天元基金影印数学丛书

CAMBRIDGE

Practical Applied Mathematics

Modelling, Analysis, Approximation

实用数学

建模、分析、逼近（影印版）

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序　言

为了更好地借鉴国外数学教育与研究的成功经验，促进我国数学教育与研究事业的发展，提高高等学校数学教育教学质量，本着“为我国热爱数学的青年创造一个较好的学习数学的环境”这一宗旨，天元基金赞助出版“天元基金影印数学丛书”。

该丛书主要包含国外反映近代数学发展的纯数学与应用数学方面的优秀书籍，天元基金邀请国内各个方向的知名数学家参与选题的工作，经专家遴选、推荐，由高等教育出版社影印出版。为了提高我国数学研究生教学的水平，暂把选书的目标确定在研究生教材上。当然，有的书也可作为高年级本科生教材或参考书，有的书则介于研究生教材与专著之间。

欢迎各方专家、读者对本丛书的选题、印刷、销售等工作提出批评和建议。

天元基金领导小组

2007 年 1 月

Preface

This book was born out of my fascination with applied mathematics as a place where the physical world meets the mathematical structures and techniques that are the cornerstones of most applied mathematics courses. I am interested largely in human-sized theatres of interaction, leaving cosmology and particle physics to others. Much of my research has been motivated by interactions with industry or by contact with scientists in other disciplines. One immediate lesson from these contacts is that it is a great asset to an interactive applied mathematician to be open to ideas from any direction at all. Almost any physical situation has some mathematical interest, but the kind of mathematics may vary from case to case. We need a strong generalist streak to go with our areas of technical expertise.

Another thing we need is some expertise in numerical methods. To be honest, this is not my strong point. That is one reason why the book does not contain much about these methods. (Another is that if it had then it would have been half as long again and would have taken five more years to write.) In the modern world, with its fast computers and plethora of easy-to-use packages, any applied mathematician has to be able to switch into numerical mode as required. At the very least, you should learn to use packages such as Maple and Matlab for their data display and plotting capabilities and for the built-in software routines for solving standard problems such as ordinary differential equations. With more confidence, you can write your own programs. In many cases, a quick and dirty first try can provide valuable information, even if this is not the finished product. Explicit finite differences (remember to use upwind differencing for first derivatives) and tiny time steps will get you a long way.

Who should read this book? Many people, I hope, but there are some prerequisites. I assume that readers have a good background in calculus up to vector calculus (grad, div, curl) and the elementary mechanics of particles. I also assume that they have done an introductory (inviscid) fluid mechanics course and a first course in partial differential equations, enough to know the basics of the heat, wave and Laplace equations (where they come from, and how to solve them in simple geometries). Linear algebra, complex analysis and probability put in an occasional appearance. High-school physics is an advantage. But the most important prerequisite is an attitude: to go out and apply your mathematics, to see it in action in the world around you, and not to worry too much about the technical aspects, focusing instead on the big picture.

Another way to assess the technical level of the book is to position it relative to the competition. From that point of view it can be thought of as a precursor to the books by Tayler [58] and Fowler [19], while being more difficult than, say, Fowkes & Mahoney [18] or Fulford & Broadbridge [22]. The edited collections [9, 38] are at the same general level, but they are organised along different lines. The books [40, 56] cover similar material but with a less industrial slant.

Organisation. The book is organised, roughly, along mathematical lines. Chapters are devoted to mathematical techniques, starting in Part I with some ideas about modelling, moving on in Part II to differential equations and distributions, and concluding with asymptotic (systematic approximation) methods in Part III. Interspersed among the chapters are case studies, descriptions of problems that illustrate the techniques; they are necessarily rather open-ended and invite you to develop your own ideas. The case studies run as strands through the book. You can ignore any of them without much impact on the rest of the book, although the more you ignore the less you will benefit from the remainder. There are long sections of exercises at the ends of the chapters; they should be regarded as an integral part of the book and at least should be read through if not attempted.

Conventions. I use 'we', as in 'we can solve this by a Laplace transform', to signal the usual polite fiction that you, the reader, and I, the author, are engaged on a joint voyage of discovery; 'we' also signifies that I am presenting ideas within a whole tradition of thought. 'You' is mostly used to suggest that *you* should get your pen out and work though some of the 'we' stuff, a good idea in view of my fallible arithmetic, or do an exercise to fill in some details. 'I' is associated with authorial opinions and can mostly be ignored if you like.

I have tried to draw together a lot of threads in this book, and in writing it I have constantly wanted to point out connections with something else or make a peripheral remark. However, I don't want to lose track of the argument. As a compromise, I have used marginal notes and footnotes[1] with slightly different purposes in mind.

Marginal notes are usually directly relevant to the current discussion, often being used to fill in details or point out a feature of a calculation. This is a book to work through: feel free to use the empty margin spaces for calculations.

[1] Footnotes are more digressional and can be ignored by readers who just want to follow the main line of argument.

Acknowledgements. I have taken examples from many sources. Some examples are very familiar and I do not apologise for this: the old ones are often the best. Much the same goes for the influence of books; if you teach a course using other people's books and then write your own, some impact is inevitable. Among the books that have been especially influential are those by Tayler [58], Fowler [19], Hinch [27] and Keener [33]. Even more influential has been the contribution of colleagues and students. Many a way of looking at a problem can be traced back to a coffee-time conversation or a Study Group meeting.[2] There are far too many of these collaborators for me to attempt the invidious task of thanking them individually. Their influence is pervasive. At a more local level, I am immensely grateful to the OCIAM students who got me out of computer trouble on various occasions and found a number of errors in drafts of the book. Any remaining errors are quite likely to have been caused by cosmic ray impact on the computer memory, or perhaps by cyber-terrorists. I will be happy to hear about them.

The book began when I was asked to give some lectures at a summer school in Siena and was continued through a similar event a year later in Pisa. I am most grateful for the hospitality extended to me during these visits. I would like to thank the editors and technical staff at Cambridge University Press for their assistance in the production of the book. In particular, I am extremely grateful to Susan Parkinson for her careful, constructive and thoughtful copy-editing of the manuscript. Lastly I would like to thank my family for their forbearance, love and support while I was locked away typing. This book is dedicated to them.

Colemanballs. At the end of each section of exercises is what would normally be a wasted space. Into each of these I have put two things. One is a depiction of a wave form and is explained on p. 212. The other is a statement made by a real live applied mathematician in full flow. In the spirit of scientific accuracy, they are wholly unedited. They are mostly there for their intrinsic qualities (and it would be a miserable publisher who would deny me that extra ink), but they make a point: interdisciplinary mathematics is a collaborative affair; it involves discussions and

[2] Study Groups are week-long intensive meetings at which academics and industrial researchers get together to work on open problems from industry, proposed by the industrial participants. Over the week, heated discussions take place involving anybody who is interested in the problem, and a short report is produced at the end. The first UK Study Group was held in Oxford in 1968, and they have been held every year since, in Oxford and other UK universities. The idea has now spread to more than 15 countries on all the habitable continents of the world. Details of forthcoming events, and reports of problems studied at past meetings, can be found on their dedicated website `www.mathematics-in-industry.org`.

arguments, the less inhibited the better. We all have to go out on a limb, in the interests of pushing the science forwards. If we are wrong, we try again. And if the mind runs ahead of the voice, our colleagues won't take it too seriously (nor will they let us forget it). Here is one to be going on with, from the collection [29] of the same title:

'If I remember rightly, $\cos \pi/2 = 1$.'

Contents

Part I
Modelling techniques

1
The basics of modelling

1.1 Introduction

This short introductory chapter is about mathematical modelling. Without trying to be too prescriptive, we discuss what we mean by the term, why we might want to do it and what kind of models are commonly used. Then we look at some very standard models, which you have almost certainly met before, and we see how their derivation is a blend of what are thought of as universal physical laws, such as conservation of mass, momentum and energy, with experimental observations and, perhaps, some ad hoc assumptions in lieu of more specific evidence.

One of the themes that run through this book is the applicability of all kinds of mathematical idea to 'real-world' problems. Some of these arise in attempts to explain natural phenomena, for example in models for water waves. We will see a number of these models as we go through the book. Other applications are found in industry, which is a source of many fascinating and non-standard mathematical problems and a big 'end-user' of mathematics. You might be surprised at how little is known of the detailed mechanics of most industrial processes, although when you see the operating conditions – ferocious temperatures, inaccessible or minute machinery, corrosive chemicals – you realise how expensive and difficult it would be to carry out detailed experimental investigations. In any case, many processes work just fine, having been designed by engineers who know their job. If it ain't broke, don't fix it; so where does mathematics come in? Some important uses are in the quality control and cost control of existing processes and in the simulation and design of new processes. We may want to understand: why does a certain type of defect occur; what is the 'rate-limiting' part of a process (the slowest ship, to be speeded up); how to improve efficiency, however marginally; whether a novel idea is likely to work at all and if so, how to control it.

It is in the nature of real-world problems that they are large, messy and often rather vaguely stated. It is very rarely worth anybody's while to produce a 'complete solution' to a problem which is complicated and whose desired outcome is not necessarily well specified (to a mathematician). Mathematicians are usually most effective in analysing a relatively small 'clean' subproblem for which more broad-brush approaches run into difficulty. Very often the analysis complements a large numerical simulation which, although effective elsewhere, has trouble with this particular aspect of the problem. Its job is to provide understanding and insight in order to complement simulation, experiment and other approaches.

We begin with a chat about what models are and what they should do for us. Then we bring some simple ideas about physical conservation laws and how to use them together with the experimental evidence about how materials behave, with the aim of formulating closed systems of equations; this is illustrated with two canonical models, for heat flow and for fluid motion. There are many other models embedded elsewhere in the book, and we will deal with these as we come to them.

1.2 What do we mean by a model?

There is no point in trying to be too precise in defining the term 'mathematical model': we all understand that it is some kind of mathematical statement about a problem originally posed in non-mathematical terms. Some models are *explicative*, that is, they explain a phenomenon in terms of simpler, more basic processes. A famous example is Newton's theory of planetary motion, whereby the whole complex motion of the solar system was shown to be a consequence of 'force equals mass times acceleration' and the inverse square law of gravitation. However, not all models aspire to explain. For example, the standard Black–Scholes model for the evolution of prices in stock markets, used by investment banks the world over, says that the percentage difference between tomorrow's stock price and today's is a lognormal random variable. Although this is a great simplification, in that it says that all we need to know are the mean and variance of this distribution, it says nothing about what will cause the price change.

All useful models, whether explicative or not, are *predictive*: they allow us to make quantitative predictions (whether deterministic or probabilistic) that can be used either to test and refine the model, should that