Graduate Texts in Mathematics

Steven Roman

# Advanced Linear Algebra

Third Edition

# 高等线性代数

第3版

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# Advanced Linear Algebra
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To Donna
and to
Rashelle, Carol and Dan

# Preface to the Third Edition

Let me begin by thanking the readers of the second edition for their many helpful comments and suggestions, with special thanks to Joe Kidd and Nam Trang. For the third edition, I have corrected all known errors, polished and refined some arguments (such as the discussion of reflexivity, the rational canonical form, best approximations and the definitions of tensor products) and upgraded some proofs that were originally done only for finite-dimensional/rank cases. I have also moved some of the material on projection operators to an earlier position in the text.

A few new theorems have been added in this edition, including the spectral mapping theorem and a theorem to the effect that $\dim(V) \leq \dim(V^*)$, with equality if and only if $V$ is finite-dimensional.

I have also added a new chapter on associative algebras that includes the well-known characterizations of the finite-dimensional division algebras over the real field (a theorem of Frobenius) and over a finite field (Wedderburn's theorem). The reference section has been enlarged considerably, with over a hundred references to books on linear algebra.

*Steven Roman* *Irvine, California, May 2007*

# Preface to the Second Edition

Let me begin by thanking the readers of the first edition for their many helpful comments and suggestions. The second edition represents a major change from the first edition. Indeed, one might say that it is a totally new book, with the exception of the general range of topics covered.

The text has been completely rewritten. I hope that an additional 12 years and roughly 20 books worth of experience has enabled me to improve the quality of my exposition. Also, the exercise sets have been completely rewritten.

The second edition contains two new chapters: a chapter on convexity, separation and positive solutions to linear systems (Chapter 15) and a chapter on the QR decomposition, singular values and pseudoinverses (Chapter 17). The treatments of tensor products and the umbral calculus have been greatly expanded and I have included discussions of determinants (in the chapter on tensor products), the complexification of a real vector space, Schur's theorem and Geršgorin disks.

*Steven Roman* *Irvine, California February 2005*

# Preface to the First Edition

This book is a thorough introduction to linear algebra, for the graduate or advanced undergraduate student. Prerequisites are limited to a knowledge of the basic properties of matrices and determinants. However, since we cover the basics of vector spaces and linear transformations rather rapidly, a prior course in linear algebra (even at the sophomore level), along with a certain measure of "mathematical maturity," is highly desirable.

Chapter 0 contains a summary of certain topics in modern algebra that are required for the sequel. *This chapter should be skimmed quickly and then used primarily as a reference.* Chapters 1–3 contain a discussion of the basic properties of vector spaces and linear transformations.

Chapter 4 is devoted to a discussion of modules, emphasizing a comparison between the properties of modules and those of vector spaces. Chapter 5 provides more on modules. The main goals of this chapter are to prove that any two bases of a free module have the same cardinality and to introduce Noetherian modules. However, the instructor may simply skim over this chapter, omitting all proofs. Chapter 6 is devoted to the theory of modules over a principal ideal domain, establishing the cyclic decomposition theorem for finitely generated modules. This theorem is the key to the structure theorems for finite-dimensional linear operators, discussed in Chapters 7 and 8.

Chapter 9 is devoted to real and complex inner product spaces. The emphasis here is on the finite-dimensional case, in order to arrive as quickly as possible at the finite-dimensional spectral theorem for normal operators, in Chapter 10. However, we have endeavored to state as many results as is convenient for vector spaces of arbitrary dimension.

The second part of the book consists of a collection of independent topics, with the one exception that Chapter 13 requires Chapter 12. Chapter 11 is on metric vector spaces, where we describe the structure of symplectic and orthogonal geometries over various base fields. Chapter 12 contains enough material on metric spaces to allow a unified treatment of topological issues for the basic

Hilbert space theory of Chapter 13. The rather lengthy proof that every metric space can be embedded in its completion may be omitted.

Chapter 14 contains a brief introduction to tensor products. In order to motivate the universal property of tensor products, without getting too involved in categorical terminology, we first treat both free vector spaces and the familiar direct sum, in a universal way. Chapter 15 (Chapter 16 in the second edition) is on affine geometry, emphasizing algebraic, rather than geometric, concepts.

The final chapter provides an introduction to a relatively new subject, called the umbral calculus. This is an algebraic theory used to study certain types of polynomial functions that play an important role in applied mathematics. We give only a brief introduction to the subject – emphasizing the algebraic aspects, rather than the applications. This is the first time that this subject has appeared in a true textbook.

One final comment. Unless otherwise mentioned, omission of a proof in the text is a tacit suggestion that the reader attempt to supply one.

*Steven Roman* *Irvine, California*

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