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Theory of Elasticity

(Third Edition)

弹性理论（第3版）

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清华大学出版社

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THIRD EDITION

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北 京

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Theory of Elasticity (Third Edition)
EISBN: 0-07-085805-5

北京市版权局著作权合同登记号 图字：01-2003-8333

图书在版编目(CIP)数据

弹性理论 = Theory of Elasticity：第 3 版/（美）铁摩辛柯（Timoshenko, S. P.)，（美）古地尔(Goodier, J. N.)著. 一影印本. 一北京：清华大学出版社，2004
（国际著名力学图书. 影印版系列）
ISBN 7-302-07975-7

Ⅰ. 弹… Ⅱ. ①铁… ②古… Ⅲ. 弹性理论一高等学校一教材一英文 Ⅳ. O343

中国版本图书馆 CIP 数据核字（2004）第 003397 号

出 版 者：清华大学出版社
http://www.tup.com.cn
社总机：（010）6277 0175
地　　址：北京清华大学学研大厦
邮　　编：100084
客户服务：（010）6277 6969
责任编辑：杨　倩
封面设计：傅瑞学
印 刷 者：北京鑫海金澳胶印有限公司
装 订 者：北京市密云县京文制本装订厂
发 行 者：新华书店总店北京发行所
开　　本：165×230 **印张**：37.25
版　　次：2004 年 2 月第 1 版 2004 年 2 月第 1 次印刷
书　　号：ISBN 7-302-07975-7/O・343
印　　数：1~3000
定　　价：49.80 元

Preface to the Third Edition

In the revision of this book for a third edition, the primary intention and plan of the first edition have been preserved—to provide for engineers, in as simple a form as the subject allows, the essential fundamental knowledge of the theory of elasticity together with a compilation of solutions of special problems that are important in engineering practice and design. The numerous footnote references indicate how the several topics may be pursued further. Since these are now readily supplemented by means of *Applied Mechanics Reviews*, new footnotes have been added sparingly with this in mind. Small print again indicates sections that can be omitted from a first reading.

The whole text has been reexamined, and many minor improvements have been made throughout by elimination and rearrangement as well as addition.

The major additions reflect developments and extensions of interest and practical applicability that have occurred since the appearance of the second edition in 1951. End effects and eigensolutions associated with the principle of Saint-Venant are treated in Chapters 3 and 4. In view of the rapid growth of the applications of dislocational elastic solutions in materials science, these discontinuous displacement solutions have been given more explicit treatment as edge dislocations and screw dislocations in Chapters 4, 8, 9, and 12. An introduction to the moiré method with a practical illustration has been added to Chapter 5. The treatment of strain energy and variational principles has been recast in three-dimensional form and embodied in Chapter 8, which now provides a basis for new sections on thermoelasticity in Chapter 13. The discussion of the use of complex potentials for two-dimensional problems has been extended by a group of new articles based on the now well-known methods of Muskhelishvili. Moreover, the approach is somewhat different, in that advantage has been taken of solutions previously developed in order to deal with analytic functions only. Further solutions for the elliptic hole, important in current fracture mechanics (cracks), are given explicit treatment. The discussion of axisymmetric stress in Chapter 12 has been simplified; and new sections have been added that replace the approximate analysis by a more exact one for the cut ring, as one turn of a helical spring. In view of its greatly increased applications, as in nuclear energy equipment, Chapter 13, on thermal stress, has been extended by inclusion of a thermoelastic reciprocal theorem and several useful results obtained from it; and by an introduction to thermal stress concentrations due to disturbance of heat flow by cavities and inclusions has also been added. In addition, treatment of two-dimensional problems has been supplemented by the two final articles, the last bringing the two-dimensional thermoelastic problems into connection with the complex potentials and Muskhelishvili procedures of Chapter 6. In Chapter 14, on wave propagation, a rearrangement gives prominence to the basic three-dimensional theory. A solution for explosive pressure in a spherical cavity has been added. The Appendix on numerical finite difference methods includes an example of the use of a digital computer to cope with a large number of unknowns.

Some of these changes offer simplifications of analysis arrived at in the experience of giving courses at Stanford University over the past twenty years. Many valuable suggestions, corrections, and even completely formulated problems with solutions, have come from numerous students and correspondents, to whom a blanket but most cordial acknowledgment is both unavoidable and inadequate.

Almost all the "Problems" are from examinations set and given at Stanford University. The reader may see roughly from these what parts of the book correspond to a course sequence occupying somewhat less than three hours per week for the academic year.

J. N. Goodier

Preface to the Second Edition

The many developments and clarifications in the theory of elasticity and its applications which have occurred since the first edition was written are reflected in numerous additions and emendations in the present edition. The arrangement of the book remains the same for the most part.

The treatments of the photoelastic method, two-dimensional problems in curvilinear coordinates, and thermal stress have been rewritten and enlarged into separate new chapters which present many methods and solutions not given in the former edition. An appendix on the method of finite differences and its applications, including the relaxation method, has been added. New articles and paragraphs incorporated in the other chapters deal with the theory of the strain gauge rosette, gravity stresses, Saint-Venant's principle, the components of rotation, the reciprocal theorem, general solutions, the approximate character of the plane stress solutions, center of twist and center of shear, torsional stress concentration at fillets, the approximate treatment of slender (*e.g.*, solid airfoil) sections in torsion and bending, and the circular cylinder with a band of pressure.

Problems for the student have been added covering the text as far as the end of the chapter on torsion.

It is a pleasure to make grateful acknowledgment of the many helpful suggestions which have been contributed by readers of the book.

S. P. Timoshenko

J. N. Goodier

Preface to the First Edition

During recent years the theory of elasticity has found considerable application in the solution of engineering problems. There are many cases in which the elementary methods of strength of materials are inadequate to furnish satisfactory information regarding stress distribution in engineering structures, and recourse must be made to the more powerful methods of the theory of elasticity. The elementary theory is insufficient to give information regarding local stresses near the loads and near the supports of beams. It fails also in the cases when the stress distribution in bodies, all the dimensions of which are of the same order, has to be investigated. The stresses in rollers and in balls of bearings can be found only by using the methods of the theory of elasticity. The elementary theory gives no means of investigating stresses in regions of sharp variation in cross section of beams or shafts. It is known that at reentrant corners a high stress concentration occurs and as a result of this cracks are likely to start at such corners, especially if the structure is submitted to a reversal of stresses. The majority of fractures of machine parts in service can be attributed to such cracks.

During recent years considerable progress has been made in solving such practically important problems. In cases where a rigorous solution cannot be readily obtained, approximate methods have been developed. In some cases solutions have been obtained by using experimental methods. As an example of this the photoelastic method of solving two-dimensional problems of elasticity may be mentioned. The photoelastic equipment may be found now at universities and also in many industrial research laboratories. The results of photoelastic experiments have proved especially useful in studying various cases of stress concentration at points of sharp variation of cross-sectional dimensions and at sharp fillets of reentrant corners. Without any doubt these results have considerably influenced the modern design of machine parts and helped in many cases to improve the construction by eliminating weak spots from which cracks may start.

Another example of the successful application of experiments in the solution of elasticity problems is the soap-film method for determining stresses in torsion and bending of prismatical bars. The difficult problems of the solution of partial differential equations with given boundary conditions are replaced in this case by measurements of slopes and deflections of a properly stretched and loaded soap film. The experiments show that in this way not only a visual picture of the stress distribution but also the necessary information regarding magnitude of stresses can be obtained with an accuracy sufficient for practical application.

Again, the electrical analogy which gives a means of investigating torsional stresses in shafts of variable diameter at the fillets and grooves is interesting. The analogy between the problem of bending of plates and the two-dimensional problem of elasticity has also been successfully applied in the solution of important engineering problems.

In the preparation of this book the intention was to give to engineers, in a simple form, the necessary fundamental knowledge of the theory of elasticity. It was also intended to bring together solutions of special problems which may be of practical importance and to describe approximate and experimental methods of the solution of elasticity problems.

Having in mind practical applications of the theory of elasticity, matters of more theoretical interest and those which have not at present any direct applications in engineering have been omitted in favor of the discussion of specific cases. Only by studying such cases with all the details and by comparing the results of exact investigations with the approximate solutions usually given in the elementary books on strength of materials can a designer acquire a thorough understanding of stress distribution in engineering structures, and learn to use, to his advantage, the more rigorous methods of stress analysis.

In the discussion of special problems in most cases the method of direct determination of stresses and the use of the compatibility equations in terms of stress components has been applied. This method is more familiar to engineers who are usually interested in the magnitude of stresses. By a suitable introduction of stress functions this method is also often simpler than that in which equations of equilibrium in terms of displacements are used.

In many cases the energy method of solution of elasticity problems has been used. In this way the integration of differential equations is replaced by the investigation of minimum conditions of certain integrals. Using Ritz's method this problem of variational calculus is reduced to a simple problem of finding a minimum of a function. In this manner useful approximate solutions can be obtained in many practically important cases.

To simplify the presentation, the book begins with the discussion of two-dimensional problems and only later, when the reader has familiarized himself with the various methods used in the solution of problems of the theory of elasticity, are three-dimensional problems discussed. The portions of the book that, although of practical importance, are such that they can be omitted during the first reading are put in small type. The reader may return to the study of such problems after finishing with the most essential portions of the book.

The mathematical derivations are put in an elementary form and usually do not require more mathematical knowledge than is given in engineering schools. In the cases of more complicated problems all necessary explanations and intermediate calculations are given so that the reader can follow without difficulty through all the derivations. Only in a few cases are final results given without complete derivations. Then the necessary references to the papers in which the derivations can be found are always given.

In numerous footnotes references to papers and books on the theory of elasticity which may be of practical importance are given. These references may be of interest to engineers who wish to study some special problems in more detail. They give also a picture of the modern development of the theory of elasticity and may be of some use to graduate students who are planning to take their work in this field.

In the preparation of the book the contents of a previous book ("Theory of Elasticity," vol. I, St. Petersburg, Russia, 1914) on the same subject, which represented a course of lectures on the theory of elasticity given in several Russian engineering schools, were used to a large extent.

The author was assisted in his work by Dr. L. H. Donnell and Dr. J. N. Goodier, who read over the complete manuscript and to whom he is indebted for many corrections and suggestions. The author takes this opportunity to thank also Prof. G. H. MacCullough, Dr. E. E. Weibel, Prof. M. Sadowsky, and Mr. D. H. Young, who assisted in the final preparation of the book by reading some portions of the manuscript. He is indebted also to Mr. L. S. Veenstra for the preparation of drawings and to Mrs. E. D. Webster for the typing of the manuscript.

S. P. Timoshenko

Notation

x, y, z Rectangular coordinates.
r, θ Polar coordinates.
ξ, η Orthogonal curvilinear coordinates; sometimes rectangular coordinates.
R, ψ, θ Spherical coordinates.
N Outward normal to the surface of a body.
l, m, n Direction cosines of the outward normal.
A Cross-sectional area.
I_x, I_y Moments of inertia of a cross section with respect to x and y axes.
I_p Polar moment of inertia of a cross section.
g Gravitational acceleration.
ρ Density.
q Intensity of a continuously distributed load.
p Pressure.
X, Y, Z Components of a body force per unit volume.
$\bar{X}, \bar{Y}, \bar{Z}$ Components of a distributed surface force per unit area.
M Bending moment.
M_t Torque.
$\sigma_x, \sigma_y, \sigma_z$ Normal components of stress parallel to x, y, and z axes.
σ_n Normal component of stress parallel to n.
σ_r, σ_θ Radial and tangential normal stresses in polar coordinates.
σ_ξ, σ_η Normal stress components in curvilinear coordinates.
$\sigma_r, \sigma_\theta, \sigma_z$ Normal stress components in cylindrical coordinates.
$\Theta = \sigma_x + \sigma_y + \sigma_z = \sigma_r + \sigma_\theta + \sigma_z$.
τ Shearing stress.
$\tau_{xy}, \tau_{xz}, \tau_{yz}$ Shearing-stress components in rectangular coordinates.
$\tau_{r\theta}$ Shearing stress in polar coordinates.
$\tau_{\xi\eta}$ Shearing stress in curvilinear coordinates.
$\tau_{r\theta}, \tau_{\theta z}, \tau_{rz}$ Shearing-stress components in cylindrical coordinates.
S Total stress on a plane. Surface tension.
u, v, w Components of displacements.
ϵ Unit elongation.
$\epsilon_x, \epsilon_y, \epsilon_z$ Unit elongations in x, y, and z directions.
$\epsilon_r, \epsilon_\theta$ Radial and tangential unit elongations in polar coordinates.

$e = \epsilon_x + \epsilon_y + \epsilon_z$	Volume expansion.
γ	Unit shear.
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	Shearing-strain components in rectangular coordinates.
$\gamma_{r\theta}, \gamma_{\theta z}, \gamma_{rz}$	Shearing-strain components in cylindrical coordinates.
E	Modulus of elasticity in tension and compression.
G	Modulus of elasticity in shear. Modulus of rigidity.
ν	Poisson's ratio.
$\mu = G, \lambda = \dfrac{\nu E}{(1+\nu)(1-2\nu)}$	Lamé's constants.
ϕ	Stress function.
$\phi(z), \psi(z), \chi(z)$	Complex potentials; functions of the complex variable $z = x + iy$.
$\bar{z}$	The conjugate complex variable $x - iy$.
C	Torsional rigidity.
θ	Angle of twist per unit length.
$F = 2G\theta$	Used in torsional problems.
V	Strain energy.
V_0	Strain energy per unit volume.
t	Time.
T	Certain interval of time. Temperature.
α	Coefficient of thermal expansion. Angle.
c_1, c_2	Wave velocities.

Contents