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VOLUME 2

ELECTRONIC METHODS

Volume 2

Electronic Methods

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FOREWORD

At the time of conceiving our series of volumes on the methods used by the experimental physicist, a somewhat arbitrary division of physics was adopted for the purpose of organizing the material. In absence of adverse criticism I assume that the subject division is acceptable to the users of these volumes, and it is in this spirit that I present our latest volume on the electronic methods used by the experimental physicist.

Preparation of this volume was not easy. In a field where the physicist is subjected to constant change and progress, the volume editor's task was the ungrateful one: to separate the ephemeral methods from the ones having lasting value and to present these in the hope that their judgment and that of the authors is the correct one. In many conferences we discussed details and I hope that the result will be satisfactory. I would like to use this opportunity to thank again Professors Bleuler and Haxby, as well as all the authors who were willing to spare their time for preparing this volume.

This may be the right time to announce two important changes in the organization of this series. One of these concerns Volume 4, Atomic and Electron Physics. The original intention of Professors Hughes and Schultz, editors of Volume 4, was to present it in two parts, Volumes 4A and 4B. What was to be Volume 4A is nearing completion and we may call it Volume 4. The second part, considerably enlarged, may be renumbered and Professors B. Bederson and W. L. Fite have taken over its editorship. They may co-opt a third volume editor.

The other important change is the organization of a problem volume, mostly oriented toward the graduate student, and already briefly mentioned in the Foreword to Volume 5A. Contrary to the opinion of some of my theoretical friends, there exist problems in experimental physics, and Professors W. Hornyak and E. A. Stern have taken over the task of organizing it.

It remains a pleasant task to express, to all those who contributed to the completion of this volume, my warmest thanks.

L. MARTON

Washington, D. C.
November, 1963

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1. EVALUATION OF MEASUREMENT*

1.1. General Rules

In a concise expression of the results of the measurement of a physical quantity, three pieces of information should be given: a number, a numerical statement of reliability, and an appropriate set of units. The number is generally an estimate expressed in a finite set of digits (the exceptions are numbers which are exact by arbitrary definition, or mathematical constants such as the base of natural logarithms) reflecting the limited accuracy of physical measurement. The statement of reliability is usually written as plus or minus one, or at *most* two digits in units of the last digit of the number, together with a sufficient explanation to allow interpretation. In particular, one should state how many measurements were employed in the determination of the number and of its reliability. As will be seen below, this is of great value in the critical comparison of the results of different experiments, and in their combination with results of previous work. The number of digits that can be read is indicated by the smallest scale division, the *least count* of an instrument. Usually, one additional digit can be estimated between scale divisions. No more—and no less—digits should be recorded than can be read reproducibly.

To remove ambiguities, a standard form may be used for the recording of data: the decimal point is put just after the first nonzero digit, and the number is multiplied by the appropriate power of ten. Every digit is then understood to be significant. The statement of reliability, or the statement of the magnitude of error, is automatically indicated by the number of significant digits. The final result will generally have one more significant digit than the individual readings. This procedure implies that one should not round off readings. Any round-off increases the error. In the course of computation, round-off may be inevitable. A brief discussion of errors so introduced, with further references, is given in Chapter 1.7.

For the estimation of the best value of the desired quantity and of the significance of the result statistical techniques are used. The terms "best" and "significant" should be understood in a technical sense: e.g., "best" and "significant" according to some statistical criterion. The criteria applied depend on assumptions which may or may not be true: attention should be paid to their validity. In the following, only a prescription of the techniques can be given. For this reason, a word of warning is in order: these techniques, properly used, can improve the understanding of the

* Part 1 is by **Sidney Reed.**

results and the judgment of their worth—but they are not a substitute for thought.[1] It should be emphasized that work in certain fields, e.g., cosmic ray or high energy physics, requires more complete attention to statistical techniques in the planning and interpretation of experiments than can be discussed here.[2,3]

1.2. Errors

1.2.1. Systematic Errors, Accuracy

Statements about reliability of a measurement require assessment of the accuracy and of the precision of the work. Lack of accuracy is considered to be due mainly to what long usage has termed *systematic errors.*

In general, systematic errors are definite functions of experimental method, instruments, or environmental conditions. If detected they can usually be corrected for. Sometimes a single correction will be adequate for the entire work and can be applied at the end. Constant, or slowly varying systematic errors are hard to detect. The crucial test is the comparison of measurements of the same quantity obtained from different experiments, using different principles.

1.2.2. Accidental Errors, Precision

Precision implies close reproducibility of the results of successive individual measurements. It is assumed that, in general, there is a variation from measurement to measurement. This scatter of data is usually considered due to accidental errors; it is imagined that the experiment is aimed at a constant quantity, superimposed on which there is a random sum of small effects independent of each other and of the quantity itself which are responsible for the variation of the results. Absence of variation is not necessarily an indication of precision; it may be due simply to an excessively large least count of the instrument used.

[1] For further references, see E. B. Wilson, Jr., "An Introduction to Scientific Research." McGraw-Hill, New York, 1952; H. Cramer, "Elements of the Theory of Probability and Its Applications." Wiley, New York, 1955; J. Cameron, *in* "Fundamental Formulas of Physics" (D. Menzel, ed.), Chapter 2. Prentice-Hall, New York, 1955; L. Parratt, *Probability and Experimental Errors in Science*, Wiley, N.Y., 1961.

[2] See, for example, L. Jánossy, "Cosmic Rays." Oxford Univ. Press, London and New York, 1953.

[3] M. Annis, W. Cheston, and H. Primakoff, *Revs. Modern Phys.* **25**, 818 (1953); J. Orear, Univ. of California Radiation Lab. Rept. UCRL-8417 (1958).

1.3. Statistical Methods

To analyze accidental errors, the actual data are imagined to be a random selection, one for each measurement, of values from a large reference distribution which could be generated by infinite repetition of the experiment. In statistical terms, this is a finite sample from a "parent distribution" (p.d.). For reasons of mathematical convenience, it is usual to assume that the p.d. can be approximated satisfactorily by an analytic function (p.d.f.) having two or three parameters. A finite data sample permits at most the assignment of odds to the values of the p.d.f. parameters which represent the best value and the significance of the measurement.

In most cases, a reasonable, explicit assumption of a definite form of the p.d.f. is desirable. Which form should be taken depends on a preliminary assessment of the probabilistic features of the experiment. If the errors are accidental in the sense described in Chapter 1.2 above, a normal (see Section 1.3.1) distribution function (n.d.f.) is appropriate. If the experiment is directly concerned with probabilistic phenomena, e.g., counting experiments in nuclear physics, the Poisson or some other discrete probability distribution function may be chosen.[1,2,3]

For the problem of estimation of the best value alone one does not need to assume any particular p.d.f.; a systematic estimation using least squares can be made.[4] A sharp quantitative statement of the statistical significance of a difference between two "best" estimates of the same quantity cannot be made, however, without assuming a definite form for the p.d.f.

1.3.1. Mean Value and Variance

The fraction of readings $dN(x)/N$ drawn from the p.d.f. $f(x)$ lying in the range between x and $x + dx$ is

$$dN(x)/N = f(x)\,dx. \tag{1.3.1}$$

The function $f(x)$ is normalized: $\int f(x)\,dx = 1$. The average of any function $g(x)$, denoted by $<g(x)>$, is defined by $<g(x)> = \int g(x)f(x)\,dx$. The range of integration may, for mathematical convenience, extend in both directions to infinity. Of special importance are the average of x called the *mean*

$$<x> = \int xf(x)\,dx \tag{1.3.2}$$

[4] E. R. Cohen, *Revs. Modern Phys.* **25**, 709 (1953).

and the average of $(x - \langle x \rangle)^2$, called the *dispersion* or *variance* of x

$$\sigma^2(x) = \int (x - \langle x \rangle)^2 f(x)\, dx. \tag{1.3.3}$$

The square root of the variance, $\sigma(x)$ is called *standard deviation* or sometimes *standard error*. It is a measure of the spread of the data and thus of the precision. An important example of a p.d.f., often assumed to apply to accidental errors, is the Gaussian or *normal distribution* (n.d.f.):

$$f_1(x) = [\sqrt{2\pi\sigma^2(x)}]^{-1} \exp[-(x - \langle x \rangle)^2/2\sigma^2(x)] \tag{1.3.4}$$

characterized by two parameters, the mean $\langle x \rangle$ and the variance $\sigma^2(x)$. N measurements x_i allow the formation of the *sample mean*

$$\bar{x} = N^{-1} \sum_{i=1}^{N} x_i \tag{1.3.5}$$

and the *sample variance*

$$s^2(x) = (N - 1)^{-1} \sum_{i=1}^{N} (x_i - \bar{x})^2. \tag{1.3.6}$$

The mean has the property of being the value of a parameter a which minimizes $\Sigma_{i=1}^{N} (x_i - a)^2$. On the grounds of consistency, one expects that in some sense $\bar{x}$ converges to $\langle x \rangle$ and s to σ as $N \to \infty$.* For computation, it is useful to subtract a constant A of the order of size of x_i, so that

$$\bar{x} - A = N^{-1} \sum_{i=1}^{N} (x_i - A) \tag{1.3.7}$$

and

$$s^2(x) = (N - 1)^{-1} \left[\sum_{i=1}^{N} (x_i - A)^2 - N(\bar{x} - A)^2 \right]. \tag{1.3.8}$$

1.3.2. Statistical Control of Measurements

The use of any p.d. implies that the data may be regarded as drawn at random from it. There are statistical tests for this,[1] but in the case of data scatter because of accidental errors, a rough "control chart" can assist in detecting systematic departures which are functions of time. Such a chart may be made by plotting, on the abscissa, the order (in time) of the reading, and on the ordinate, the reading itself. If there is previous information on the scatter of the data using the same instrument under similar conditions, so that $\sigma(x)$ is known, one can, at least tentatively, draw lines on the chart at $\bar{x} \pm 3\sigma$ which should, if the data

* This is so in the technical sense of convergence in probability; see, e.g., Cramer, reference 1.

are in control, bracket practically all the points. It is quite valuable to have such a chart associated with a precise instrument.

If no previous information is available, one should take a number of points, draw lines at $\bar{x} \pm 3s$ and continue for a few more readings in order to see whether the additional data fall between these lines. If it appears that randomness is a fair assumption, one can use the function "chi-square"[5] to test the overall fit of an assumed p.d.f. The chi square, χ^2, function can be defined as

$$\chi^2 = \text{sum of } \left\{ \frac{(\text{observed values} - \text{values expected from p.d.f.})^2}{\text{p.d.f. variance}} \right\}$$

and is tabulated as function of the number of degrees of freedom. Here the number of degrees of freedom equals the number of terms in the sum minus the quantity: one plus the number of p.d.f. parameters which must be estimated from the data. In the case of a n.d.f., this is the number of terms minus three. It is generally necessary to group the observed data and the corresponding values from the p.d.f. into cells.[6] For moderate numbers of readings, say 20 or so, χ^2 will only show *marked* discrepancies between the data and the proposed p.d.f. The χ^2 tables give the probability that tabulated values of χ^2 would be exceeded by those computed from a random sample from the assumed p.d.f.

1.4. Direct Measurements

It is useful to distinguish between direct measurements, such as can be made of length, time, or electrical current; and indirect measurements, in which the quantity in question can be calculated from measurement of other quantities. In the latter case the law of connection between the quantities measured and sought may also be in question. In such a case, one has first to decide whether the proposed relation holds for any values of the quantities (establishment of the law of connection), and then to make as good an estimate as possible of the quantity desired.[7] In the case

[5] See any standard statistical tables, e.g., R. A. Fisher and G. Yates, "Statistical Tables," Oliver & Boyd, Edinburgh and London, 1953; C. D. Hodgman, ed., "Handbook of Chemistry and Physics." Chemical Rubber Publ., Cleveland. (New editions of the latter volume are published frequently.)

[6] W. G. Cochran, *Ann Math. Statistics* **23**, 315 (1952); also Parratt, reference 1.

[7] See Wilson or Parratt, reference 1; Annis *et al.*, reference 3; and Cohen *et al.*, reference 12.

of direct measurements only the latter problem needs to be solved. This simple situation will be discussed first.[8] There are several cases, depending on what information is available at the start.

1.4.1. Errors of Direct Measurements

If one has information at the start of the experiment regarding the variance of readings of the measuring instrument under similar conditions, the following procedure can be employed: One can draw up a control chart, using the previous $\sigma(x)$ together with the mean $\bar{x}$ of a short preliminary run. If subsequent readings appear to be in statistical control, i.e., if the points fall between the lines at $\bar{x} \pm 3\sigma$, one can terminate the process at a definite number of readings which depends on the precision desired. One can then say that the most likely value of $<x>$ is given by the mean $\bar{x}$, and that the reliability of this estimate is such that the probability is one-half that the interval $\bar{x} - 0.67\sigma/\sqrt{N} \leq x \leq \bar{x} + 0.67\sigma/\sqrt{N}$ contains $<x>$. The precision increases with N in the sense that the interval having a definite probability of containing $<x>$ narrows proportional to $N^{-1/2}$.* In this case, the interval length is sharply defined (for fixed N and probability) and if the results are quoted as

$$\bar{x} \pm \epsilon; \qquad N \text{ measurements}$$

the meaning of this statement is as stated above.

Frequently the only information available at the start is that provided by the data itself. If the data seem to be in statistical control, one can make statements about the probability of bracketing the p.d.f. mean which differ from those possible when $\sigma(x)$ is known. The levels of probability now depend on the number of data points in the sample and the intervals bracketing $<x>$ can now vary in length from sample to sample. The type of statement that can be made for this case is that the best estimate of $<x>$ is $\bar{x}$ and that the probability is $1 - P$ that the interval

$$\bar{x} - \frac{t(P,f)s(x)}{\sqrt{N}} \leq x \leq \bar{x} + \frac{t(P,f)s(x)}{\sqrt{N}} \tag{1.4.1}$$

will include, on the average of many samples of size N, the p.d.f. mean $<x>$. The function $t(P,f)$ is called Student's or Fisher's t and is tabulated

* An interval of this type is called a *confidence interval*. It should be distinguished from a *tolerance interval* which will contain a definite fraction of the population, e.g., a single observation. See, e.g., reference 1; or N. Arley and K. Buch, "Probability and Statistics," p. 168. Wiley, New York, 1950.

[8] A valuable, readable discussion is given by W. E. Deming and R. T. Birge, *Revs. Modern Phys.* **6**, 119 (1934).

giving probability levels P that a value of t should be found at least as large as that tabulated, as function of the number of degrees of freedom f, which in this case is $N - 1$.[5,9]

Equation (1.4.1) indicates that the precision increases, i.e., that the interval narrows as N increases. From this it might be inferred that with repetition of the experiment the precision of the mean would improve indefinitely. That would be so under ideal circumstances but in actual experiments the probability of occurrence of systematic errors increases with the length of the investigation.

Another limitation is due to the least count of the instruments used. All the statements above depend on the approximation of a continuous parent distribution function. This approximation improves with the ratio of σ to the least count w. ($w \leq \frac{1}{4}\sigma$ is usually considered reasonable; see Eisenhart *et al.*[10]) If w is large, and the readings come out constant, statistical methods cannot be applied. If there are random errors in the data, one can improve the precision by repetition, but a very large number of measurements are required if $\sigma \approx w$.*

In order to be fairly sure the accidental errors are in a state of statistical control, a certain number of measurements is necessary (how many depends largely on the investigation, but the number must be fairly large). Once such a state has been reached, further measurements can be added to refine the precision of the mean, but on the hypothesis that each added measurement can be joined with the preceding ones. For this reason a continuously running control chart is very useful.

1.4.2. Rejection of Data

A closely related problem is the rejection of data. Often some criterion for doing so is given, based on analysis of the data alone. This is not recommended. A measured point should not be excluded on statistical grounds alone. If a control chart as suggested in the previous paragraph is used, and if a point or set of points seem out of control, the experimental conditions should be carefully re-examined, and if no assignable causes can be found, the point should be remeasured after settings have been remade, etc. This presumes that the analysis is made in the course of the

* If the least count is large, attention must be paid to providing a random character for the data. For example, an improvement in the precision of a measurement of length can be obtained by putting the ends of an object at random between the least count marks of a scale rather than always setting one end to coincide with one of the markers.

[9] R. A. Fisher and F. Yates, "Statistical Tables." Oliver & Boyd, Edinburgh and London, 1953.

[10] C. Eisenhart, M. W. Hastay, and W. A. Wallis, eds., "Techniques of Statistical Analysis." McGraw-Hill, New York, 1947.