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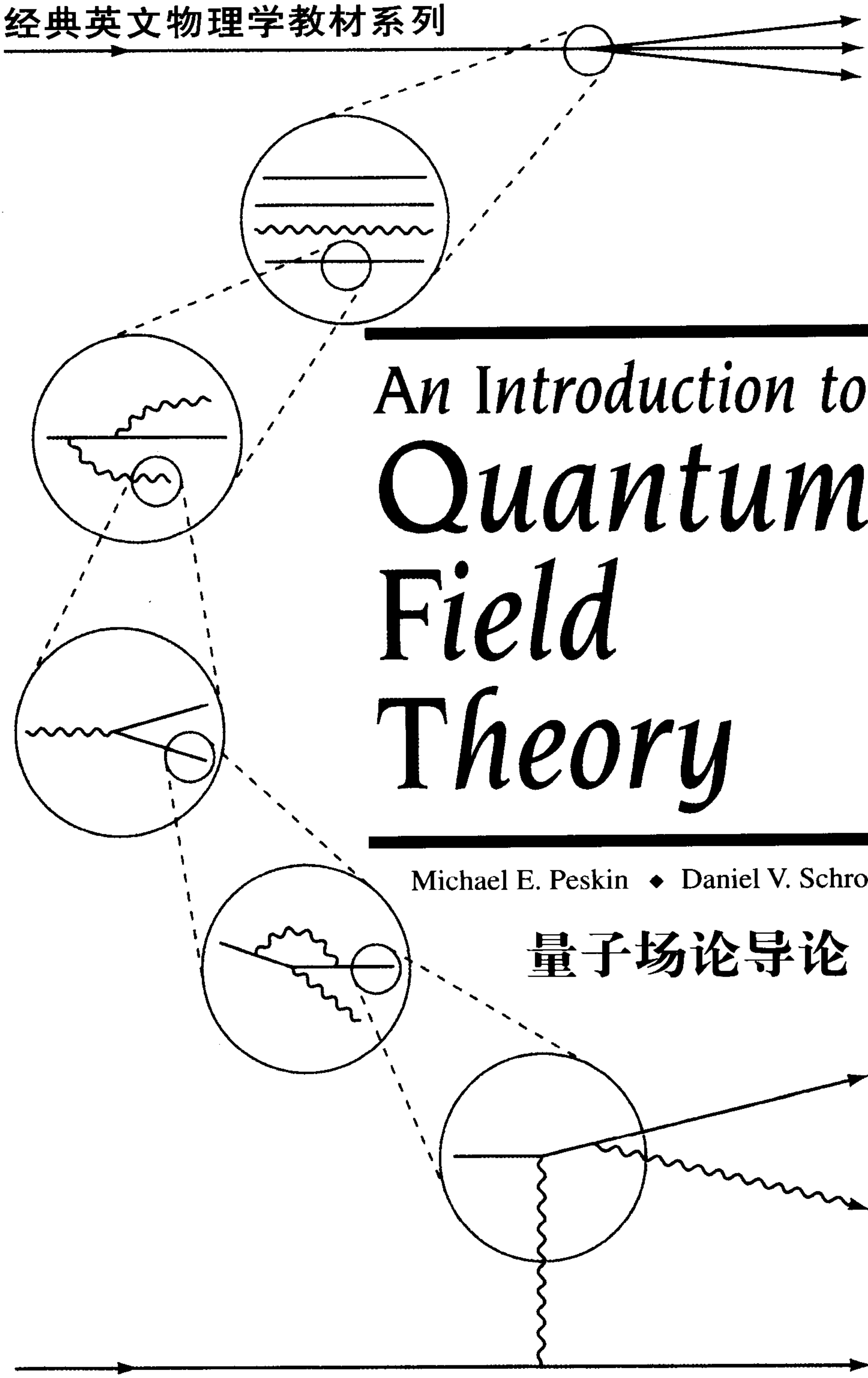

An Introduction to Quantum Field Theory

Michael E. Peskin ◆ Daniel V. Schroeder

量子场论导论

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这本书对于我的书是如此厉害的竞争对手，我不能不担心我们今后的版税了。

I. I. 德雷尔(Bjorken)，斯坦福直线加速器中心教授

我用佩斯金（Peskin）和斯科罗德（Schroeder）的书讲了几次研究生的课，它给学生提供了现代场论几乎所有的工具。它是唯一一本包括对于重整化和重整化群完全现代的Wilson处理的场论教材。学生觉得它很好地满足从事QCD和电弱理论研究的需要。

M. 戴恩（Dine），加利福尼亚大学圣克鲁兹分校教授

佩斯金和斯科罗德完全采用现代的观点写了一本导论性的场论教科书。该书难易程度适中，内容取舍合理，对于场论的工具和概念给出了一种适合于教学的介绍。它对于凝聚态、宇宙学和粒子物理专业的学生都是有用的。

J.哈维（Harvey），芝加哥大学教授

本书是一部曾被美国许多大学选用的研究生教材，并受到普遍好评。与同类教材相比，该书的内容非常丰富。全书分三个部分。第一部分集中介绍场的正则量子化方法、量子电动力学和费曼图。第三部分是关于非阿贝尔规范场的详细讨论。而第二部分是在这两个部分之间搭建的一个桥梁，着重阐述泛函方法、重整化和重整化群以及临界指数等问题。作者从教学角度对于这三个部分的安排提出了详细的建议。鉴于作者的背景，这三个部分的全部内容是针对粒子物理专业研究生的需要而编排的。对于凝聚态和实验物理专业的研究生，作者建议可以把后两部分合并而舍弃用星号标记的章节即可。

作为一本教科书，作者很注重使其易读易懂和富于启发性，公式的推导和例题的分析尽可能地详尽。每一章都给出了几个习题，它们的总量虽然不大，但每个题目都经过了精心挑选，使其对深入理解课程内容和应用其解决实际问题有实质性的帮助。

我们相信，这本书不仅对于量子场论的教学（特别是双语教学）很有实际的应用价值，对于相关专业的科研人员也是一本很好的参考书。

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影印版前言

量子场论是物理学专业研究生的基础理论课，对于粒子物理与核物理、原子和分子物理、凝聚态物理和天体物理的学生通常都列为必修课程。

量子场论集 Maxwell 的经典电磁场理论、Einstein 的相对论和量子力学之大成，在上世纪的 40 年代末和 50 年代初，成功地解决了带电粒子和光子的相互作用中遇到的一系列困难问题，这主要归功于 Feynman、 Schwinger 和朝永的卓越贡献而取得了突破性的进展。从那时起，量子电动力学成了量子场论的主体，而 Feynman 图方法则成为微扰计算的不可缺少的工具。此后大约过了 20 年左右，在非阿贝尔规范场的基础上，建立了电磁相互作用和弱作用的统一理论，发展了描写夸克之间强相互作用的量子色动力学，由它们构成了基本粒子的标准模型。利用这一模型，近 20 年来高能物理的理论与实验研究取得了难以计数的丰硕成果。与此同时，Wilson 提出了重整化群理论，把量子场论方法与统计力学更密切地联系起来。所有这一切都使量子场论从概念框架到应用范围再一次取得了突破性的进展，成为揭示微观世界奥秘，搭建从微观到宏观现象桥梁的强大的理论工具。

在我国，从上世纪 50 年代末开始开设量子场论课程，引入了一些前苏联的著名教材作为主要参考书。1960 年朱洪元先生的《量子场论》一书问世，对于其后相当长的一段时间国内量子场论的教学有着重要的影响。70 年代末和 80 年代初，西方出版的许多优秀教材开始进入我国，其中的量子场论教材中影响较大的是 Bjorken 和 Drell 的“Relativistic Quantum Field Theory”及 Itzykson 和 Zuber 合著的“Quantum Field Theory”（这两部书均有中文译本）。其后，我们还先后见到了一些由国内外的著名专家撰写的以粒子物理应用为主要对象的规范场理论和高等量子场论的专著。

现在我们向读者推荐的是由 Peskin 和 Schroeder 合著的一本量子场论教科书。它是 1995 年由 Westview 出版社出版的。曾被美国许多大学选用为一个学年的研究生课程的教材，受到普遍好评。作者十分推崇 Bjorken 和 Drell 的那部名著，希望这部新书能够承袭他们的风格，把量子场论抽象的形式、直观的解释和实际的应用理想地结合在一起，同时反映出从上世纪 60 年代起

量子场论取得的新进展，给读者提供像那部名著一样的深度和容易接受的全面的阐述。

与同类教材相比，该书的内容非常丰富。全书分三个部分。第一部分集中介绍场的正则量子化方法、量子电动力学和费曼图。第三部分是关于非阿贝尔规范场的详细讨论。而第二部分是在这两个部分之间搭建的一个桥梁，着重阐述泛函方法、重整化和重整化群以及临界指数等问题。作者从教学角度对于这三个部分的安排提出了详细的建议。鉴于作者的背景，这三个部分的全部内容是针对粒子物理专业研究生的需要而编排的。对于凝聚态和实验物理专业的研究生，作者建议可以把后两部分合并而舍弃许多用星号标记的章节即可。通常，国内相应专业研究生的量子场论课安排在两个学期（分别称为量子场论 I 和 II，或者称为量子场论和规范场理论），基本上相当于上述的后一种情况。

本书的定位是一本量子场论的导论，因此不要求读者有这一课题的知识基础。但是假定了学生们学过研究生水平的经典力学，经典电动力学和量子力学，而第二部分要求有统计力学的相关知识。尽管并不要求学生掌握所有这些课程的全部知识，但至少应该熟悉动力学的拉格朗日和哈密顿形式、电磁学的相对论协变形式以及非相对论量子力学简谐振子的量子化。此外，关于转动群以及复平面上的回路积分等数学知识也是必须要求的。

作为一本教科书，作者很注重使其易读易懂和富于启发性，公式的推导和例题的分析尽可能地详尽。每一章都给出了几个习题，它们的总量虽然不大，但每个题目都经过了精心挑选，使其对深入理解课程内容和应用其解决实际问题有实质性的帮助。考虑到量子场论的很多问题解决起来难度较大，需要很长的推导过程，相当于一个小的科研课题而不适合作为家庭作业，在每一个部分的结尾作者都给出了一个示例，演示如何把这样的问题进行适当地分割，并对每一小段给出解题的提示和指南。

我们相信，这本书不仅对于量子场论的教学（特别是双语教学）很有实际的应用价值，对于相关专业的科研人员也是一本很好的参考书。

中国科学院研究生院物理科学学院教授　丁亦兵

20006 年 5 月

Preface

Quantum field theory is a set of ideas and tools that combines three of the major themes of modern physics: the quantum theory, the field concept, and the principle of relativity. Today, most working physicists need to know some quantum field theory, and many others are curious about it. The theory underlies modern elementary particle physics, and supplies essential tools to nuclear physics, atomic physics, condensed matter physics, and astrophysics. In addition, quantum field theory has led to new bridges between physics and mathematics.

One might think that a subject of such power and widespread application would be complex and difficult. In fact, the central concepts and techniques of quantum field theory are quite simple and intuitive. This is especially true of the many pictorial tools (Feynman diagrams, renormalization group flows, and spaces of symmetry transformations) that are routinely used by quantum field theorists. Admittedly, these tools take time to learn, and tying the subject together with rigorous proofs can become extremely technical. Nevertheless, we feel that the basic concepts and tools of quantum field theory can be made accessible to all physicists, not just an elite group of experts.

A number of earlier books have succeeded in making parts of this subject accessible to students. The best known of these is the two-volume text written in the 1960s by our Stanford colleagues Bjorken and Drell. In our opinion, their text contains an ideal mixture of abstract formalism, intuitive explanations, and practical calculations, all presented with great care and clarity. Since the 1960s, however, the subject of quantum field theory has developed enormously, both in its conceptual framework (the renormalization group, new types of symmetries) and in its areas of application (critical exponents in condensed matter systems, the standard model of elementary particle physics). It is long overdue that a textbook of quantum field theory should appear that provides a complete survey of the subject, including these newer developments, yet still offers the same accessibility and depth of treatment as Bjorken and Drell. We have written this book with that goal in mind.

An Outline of the Book

This textbook is composed of three major sections. The first is mainly concerned with the quantum theory of electromagnetism, which provided the first example of a quantum field theory with direct experimental applications. The third part of the book is mainly concerned with the particular quantum field theories that appear in the standard model of particle interactions. The second part of the book is a bridge between these two subjects; it is intended to introduce some of the very deep concepts of quantum field theory in a context that is as straightforward as possible.

Part I begins with the study of fields with linear equations of motion, that is, fields without interactions. Here we explore the combined implications of quantum mechanics and special relativity, and we learn how particles arise as the quantized excitations of fields. We then introduce interactions among these particles and develop a systematic method of accounting for their effects. After this introduction, we carry out explicit computations in the quantum theory of electromagnetism. These illustrate both the special features of the behavior of electrons and photons and some general aspects of the behavior of interacting quantum fields.

In several of the calculations in Part I, naive methods lead to infinite results. The appearance of infinities is a well-known feature of quantum field theory. At times, it has been offered as evidence for the inconsistency of quantum field theory (though a similar argument could be made against the classical electrodynamics of point particles). For a long time, it was thought sufficient to organize calculations in such a way that no infinities appear in quantities that can be compared directly to experiment. However, one of the major insights of the more recent developments is that these formal infinities actually contain important information that can be used to predict the qualitative behavior of a system. In Part II of the book, we develop this theory of infinities systematically. The development makes use of an analogy between quantum-mechanical and thermal fluctuations, which thus becomes a bridge between quantum field theory and statistical mechanics. At the end of Part II we discuss applications of quantum field theory to the theory of phase transitions in condensed matter systems.

Part III deals with the generalizations of quantum electrodynamics that have led to successful models of the forces between elementary particles. To derive these generalizations, we first analyze and generalize the fundamental symmetry of electrodynamics, then work out the consequences of quantizing a theory with this generalized symmetry. This analysis leads to intricate and quite nontrivial applications of the concepts introduced earlier. We conclude Part III with a presentation of the standard model of particle physics and a discussion of some of its experimental tests.

The Epilogue to the book discusses qualitatively the frontier areas of research in quantum field theory and gives references that can guide a student to the next level of study.

Where a choice of viewpoints is possible, we have generally chosen to explain ideas in language appropriate to the applications to elementary particle physics. This choice reflects our background and research interests. It also reflects our strongly held opinion, even in this age of intellectual relativism, that there is something special about unraveling the behavior of Nature at the deepest possible level. We are proud to take as our subject the structure of the fundamental interactions, and we hope to convey to the reader the grandeur and continuing vitality of this pursuit.

How to Use This Book

This book is an *introduction* to quantum field theory. By this we mean, first and foremost, that we assume no prior knowledge of the subject on the part of the reader. The level of this book should be appropriate for students taking their first course in quantum field theory, typically during the second year of graduate school at universities in the United States. We assume that the student has completed graduate-level courses in classical mechanics, classical electrodynamics, and quantum mechanics. In Part II we also assume some knowledge of statistical mechanics. It is not necessary to have mastered every topic covered in these courses, however. Crucially important prerequisites include the Lagrangian and Hamiltonian formulations of dynamics, the relativistic formulation of electromagnetism using tensor notation, the quantization of the harmonic oscillator using ladder operators, and the theory of scattering in nonrelativistic quantum mechanics. Mathematical prerequisites include an understanding of the rotation group as applied to the quantum mechanics of spin, and some facility with contour integration in the complex plane.

Despite being an "introduction", this book is rather lengthy. To some extent, this is due to the large number of explicit calculations and worked examples in the text. We must admit, however, that the total number of topics covered is also quite large. Even students specializing in elementary particle theory will find that their first research projects require only a part of this material, together with additional, specialized topics that must be gleaned from the research literature. Still, we feel that students who want to become experts in elementary particle theory, and to fully understand its unified view of the fundamental interactions, should master every topic in this book. Students whose main interest is in other fields of physics, or in particle experimentation, may opt for a much shorter "introduction", omitting several chapters.

The senior author of this book once did succeed in "covering" 90% of its content in a one-year lecture course at Stanford University. But this was a mistake; at such a pace, there is not enough time for students of average preparation to absorb the material. Our saner colleagues have found it more reasonable to cover about one Part of the book per *semester*. Thus, in planning a one-year course in quantum field theory, they have chosen either to reserve

Part III for study at a more advanced level or to select about half of the material from Parts II and III, leaving the rest for students to read on their own.

We have designed the book so that it can be followed from cover to cover, introducing all of the major ideas in the field systematically. Alternatively, one can follow an accelerated track that emphasizes the less formal applications to elementary particle physics and is sufficient to prepare students for experimental or phenomenological research in that field. Sections that can be omitted from this accelerated track are marked with an asterisk in the Table of Contents; none of the unmarked sections depend on this more advanced material. Among the unmarked sections, the order could also be varied somewhat: Chapter 10 does not depend on Chapters 8 and 9; Section 11.1 is not needed until just before Chapter 20; and Chapters 20 and 21 are independent of Chapter 17.

Those who wish to study some, but not all, of the more advanced sections should note the following table of dependencies:

Before reading . . .	one should read all of . . .
Chapter 13	Chapters 11, 12
Section 16.6	Chapter 11
Chapter 18	Sections 12.4, 12.5, 16.5
Chapter 19	Sections 9.6, 15.3
Section 19.5	Section 16.6
Section 20.3	Sections 19.1–19.4
Section 21.3	Chapter 11

Within each chapter, the sections marked with an asterisk should be read sequentially, except that Sections 16.5 and 16.6 do not depend on 16.4.

A student whose main interest is in statistical mechanics would want to read the book sequentially, confronting the deep formal issues of Part II but ignoring most of Part III, which is mainly of significance to high-energy phenomena. (However, the material in Chapters 15 and 19, and in Section 20.1, does have beautiful applications in condensed matter physics.)

We emphasize to all students the importance of working actively with the material while studying. It probably is not possible to understand any section of this book without carefully working out the intermediate steps of every derivation. In addition, the problems at the end of each chapter illustrate the general ideas and often apply them in nontrivial, realistic contexts. However, the most illustrative exercises in quantum field theory are too long for ordinary homework problems, being closer to the scale of small research projects. We have provided one of these lengthy problems, broken up into segments with hints and guidance, at the end of each of the three Parts of the book. The volume of time and paper that these problems require will be well invested.

At the beginning of each Part we have included a brief "Invitation" chapter, which previews some of the upcoming ideas and applications. Since these chapters are somewhat easier than the rest of the book, we urge all students to read them.

What This Book is Not

Although we hope that this book will provide a thorough grounding in quantum field theory, it is in no sense a complete education. A dedicated student of physics will want to supplement our treatment in many areas. We summarize the most important of these here.

First of all, this is a book about theoretical methods, not a review of observed phenomena. We do not review the crucial experiments that led to the standard model of elementary particle physics or discuss in detail the more recent experiments that have confirmed its predictions. Similarly, in the chapters that deal with applications to statistical mechanics, we do not discuss the beautiful and varied experiments on phase transitions that led to the confirmation of field theory models. We strongly encourage the student to read, in parallel with this text, a modern presentation of the experimental development of each of these fields.

Although we present the elementary aspects of quantum field theory in full detail, we state some of the more advanced results without proof. For example, it is known rigorously, to all orders in the standard expansion of quantum electrodynamics, that formal infinities can be removed from all experimental predictions. This result, known as *renormalizability*, has important consequences, which we explore in Part II. We do not present the general proof of renormalizability. However, we do demonstrate renormalizability explicitly in illustrative, low-order computations, we discuss intuitively the issues that arise in the complete proof, and we give references to a more complete demonstration. More generally, we have tried to motivate the most important results (usually through explicit examples) while omitting lengthy, purely technical derivations.

Any introductory survey must classify some topics as beyond its scope. Our philosophy has been to include what can be learned about quantum field theory by considering weakly interacting particles and fields, using series expansions in the strength of the interaction. It is amazing how much insight one can obtain in this way. However, this definition of our subject leaves out the theory of bound states, and also phenomena associated with nontrivial solutions to nonlinear field equations. We give a more complete listing of such advanced topics in the Epilogue.

Finally, we have not attempted in this book to give an accurate record of the history of quantum field theory. Students of physics do need to understand the history of physics, for a number of reasons. The most important is to acquire a precise understanding of the experimental basis of the subject. A second important reason is to gain an idea of how science progresses as a human endeavor, how ideas develop as small steps taken by individuals to become the major achievements of the community as a whole.*

In this book we have not addressed either of these needs. Rather, we have included only the kind of mythological history whose purpose is to motivate new ideas and assign names to them. A principle of physics usually has a name that has been assigned according to the community's consensus on who deserves credit for its development. Usually the real credit is only partial, and the true historical development is quite complex. But the clear assignment of names is essential if physicists are to communicate with one another.

Here is one example. In Section 17.5 we discuss a set of equations governing the structure of the proton, which are generally known as the Altarelli-Parisi equations. Our derivation uses a method due to Gribov and Lipatov (GL). The original results of GL were rederived in a more abstract language by Christ, Hasslacher, and Mueller (CHM). After the discovery of the correct fundamental theory of the strong interactions (QCD), Georgi, Politzer, Gross, and Wilczek (GPGW) used the technique of CHM to derive formal equations for the variation of the proton structure. Parisi gave the first of a number of independent derivations that converted these equations into a useful form. The combination of his work with that of GPGW gives the derivation of the equations that we present in Section 18.5. Dokhshitzer later obtained these equations more simply by direct application of the method of GL. Sometime later, but independently, Altarelli and Parisi obtained these equations again by the same route. These last authors also popularized the technique, explaining it very clearly, encouraging experimentalists to use the equations in interpreting their data, and prodding theorists to compute the systematic higher-order corrections to this picture. In Section 17.5 we have presented the shortest path to the end of this convoluted historical road and hung the name 'Altarelli-Parisi' on the final result.

There is a fourth reason for students to read the history of physics: Often the original breakthrough papers, though lacking a textbook's advantages of hindsight, are filled with marvelous personal insights. We strongly encourage students to go back to the original literature whenever possible and see what the creators of the field had in mind. We have tried to aid such students with references provided in footnotes. Though occasionally we refer to papers merely to give credit, most of the references are included because we feel the reader should not miss the special points of view that the authors put forward.

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*The history of the development of quantum field theory and particle physics has recently been reviewed and debated in a series of conference volumes: *The Birth of Particle Physics*, L. M. Brown and L. Hoddeson, eds. (Cambridge University Press, 1983); *Pions to Quarks*, L. M. Brown, M. Dresden, and L. Hoddeson, eds. (Cambridge University Press, 1989); and *The Rise of the Standard Model*, L. M. Brown, M. Dresden, L. Hoddeson, and M. Riordan, eds. (Cambridge University Press, 1995). The early history of quantum electrodynamics is recounted in a fascinating book by Schweber (1994).

Acknowledgments

The writing of this book would not have been possible without the help and encouragement of our many teachers, colleagues, and friends. It is our great pleasure to thank these people.

Michael has been privileged to learn field theory from three of the subject's contemporary masters—Sidney Coleman, Steven Weinberg, and Kenneth Wilson. He is also indebted to many other teachers, including Sidney Drell, Michael Fisher, Kurt Gottfried, John Kogut, and Howard Georgi, and to numerous co-workers, in particular, Orlando Alvarez, John Preskill, and Edward Witten. His association with the laboratories of high-energy physics at Cornell and Stanford, and discussions with such people as Gary Feldman, Martin Perl, and Morris Swartz, have also shaped his viewpoint. To these people, and to many other people who have taught him points of physics over the years, Michael expresses his gratitude.

Dan has learned field theory from Savas Dimopoulos, Leonard Susskind, Richard Blankenbecler, and many other instructors and colleagues, to whom he extends thanks. For his broader education in physics he is indebted to many teachers, but especially to Thomas Moore. In addition, he is grateful to all the teachers and friends who have criticized his writing over the years.

For their help in the construction and writing of this book, many people are due more specific thanks. This book originated as graduate courses taught at Cornell and at Stanford, and we thank the students in these courses for their patience, criticism, and suggestions. During the years in which this book was written, we were supported by the Stanford Linear Accelerator Center and the Department of Energy, and Dan also by the Pew Charitable Trusts and the Departments of Physics at Pomona College, Grinnell College, and Weber State University. We are especially grateful to Sidney Drell for his support and encouragement. The completion of this book was greatly assisted by a sabbatical Michael spent at the University of California, Santa Cruz, and he thanks Michael Nauenberg and the physics faculty and students there for their gracious hospitality.

We have circulated preliminary drafts of this book widely and have been continually encouraged by the response to these drafts and the many suggestions and corrections they have elicited. Among the people who have helped us in this way are Curtis Callan, Edward Farhi, Jonathan Feng, Donald Finnell, Paul Frampton, Jeffrey Goldstone, Brian Hatfield, Barry Holstein, Gregory Kilcup, Donald Knuth, Barak Kol, Phillip Nelson, Matthias Neubert, Yossi Nir, Arvind Rajaraman, Pierre Ramond, Katsumi Tanaka, Xerxes Tata, Arkady Vainshtein, Rudolf von Bunau, Shimon Yankielowicz, and Eran Yehudai. For help with more specific technical problems or queries, we are grateful to Michael Barnett, Marty Breidenbach, Don Groom, Toichiro Kinoshita, Paul Langacker, Wu-Ki Tung, Peter Zerwas, and Jean Zinn-Justin. We thank our Stanford colleagues James Bjorken, Stanley Brodsky, Lance Dixon, Bryan Lynn, Helen Quinn, Leonard Susskind, and Marvin Weinstein,

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We pause to remember two friends of this book who did not see its completion—Donald Yennie, whose long career made him one of the heroes of Quantum Electrodynamics, and Brian Warr, whose brilliant promise in theoretical physics was cut short by AIDS.

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Finally, we thank you, the reader, for your time and effort spent studying this book. Though we have tried to cleanse this text of conceptual and typographical errors, we apologize in advance for those that have slipped through. We will be glad to hear your comments and suggestions for further improvements in the presentation of quantum field theory.

Michael E. Peskin
Daniel V. Schroeder

Notations and Conventions

Units

We will work in "God-given" units, where

$$\hbar = c = 1.$$

In this system,

$$[\text{length}] = [\text{time}] = [\text{energy}]^{-1} = [\text{mass}]^{-1}.$$

The mass (m) of a particle is therefore equal to its rest energy (mc^2), and also to its inverse Compton wavelength ($mc/\hbar$). For example,

$$m_{\text{electron}} = 9.109 \times 10^{-28}\,\text{g} = 0.511\,\text{MeV} = \left(3.862 \times 10^{-11}\,\text{cm}\right)^{-1}.$$

A selection of other useful numbers and conversion factors is given in the Appendix.

Relativity and Tensors

Our conventions for relativity follow Jackson (1975), Bjorken and Drell (1964, 1965), and nearly all recent field theory texts. We use the metric tensor

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

with Greek indices running over 0, 1, 2, 3 or t, x, y, z. Roman indices—i, j, etc.—denote only the three spatial components. Repeated indices are summed in all cases. Four-vectors, like ordinary numbers, are denoted by light italic type; three-vectors are denoted by boldface type; unit three-vectors are denoted by a light italic label with a hat over it. For example,

$$x^\mu = (x^0, \mathbf{x}), \qquad x_\mu = g_{\mu\nu}x^\nu = (x^0, -\mathbf{x});$$

$$p \cdot x = g_{\mu\nu}p^\mu x^\nu = p^0x^0 - \mathbf{p} \cdot \mathbf{x}.$$

A massive particle has

$$p^2 = p^\mu p_\mu = E^2 - |\mathbf{p}|^2 = m^2.$$

Note that the displacement vector x^μ is "naturally raised", while the derivative operator,

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial x^0}, \nabla\right),$$

is "naturally lowered".

We define the totally antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$ so that

$$\epsilon^{0123} = +1.$$

Be careful, since this implies $\epsilon_{0123} = -1$ and $\epsilon^{1230} = -1$. (This convention agrees with Jackson but not with Bjorken and Drell.)

Quantum Mechanics

We will often work with the Schrödinger wavefunctions of single quantum-mechanical particles. We represent the energy and momentum operators acting on such wavefunctions following the usual conventions:

$$E = i\frac{\partial}{\partial x^0}, \qquad \mathbf{p} = -i\nabla.$$

These equations can be combined into

$$p^\mu = i\partial^\mu;$$

raising the index on ∂^μ conveniently accounts for the minus sign. The plane wave $e^{-ik\cdot x}$ has momentum k^μ, since

$$i\partial^\mu\left(e^{-ik\cdot x}\right) = k^\mu\, e^{-ik\cdot x}.$$

The notation 'h.c.' denotes the Hermitian conjugate.

Discussions of spin in quantum mechanics make use of the Pauli sigma matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Products of these matrices satisfy the identity

$$\sigma^i\sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma^k.$$

It is convenient to define the linear combinations $\sigma^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$; then

$$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Fourier Transforms and Distributions

We will often make use of the Heaviside step function $\theta(x)$ and the Dirac delta function $\delta(x)$, defined as follows:

$$\theta(x) = \begin{cases} 0 & x < 0, \\ 1 & x > 0; \end{cases} \qquad \delta(x) = \frac{d}{dx}\theta(x).$$

The delta function in n dimensions, denoted $\delta^{(n)}(\mathbf{x})$, is zero everywhere except at $\mathbf{x} = 0$ and satisfies

$$\int d^n x \, \delta^{(n)}(\mathbf{x}) = 1.$$

In Fourier transforms the factors of 2π will always appear with the momentum integral. For example, in four dimensions,

$$f(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \tilde{f}(k);$$

$$\tilde{f}(k) = \int d^4x \, e^{ik\cdot x} f(x).$$

(In three-dimensional transforms the signs in the exponents will be + and −, respectively.) The tilde on $\tilde{f}(k)$ will sometimes be omitted when there is no potential for confusion. The other important factors of 2π to remember appear in the identity

$$\int d^4x \, e^{ik\cdot x} = (2\pi)^4 \delta^{(4)}(k).$$

Electrodynamics

We use the Heaviside-Lorentz conventions, in which the factors of 4π appear in Coulomb's law and the fine-structure constant rather than in Maxwell's equations. Thus the Coulomb potential of a point charge Q is

$$\Phi = \frac{Q}{4\pi r},$$

and the fine-structure constant is

$$\alpha = \frac{e^2}{4\pi} = \frac{e^2}{4\pi\hbar c} \approx \frac{1}{137}.$$

The symbol e stands for the charge of the electron, a negative quantity (although the sign rarely matters). We generally work with the relativistic form of Maxwell's equations:

$$\epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma} = 0, \qquad \partial_\mu F^{\mu\nu} = ej^\nu,$$

where

$$A^\mu = (\Phi, \mathbf{A}), \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and we have extracted the e from the 4-vector current density j^μ.

Dirac Equation

Some of our conventions differ from those of Bjorken and Drell (1964, 1965) and other texts: We use a chiral basis for Dirac matrices, and relativistic normalization for Dirac spinors. These conventions are introduced in Sections 3.2 and 3.3, and are summarized in the Appendix.