianna L. Zook contains step-by-step solutions to the odd-numbered exercises. Answers to the even-numbered exercises are given in the *Instructors Guide* by Ann R. Kraus.

#### CHANGES IN THE SECOND EDITION

In writing this edition, we were guided by suggestions given by instructors and students who used the first edition.

- Several new sections were added to the text: Section 2.8 (Related Rates), Section 3.8 (Differentials), Section 4.5 (The Definite Integral as the Limit of a Sum), Section 6.7 (Random Variables and Probability), Section 6.8 (Expected Value, Standard Deviation, and Median), Section 7.3 (First-Order Linear Differential Equations), and Section 9.3 (p-Series and the Ratio Test). Also, the first section in which surfaces in space are introduced was expanded to two sections—Section 8.1 (The Three-Dimensional Coordinate System) and Section 8.2 (Surfaces in Space).
- Many of the existing sections in the first edition were rewritten. For instance, the
  presentation of limits and continuity in Sections 1.5 and 1.6 is essentially new.
- Chapter summaries and review exercises were added to the ends of the chapters (beginning with Chapter 1).
- Reference tables were added to the back of the text.
- Computer software is now available to supplement the text. The software consists

of exploratory and directed activities that reinforce the learning and enhance the teaching of calculus. The programs feature a carefully designed user interface and are structured as reusable tools that a student will be able to use in other mathematics courses.

#### **ACKNOWLEDGMENTS**

We would like to thank the many people who have helped us at various stages of this project. Their encouragement, criticisms, and suggestions have been invaluable to us. Special thanks go to the reviewers of the first and second editions: Miriam E. Connellan, Marquette University; Bruce H. Edwards, University of Florida; Roger A. Engle, Clarion University of Pennsylvania; William C. Huffman, Loyola University of Chicago; James A. Kurre, Pennsylvania State University; Norbert Lerner, State University of New York at Cortland; Earl H. McKinney, Ball State University; Eldon L. Miller, University of Mississippi; Maurice L. Monahan, South Dakota State University; Stephen B. Rodi, Austin Community College; DeWitt L. Sumners, Florida State University; Jonathan Wilkin, Northern Virginia Community College; Melvin R. Woodard, Indiana University of Pennsylvania; and Robert A. Yawin, Springfield Technical Community College.

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Several other people worked on this project with us and we appreciate their help. David E. Heyd assisted us in writing the text. Dianna L. Zook wrote the *Student Solutions Guide*. Ann R. Kraus wrote the *Instructors Guide*. Linda L. Matta proofread the galleys. Timothy R. Larson prepared the art and proofread the galleys. Linda M. Bollinger typed part of the manuscript and proofread the galleys. Nancy K. Stout typed part of the manuscript. Helen Medley proofread the manuscript.

On a personal level, we are grateful to our children for their support during the past several years, and to our wives, Deanna Gilbert Larson and Eloise Hostetler, for their love, patience, and understanding.

If you have suggestions for improving this text, please feel free to write to us. Over the past 15 years we have received many useful comments from both instructors and students and we value these very much.

Roland E. Larson Robert P. Hostetler

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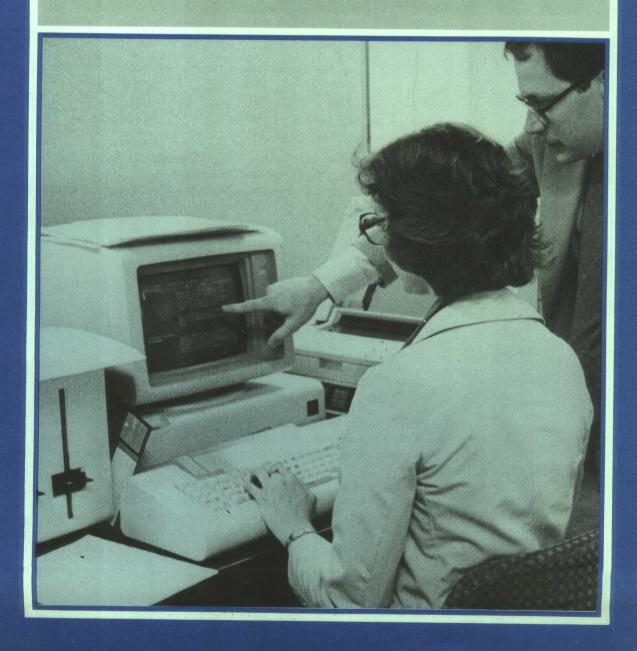
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## A Precalculus Review

CHAPTER



# The Real Line and Order

#### SECTION TOPICS

- The Real Line
- Order and Inequalities
- Intervals on the Real Line

To represent the real numbers, we use a coordinate system called the **real line** (or x-axis), as shown in Figure 0.1. The **positive direction** (to the right) is denoted by an arrowhead and indicates the direction of increasing values of x. The real number corresponding to a particular point on the real line is called the **coordinate** of the point. As shown in Figure 0.1, it is customary to label those points whose coordinates are integers.

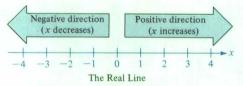
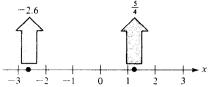


FIGURE 0.1

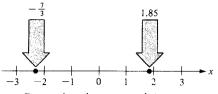
The point on the real line corresponding to zero is called the **origin**. Numbers to the right of the origin are **positive**, and numbers to the left of the origin are **negative**. We use the term **nonnegative** to describe a number that is either positive or zero.

The importance of the real line is that it provides us with a conceptually perfect picture of the real numbers. That is, each point on the real line corresponds to one and only one real number, and each real number corresponds to one and only one point on the real line. This type of relationship is called a **one-to-one correspondence**, as indicated in Figure 0.2.

Each of the four points in Figure 0.2 corresponds to a real number that can be expressed as the ratio of two integers. (Note that  $1.85 = \frac{37}{20}$  and  $-2.6 = -\frac{13}{5}$ .) We



Every point on the real line corresponds to a real number.



Every real number corresponds to a point on the real line.

FIGURE 0.2

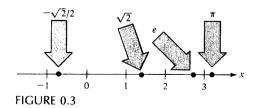
call such numbers **rational**. Rational numbers have either terminating or infinite repeating decimal representations.

Terminating decimals	Infinite repeating decimals
$\frac{2}{5} = 0.4$	$\frac{1}{3} = 0.333 \dots = 0.\overline{3}*$
$\frac{7}{8} = 0.875$	$\frac{12}{7} = 1.714285714285 \dots = 1.\overline{714285}$

Real numbers that are not rational are called **irrational**, and they cannot be represented as the ratio of two integers (or as terminating or infinite repeating decimals). To represent an irrational number, we usually resort to a decimal approximation. Some irrational numbers occur so frequently in applications that mathematicians have invented special symbols to represent them. For example, the symbols  $\sqrt{2}$ ,  $\pi$ , and e represent irrational numbers whose decimal approximations are as follows:

$$\sqrt{2} \approx 1.4142135623$$
 $\pi \approx 3.1415926535$ 
 $e \approx 2.7182818284$ 

Remark: We use  $\approx$  to mean approximately equal to. Remember that even though we cannot represent irrational numbers exactly as terminating decimals, they can be represented exactly by points on the real line, as shown in Figure 0.3.

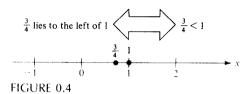


<sup>\*</sup>The bar indicates which digits repeat.

#### Order and Inequalities

One important property of the real numbers is that they are **ordered:** 0 is less than 1, -3 is less than -2.5,  $\pi$  is less than  $\frac{22}{7}$ , and so on. We can visualize this property on the real line by observing that a is less than b if and only if a lies to the left of b. Symbolically, we denote "a is less than b" by the inequality

For example, the inequality  $\frac{3}{4} < 1$  follows from the fact that  $\frac{3}{4}$  lies to the left of 1 on the real line, as shown in Figure 0.4.

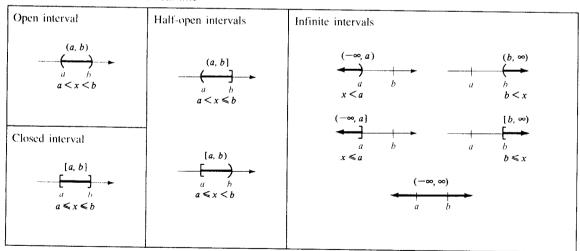


When three real numbers a, x, and b are ordered such that a < x and x < b, we say that x is **between** a and b and write

$$a < x < b$$
 x is between a and b

The set of *all* real numbers between a and b is called the **open interval** between a and b and is denoted by (a, b). An interval of the form (a, b) does not contain the "endpoints" a and b. Intervals that include their endpoints are called **closed** and are denoted by [a, b]. Intervals of the form [a, b) and (a, b] are called **half-open intervals**. Table 0.1 pictures the nine types of intervals on the real line.

TABLE 0.1 Intervals on the real line



Remark: Note that a square bracket is used to denote "less than or equal to" ( $\leq$ ). Furthermore, we use the symbols  $\infty$  and  $-\infty$  to denote positive and negative infinity. These

symbols do not denote real numbers; they merely enable us to describe unbounded conditions more concisely. For instance, the interval  $[b, \infty)$  is unbounded to the right since it includes *all* real numbers that are greater than b.

#### **EXAMPLE 1**

#### Intervals on the Real Line

Describe the intervals on the real line that correspond to the temperature ranges (in degrees Fahrenheit) for water in the following two states:

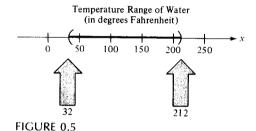
(a) liquid

(b) gas

#### **SOLUTION**

(a) Since water is in a liquid state at temperatures that are greater than 32° and less than 212°, we have the open interval (32, 212). If we let x represent the temperature of water, this interval consists of all x such that

as shown in Figure 0.5.



(b) Since water is in a gaseous state (steam) at temperatures that are greater than or equal to  $212^{\circ}$ , we have the interval [212,  $\infty$ ). This interval consists of all temperatures x such that

$$212 \le x$$

as shown in Figure 0.6.

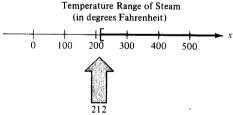


FIGURE 0.6

In calculus we are frequently required to "solve inequalities" involving variable expressions such as 3x - 4 < 5. We say x = a is a **solution** to an inequality if the inequality is true when a is substituted for x. The set of all values of x that satisfy

an inequality is called the **solution set** of the inequality. The following properties are useful for solving inequalities. (Similar properties are obtained if < is replaced by  $\le$  and > is replaced by  $\ge$ .)

### Properties of inequalities

- 1. Transitive Property: a < b and  $b < c \implies a < c$
- 2. Adding inequalities: a < b and  $c < d \Rightarrow a + c < b + d$
- 3. Multiplying by a (positive) constant:  $a < b \implies ac < bc$
- 4. Multiplying by a (negative) constant:  $a < b \implies ac > bc$
- 5. Subtracting a constant:  $a < b \implies a c < b c$
- 6. Adding a constant:  $a < b \implies a + c < b + c$

**Remark:** Note that we *reverse the inequality* when we multiply by a negative number. For example, if x < 3, then -4x > -12. This principle also applies to division. Thus, if -2x > 4, then x < -2.

#### **EXAMPLE 2**

#### Solving an Inequality

Find the solution set of the inequality 3x - 4 < 5.

#### SOLUTION

To find the solution set, we attempt to rewrite the inequality in such a way that x appears alone on either the right or the left side. To do this, we use appropriate properties of inequalities as follows. First, we can add 4 to both sides of the inequality to obtain

$$3x - 4 < 5$$
$$3x - 4 + 4 < 5 + 4$$
$$3x < 9$$

Now, multiplying both sides of this inequality by  $\frac{1}{3}$  gives us

$$\frac{1}{3}(3x) < \frac{1}{3}(9)$$

$$x < 3$$

Thus, the solution set is given by the interval  $(-\infty, 3)$ , as shown in Figure 0.7.

For 
$$x = 0$$
,  $3(0) - 4 = -4$ .  
For  $x = 2$ ,  $3(2) - 4 = 2$ .  
For  $x = 4$ ,  $3(4) - 4 = 8$ .

Solution set for 3x - 4 < 5.

FIGURE 0.7

Remark: In Example 2, all five inequalities listed as steps in the solution have the same solution set, and we call them equivalent.

Once you have solved an inequality, it is a good idea to check some x-values in your solution interval to see if they satisfy the original inequality. You might also check some values outside your solution interval to verify that they do not satisfy the inequality. For example, in Figure 0.7 we see that when x = 0 or x = 2 the inequality is satisfied, but when x = 4 the inequality is not satisfied.

#### **EXAMPLE 3**

#### Solving a Double Inequality

Find the interval corresponding to the set of x-values that satisfy the inequality

$$-3 \le 2 - 5x \le 12$$

Note that this double inequality means that  $-3 \le 2 - 5x$  and  $2 - 5x \le 12$ .

#### SOLUTION

Although two inequalities are involved in this problem, we can work with both simultaneously. We begin by subtracting 2 from all three expressions to obtain

$$-3 - 2 \le 2 - 5x - 2 \le 12 - 2$$
$$-5 \le -5x \le 10$$

Now, we divide all three expressions by -5 (making sure to reverse both inequalities) to obtain

$$\frac{-5}{-5} \ge \frac{-5x}{-5} \ge \frac{10}{-5}$$

$$1 \ge x \ge -2$$

Thus, the interval representing the solution is [-2, 1], as shown in Figure 0.8.

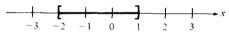


FIGURE 0.8

The inequalities in Examples 2 and 3 involve first-degree polynomials. To solve inequalities involving polynomials of higher degree, we use a result from algebra—a polynomial can change signs *only* at its zeros (the values that make the polynomial zero). Between two consecutive zeros a polynomial must be entirely positive or entirely negative. This means that when the real zeros of a polynomial are put in order, they divide the real line into **test intervals** in which the polynomial has no sign changes. For example, the polynomial

$$x^2 - x - 6 = (x - 3)(x + 2)$$

can change signs only at x = -2 and x = 3. This means that to determine the sign of the polynomial in the intervals  $(-\infty, -2)$ , (-2, 3), and  $(3, \infty)$ , we need to test only *one value* from each interval. This procedure is demonstrated in the next two examples.