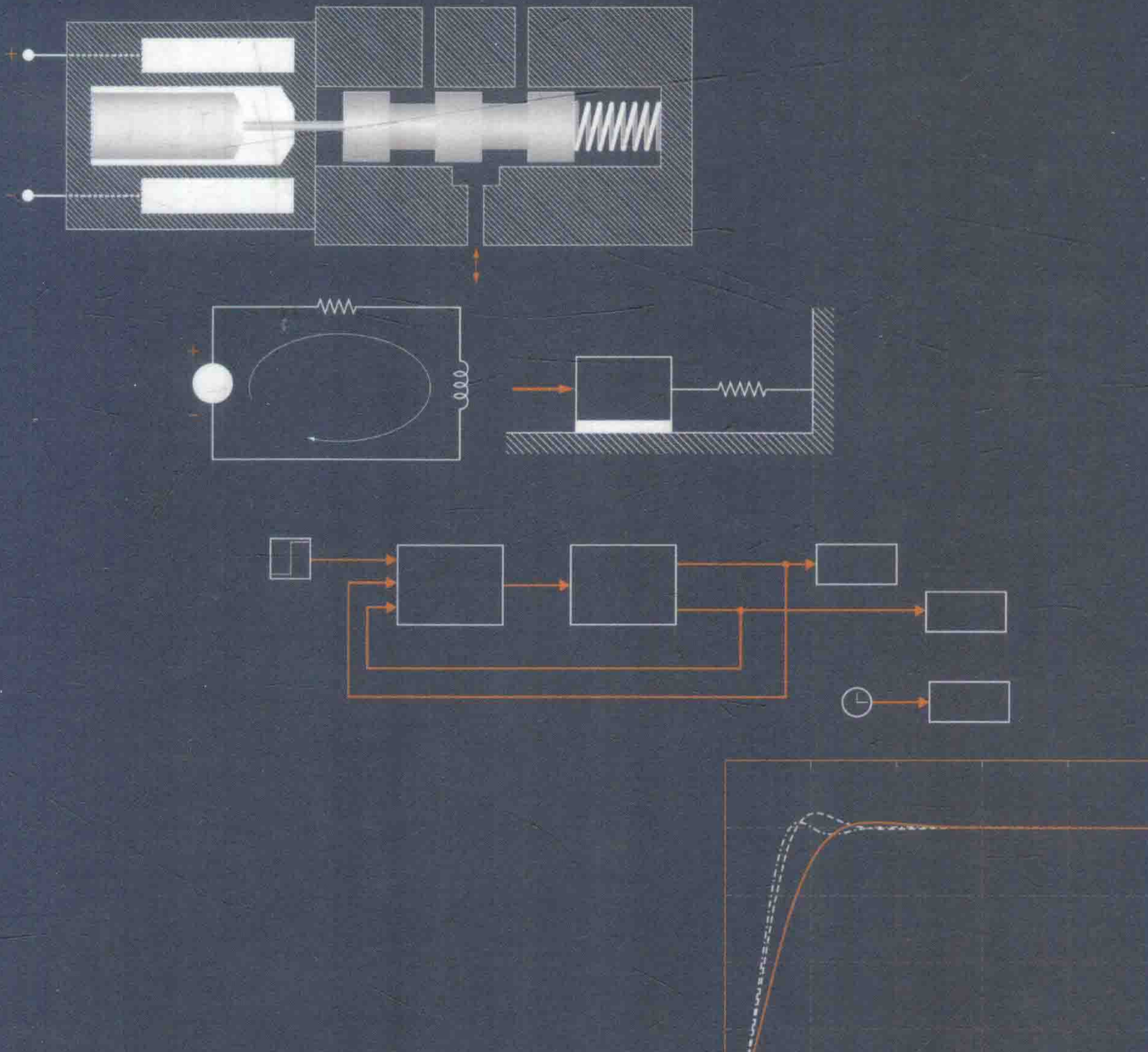
DYNAMIC SYSTEMS

Modeling, Simulation, and Control



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Dynamic Systems: Modeling, Simulation, and Control

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Preface

This textbook is intended for an introductory course in dynamic systems and control, typically required in undergraduate mechanical engineering and some aerospace engineering curricula. Such a course is usually taken in the junior or senior year, after the student has completed courses in mechanics, differential equations, and electrical circuits. The major topics of a dynamic systems and control course include (1) mathematical modeling, (2) system-response analysis, and (3) an introduction to feedback control systems. The primary objective of this textbook is a comprehensive yet concise treatment of these major topics with an emphasis on demonstrating physical engineering applications. It has been my experience that undergraduate students remain engaged in a system dynamics course when the concepts are presented in terms of real engineering systems (such as a hydraulic actuator) instead of academic examples. This textbook is a distillation of 20 years of course notes and strategies for teaching system dynamics in the Mechanical and Aerospace Engineering Department at the University of Missouri-Columbia. It is thus based on my extensive classroom experience and student feedback, and the end result is a text whose key features differ from current system dynamics textbooks.

Chapter 1 introduces dynamic systems and control, including definitions of the relevant terms and categories of dynamic systems. The following three chapters all deal with developing mathematical models of physical engineering systems. Chapter 2 introduces the fundamental techniques used to derive the modeling equations for mechanical systems. Mechanical systems, perhaps more intuitively understood by undergraduate students as they involve Newton's laws of motion, are treated first. Chapter 3 introduces the fundamental methods of developing the mathematical models for electrical and electromechanical systems. Models in this chapter are derived by applying Kirchhoff's and Faraday's laws and the electrical element laws that govern the interactions among electrical charge, current, magnetic flux, and voltage. Chapter 4 presents the fundamental techniques involved in developing models for fluid and thermal systems. It is the last chapter dedicated to the derivation of mathematical models. Fluid (hydraulic and pneumatic) models are based on the conservation of mass, while thermal models are derived using the conservation of energy.

Chapter 5 presents standard forms for representing the mathematical models of dynamic systems: state-variable equations, state-space representation, input-output equations, transfer functions, and block diagrams. The linearization process is also described in this chapter. The key concept emphasized in Chapter 5 is that each standard form is simply a convenient representation of the system model (i.e., the differential equations) that lends itself to analysis of the system's dynamic response. Therefore Chapter 5 serves as a transition between developing the mathematical model (Chapters 2–4) and obtaining the system's response using either numerical simulation or analytical techniques (Chapters 6–10).

Chapter 6 presents numerical simulation methods for obtaining the response of dynamic systems. Here, the MATLAB simulation software is used exclusively as it has become the standard computational platform for academia and industry. Simulating the response of linear systems using MATLAB commands is presented first. The graphical software Simulink is presented next, and it is the focus of this chapter and the primary simulation tool used throughout the remainder of the textbook. Simulink is used to simulate linear and nonlinear systems using the standard forms presented in Chapter 5.

The next three chapters involve the analytical solution of linear systems. Chapter 7 covers analytical methods for obtaining the system's response in the time domain with an emphasis on first- and second-order system response. Here, the two key concepts are (1) the correlation between the roots of the characteristic equation and the form of the free (or transient) response; and (2) the equivalence of the characteristic roots, poles of the transfer function, and eigenvalues of the system matrix. Chapter 8 presents a brief overview of Laplace transform theory and its use in obtaining the response of linear dynamic systems. Chapter 9 involves frequency response, or the system response to periodic input functions. In this chapter, the emphasis is on the Bode diagram as a graphical depiction of the information required for complete frequency-response analysis.

Chapter 10 introduces feedback control systems where the PID controller (and its variants) is emphasized. Two graphical techniques, the root-locus method and the Bode diagram, are used to analyze the closed-loop response, design controllers, and assess stability. The chapter closes with a brief discussion of how controllers are implemented as discrete-time algorithms in a digital computer.

Serving as a capstone for the textbook, Chapter 11 presents case studies in dynamic systems and control. Mixed-discipline, integrated engineering systems, inspired by the research literature, serve as the case studies. The five case studies illustrate the major topics of the textbook: (1) developing mathematical models, (2) predicting the system's behavior using analytical and numerical methods, and (3) selecting the important system parameters in order to improve performance. Simulink is used extensively to obtain the dynamic response of these systems that often involve nonlinearities and other complexities.

Numerous examples are provided at key locations throughout Chapters 2 to 10 in order to illustrate the topic discussed by the particular section. Chapters 2–10 contain end-of-chapter problems that are grouped into three categories: (1) conceptual problems, (2) MATLAB problems, and (3) engineering applications. In many cases, physical engineering systems (such as a suspension system, solenoid actuator, or filter circuit) are revisited throughout the textbook in the chapter examples and problems. As with the capstone Chapter 11, many of the example and end-of-chapter problems illustrate concepts in dynamic systems and control by presenting "real-world," physical engineering systems.

Appendix A presents the basic and derived units used in this textbook. Appendix B provides a brief introduction to MATLAB, M-files, and the commands that pertain to solving problems involving dynamic systems and control. Appendix C is a primer on Simulink and expands on the brief Simulink tutorial covered in Chapter 6.

As previously mentioned, this textbook is an outgrowth of 20 years of teaching a system dynamics course. In the long time that I have taught this course, I have often employed a trial-and-error approach to determining an optimum set of strategies for maximizing student understanding of key topics. Some of these strategies are unique to my teaching, and I outline them here so that the reader can see how this textbook marks a departure from others in the field.

1. Because system analysis and control system design begin with developing the appropriate mathematical models, the foundations for modeling all physical systems are presented in sequence by Chapters 2–4. The goal of these chapters is to present modeling explanations that are concise and clear, and this objective is achieved by focusing on essential, representative systems. Chapter 5 succinctly presents all standard forms for representing system models so that the fundamentals of system modeling are covered early in the textbook and not spread throughout the manuscript.

2. Numerical simulation of linear and nonlinear systems using MATLAB and Simulink (Chapter 6) are covered before analytical methods (Chapters 7–9). My experience is that presenting modeling and simulation of real engineering applications early in the semester engages the students in the subject material. Industry has long used Simulink to model and analyze real engineering systems. My classroom experience has shown that undergraduate students are quite capable of – and enthusiastic about! – using Simulink to simulate complex, integrated systems that involve real effects such as nonlinear friction, turbulent flow, and modeling discontinuities. Instructors who wish to present analytical methods before numerical simulation may simply cover Chapter 6 after Chapters 7 and 8.

3. The methods for obtaining analytical solutions do not rely on Laplace transform theory. In Chapter 7 (time-domain response) the characteristic equation and the associated characteristic roots (or transfer function poles or eigenvalues) are used to determine the transient response. The frequency response is derived in Chapter 9 by determining the forced response to the input e^{st} where $s = \sigma + j\omega$ is a complex variable. Transfer functions are used extensively in this textbook and are derived without using Laplace transforms. I believe that using Laplace transforms (and the associated inversion process) to obtain the response is unduly tedious; furthermore, my experience is that most students find their

use nonintuitive. Therefore, my approach is to focus on the characteristic roots r_i and the *form* of the system response in terms of $e^{r_i t}$. Chapter 8 presents the Laplace transformation and its use to obtain the dynamic response. Therefore, instructors who wish to utilize Laplace transform methods may do so. However, Chapter 8 may be omitted if the instructor does not care to include the Laplace transform method.

4. Chapter 11 presents five engineering case studies in dynamic systems and control. The five case studies, inspired by research articles, are (1) vibration isolation in a vehicle suspension system, (2) an electromechanical (solenoid) actuator, (3) a pneumatic air-brake system, (4) hydraulic servomechanism control, and (5) feedback control of a magnetic levitation system. These cases illustrate topics from the previous chapters: modeling, simulation, linearization, analytical methods, and control. Instructors may wish to introduce the various applications contained in Chapter 11 early in the semester as certain topics are covered and then revisit the cases studies as the course progresses. Again, I cannot overstate the importance of presenting analysis of *real*, complex engineering systems to undergraduate students in order to motivate them, engage their interest, and illustrate key concepts in dynamic systems and control.

Several people have contributed to the production of this textbook. I would like to thank Roger C. Fales, my colleague at the University of Missouri, for his expert knowledge and comments on fluid systems. Two students at the University of Missouri provided tremendous assistance: Annemarie Hoyer searched the engineering literature and helped with the solution manual, and James Smith created illustrations for the draft manuscript. Many reviewers provided valuable suggestions for improving this textbook and they are listed here:

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Chapter 1

Introduction to Dynamic Systems and Control

1.1 INTRODUCTION

In solving engineering problems, there is a need to understand and determine the dynamic response of a physical system that may consist of several components. These efforts involve *modeling*, *analysis*, and *simulation* of physical systems. Typically, building a prototype system and conducting experimental tests are either infeasible or are too expensive for a preliminary design. Therefore, mathematical modeling, analysis, and simulation of engineering systems aid the design process immensely.

Dynamic systems and control involves the analysis, design, and control of physical engineering systems that are often composed of interacting mechanical, electrical, and fluid subsystem components. One example is an electrically controlled hydraulic actuator that is used to change the position of an aerodynamic surface (e.g., rudder) on an airplane. This system consists of several interacting components: an electromagnet circuit is used to open a mechanical valve that allows high-pressure hydraulic fluid to flow into a cylinder chamber; the fluid pressure causes a mechanical piston to move; and mechanical linkages connecting the hydraulic piston to the aerodynamic surface (e.g., rudder) cause the rudder to change position. Finally, an onboard digital computer (an "autopilot") uses feedback from sensors to adjust the operation of the hydraulic actuator so that the rudder position (and the subsequent response of the airplane) matches the desired value. This example demonstrates why it is advantageous for the engineer to understand the dynamic response of this interconnected system without relying on experimentation with a physical prototype.

Here are definitions of the important terms that we use throughout the book:

System: A combination of components acting together to perform a specified objective. The components or interacting elements have cause-and-effect (or input-output) relationships. One example of a system is a direct-current (DC) motor where a voltage input causes angular velocity (the output) of the mechanical load attached to the motor's shaft.

Dynamic system: A system where the current output variables (or *dynamic variables*) depend on the initial conditions (or stored energy) of the system and/or the previous input variables. The dynamic variables of the system (e.g., displacement, velocity, voltage, pressure) vary with time. For the DC motor example, the angular velocity of the motor is the dynamic variable and the circuit voltage is the input.

Modeling: The process of applying the appropriate fundamental physical laws in order to derive mathematical equations that adequately describe the physics of the engineering system. Dynamic systems are represented by differential equations. For the DC motor example, the electrical circuit is modeled by using Kirchhoff's voltage law and the mechanical motion is modeled by using Newton's second law.

Mathematical model: A mathematical description of a dynamic system's behavior, which is usually a set of linear or nonlinear ordinary differential equations (ODEs). For the DC motor example, the mathematical model consists of a differential equation for the electrical current and a differential equation for mechanical motion.

Simulation: The process of obtaining the system's dynamic response by numerically solving the governing modeling equations. Simulation involves numerical integration of the model's differential equations and is performed by digital computers and simulation software.

System analysis: The use of analytical calculations or numerical simulation tools to determine the system response in order to assess its performance. Repeated analysis aids the design process where the system's configuration or parameters are altered to improve performance or meet desired constraints. For the DC motor example, we might apply a constant voltage input and determine the characteristics of the angular velocity response by using analytical calculations ("by hand") or numerical simulations. If the angular velocity response is inadequate, we could alter the system's parameters in order to improve performance.

1.2 CLASSIFICATION OF DYNAMIC SYSTEMS

In general, we can classify dynamic systems according to the following four categories: (1) distributed vs. "lumped" systems, (2) continuous-time vs. discrete-time systems, (3) time-varying vs. time-invariant systems, and (4) linear vs. nonlinear systems.

Distributed vs. Lumped Systems

A *distributed system* requires an infinite number of "internal" variables, and, therefore, the system is governed by partial differential equations (PDEs). A *lumped system* involves a finite number of "internal" variables, and, therefore, the system is governed by ODEs. For example, if we want to model a hydraulic piston, we would "lump" all pressure distributions in a cylinder chamber into one single pressure term. Therefore, we would have one ODE for the time derivative of pressure (dP/dt or $\dot{P}$) for each "lump" of fluid in a particular chamber. In this textbook we work exclusively with lumped systems and ODEs.

Continuous-Time vs. Discrete-Time Systems

A *continuous-time system* involves variables and functions that are defined for all time, whereas a *discrete-time system* involves variables that are defined only at discrete time points. We may think of continuous-time systems consisting of variables in the "analog" domain, such as position $x(t)$. Discrete-time systems consist of variables in the "digital" domain, such as the sampled (measured) position $x(kT_s)$ that exists only at the discrete-time points $t = T_s, t = 2T_s, \ldots, t = kT_s$ where T_s is the sampling interval. Continuous-time systems are described by differential equations while discrete-time systems are described by difference equations. In this volume we work with continuous-time systems and differential equations. We introduce discrete-time systems in Chapter 10 when we examine the role of digital computers in automatic control systems and the need to convert analog signals to digital signals and vice versa.

Time-Varying vs. Time-Invariant Systems

In a *time-varying system* the system parameters change with time (e.g., the friction coefficient changes with time). In a *time-invariant system* the parameters remain constant. The reader should not confuse the variation of the system parameters with the variation of the dynamic variables. For the DC motor example, the system parameters would be electrical resistance of the circuit, inductance of the coil windings around the rotor, friction coefficient for the rotor bearings, and moment of inertia of the rotor. If these system parameters do not change with time (i.e., they are constants for the system model), then the DC motor is a time-invariant system. Of course, the dynamic variables associated with the DC motor (electrical current of the circuit and angular velocity of the output shaft) can change with time. We focus primarily on time-invariant systems in this text.

Linear vs. Nonlinear Systems

Suppose we have a system or input-output relationship that is described by the function $y = f(u)$ where u is the input and y is the output. Linear systems obey the *superposition property*:

1. If $y_1 = f(u_1)$, then $ay_1 = f(au_1)$, where $a =$ any constant.
2. If $y_1 = f(u_1)$ and $y_2 = f(u_2)$, then $y_1 + y_2 = f(u_1 + u_2)$.

Consider again the DC motor example: suppose we apply 12 volts (V) to a motor and through measurements determine the steady-state (constant) angular velocity to be 1600 revolutions per minute (rpm). Next, if we apply 6 V to the motor and the measured steady-state angular velocity is 800 rpm then the system obeys the first superposition property and the DC motor system is linear. Of course, a physical system that demonstrates linearity (such as the DC motor) has a limited linear range of operation; that is, we cannot increase the input voltage by a factor of 100 and expect the corresponding angular velocity to increase by a factor of 100. Increasing the system input beyond a threshold may cause the output to saturate (i.e., reach a limit) and, therefore, the system is no longer linear.

The second superposition property shows that the total dynamic response of a linear system can be obtained by adding or superimposing the responses (or solutions) to individual input functions. Nonlinear systems do not obey either superposition property.

The following equations are examples of *linear* ODEs:

$$\ddot{x} + 3\dot{x} - 40x = 6u \tag{1.1}$$

$$2\ddot{x} + 0.4\dot{x} + 0.6e^{-2t}x = -8u \tag{1.2}$$

Equation (1.1) is a second-order linear ODE because the dynamic variable x and its derivatives appear as linear combinations (we will use the over-dot notation to denote derivatives with respect to time; hence $\dot{x} = dx/dt$, $\ddot{x} = d^2x/dt^2$, etc). Equation (1.1) involves constant coefficients and hence it is a *linear time-invariant* (LTI) differential equation. Equation (1.2) is linear as x and its derivatives appear in linear combinations. Because the coefficient $0.6e^{-2t}$ changes with time, Eq. (1.2) is a *linear time-varying* ODE. The following equation

$$2\ddot{x} + 3\dot{x} + 16x^2 = 5u \tag{1.3}$$

is a *nonlinear* ODE because of the x^2 term.

All physical systems are nonlinear. However, if we confine the input-output variables to a restricted (nominal) range, then we can often replace a nonlinear system with a *linear* model comprising linear differential equations. This important process is called *linearization*. Obtaining a linear model is extremely important and advantageous in system analysis because it is possible to obtain the analytical (closed-form) solution to linear ODEs. Nonlinear systems must be solved by using numerical methods to integrate the ODEs.

1.3 MODELING DYNAMIC SYSTEMS

A major focus of this book is mathematical modeling of dynamic systems. Developing an appropriate model is always the first step in system analysis because it is impossible to determine the system's response without a mathematical representation of the system dynamics. Mathematical models are obtained by applying the appropriate laws of physics to each element of a system. Some system parameters (such as friction characteristics) may be unknown, and these parameters are often determined through experimentation and observation, which lead to empirical relations. Engineering judgment must be used to trade model complexity with the accuracy of the analysis. Nonlinearities (such as gear backlash) are often ignored in preliminary design studies in order to derive linear models. Sometimes, low-order approximate linear models can be developed to accurately represent the system dynamics. These low-order linear models can be solved analytically ("by hand"), which gives the engineer an intuitive feel for the nature of the dynamic system. Furthermore, simulations are easier to construct with low-order linear models and therefore the time required to perform system analysis is reduced. Nonlinear models, on the other hand, require numerical solutions using simulation software. Extremely complex nonlinear models typically require small integration time steps to accurately solve the ODEs thus increasing computer-run time. Consequently, there is usually a trade-off between model complexity and analysis time.

Engineers must remember that the results obtained for a particular mathematical model are only approximate and are valid only to the extent of the assumptions used to derive the model. The model must be sufficiently sophisticated to demonstrate the significant features of the dynamic response without becoming too cumbersome for the available analysis tools. The validity of a mathematical model can often be verified by comparing the model solution (such as simulation results) with experimental results. The Shuttle Avionics Integration Laboratory (SAIL) was a hardware-in-the-loop test facility at NASA Johnson Space Center [1]. SAIL consisted of actual Space Shuttle hardware (such as the flight deck, cockpit displays, sensors, and electronic wiring) and mathematical models of the physical forces due to aerodynamics, gravity, and propulsion. Engineers and astronauts used SAIL to perform "real-time" simulations of Space Shuttle missions in order to test and validate the flight software. The simulation results from SAIL tests showed an excellent match with actual Shuttle flight data. SAIL tests, however, could occasionally be intermittent owing to their reliance on using very complex mathematical models (i.e., computer software) to interface with and drive all of the physical Shuttle hardware. The SAIL facility is an example of one extreme end of the mathematical modeling spectrum: a complex, "high-fidelity" simulation that was at times prone to sporadic testing. This trade-off was necessary in order to accurately model the Shuttle's flight dynamics.

Simulation Tools

Several commercial simulation tools have been developed to help engineers design and analyze dynamic systems. We briefly discuss a few of these software tools as examples of mathematical modeling and system analysis.

Simulink is a numerical simulation tool that is part of the MATLAB software package developed by MathWorks [2]. It uses a graphical user interface (GUI) to develop a block diagram representation of dynamic systems. Simulink is used by engineers in industry and academia. Constructing system models with Simulink is relatively easy and, therefore, it is often used to build simple models during the preliminary design stage. However, Simulink can be used to simulate complex, highly nonlinear systems. In this book, we use it extensively to simulate and analyze dynamic systems.

Caterpillar Inc. has developed the simulation tool Dynasty that allows engineers to construct complex models of large off-road vehicles [3]. The engineer can build software models of integrated machines by "dragging-and-dropping" subsystem models from a library. These subsystems include engines, linkages, drive trains, hydraulics, and controls. The underlying physics of each subsystem are contained within the mathematical model of the individual component. The Dynasty software simulates the dynamics of the integrated vehicle model and allows engineers to perform tests, analyze the dynamic response, and vary the subsystem components in order to improve overall system performance. Caterpillar engineers used Dynasty to analyze and design the 797B mining truck (its largest vehicle) and bring it to production in less than half the time it would take by building physical prototypes of the truck.

EASY5, originally developed by Boeing, is a graphics-based simulation tool for constructing virtual prototypes of engineering systems [4]. As in Dynasty, the user can select prebuilt components from libraries that include models of mechanical, electrical, hydraulic, pneumatic, and thermal subsystems. EASY5 can interface with Simulink and other computer-aided engineering software tools. Engineers have used EASY5 to analyze and design aerospace vehicles, for example.

In summary, it should be noted that all numerical simulation tools are constructed using the basic principles of mathematical modeling that are presented in this book. That is, the appropriate physical law (e.g., Newton's second law, Kirchhoff's voltage law) is applied to the particular system (mechanical, electrical, fluid, etc.) in order to develop the differential equations that describe the system dynamics. The differential equations are then solved using numerical integration methods. The solution to the differential equations is the system's dynamic response.

1.4 OBJECTIVES AND TEXTBOOK OUTLINE

The objective of this book is to present a comprehensive yet concise treatment of dynamic systems and control. In particular, on completing this volume, the reader should be able to accomplish the following tasks: (1) develop the mathematical models for mechanical, electrical, fluid, or thermal systems; (2) obtain the system's dynamic response (due to input functions and/or initial energy storage) by using numerical simulation tools and analytical techniques; and (3) analyze and design feedback control systems in order to achieve a desirable system response. This book primarily emphasizes lumped, continuous-time, LTI systems. Hence, all mathematical models involve ODEs and the majority have constant coefficients. Nonlinear systems are given considerable attention, and, therefore, we make frequent use of Simulink to obtain the dynamic response. Furthermore, we often utilize the linearization process in order to approximate the nonlinear dynamics with linear system dynamics. As is demonstrated, obtaining a linear mathematical model allows us to use a wealth of analytical tools for system analysis and graphical techniques for designing feedback control systems.

This book is organized according to its three major objectives. Chapters 2–4 deal with developing the mathematical models of physical engineering systems. In particular, Chapter 2 treats mechanical systems and the derivation of the modeling equations by applying Newton's laws of motion. Chapter 3 deals with mathematical models for electrical and electromechanical systems. Here we apply the element laws that govern the interaction between electrical charge, current, magnetic flux, and voltage. Mathematical models for fluid and thermal systems are developed in Chapter 4 by utilizing the conservation of mass and the conservation of energy, respectively. In Chapters 2–4 we focus on developing mathematical models of "real-world" physical engineering systems such as vehicle suspension systems, energy-transmission devices, and systems with mixed disciplines such as electromechanical, hydromechanical, and pneumatic actuators. Chapter 5 deals with the standard formats for representing the various mathematical models derived in the previous three chapters. These standard formats facilitate the second major topic of the book – obtaining the system response – whether we use numerical or analytical techniques.

Chapter 6 begins the system-analysis section of the book. This chapter introduces Simulink as the numerical simulation tool of choice for obtaining the response of linear and nonlinear dynamic systems. Chapter 7 presents analytical techniques for solving the mathematical modeling equations "by hand." Here we analyze the total system response (comprising the transient and steady-state responses) to input functions. In Chapter 8 we introduce the Laplace transformation method for obtaining the response of dynamic systems that are modeled by LTI differential equations. Chapter 9 deals with obtaining the response of a dynamic system that is driven by an oscillating or harmonic input function. This frequency-response analysis is aided by graphical techniques such as the Bode diagram.

Chapter 10 introduces the reader to the third major objective of the textbook: the analysis and design of feedback control systems. Here we investigate the use of feedback (from measurement sensors) to shape the system input function in order to achieve a desirable output response. Although different control schemes are discussed, this chapter emphasizes the proportional-integral-derivative (PID) controller (and its variants) because it is the most widely used control scheme in industry. Control-system design is aided by two graphical techniques, the root-locus method and the Bode diagram, that are discussed in this chapter.

Chapter 11 presents five engineering case studies that demonstrate the three major objectives of this textbook: modeling, analysis, and control of dynamic systems. These examples are inspired by research from the engineering literature and involve physical systems such as vehicle suspensions and actuators. The final chapter serves as a "capstone" for this book.

Appendix A presents units and Appendix B gives a brief overview of MATLAB usage, its commands, and programming with MATLAB. Only the MATLAB commands that pertain to solving problems in dynamic systems and control are presented in Appendix B. Appendix C is a tutorial on using Simulink to simulate linear and nonlinear dynamic systems.

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Chapter 2

Modeling Mechanical Systems

2.1 INTRODUCTION

The objective of this and the next two chapters is to develop the mathematical models of physical engineering systems. This chapter introduces the fundamental techniques for deriving the modeling equations for mechanical systems. These systems are composed of inertia, stiffness, and friction elements. Our mathematical models of mechanical systems are developed by applying Newton's laws of motion, which govern the interaction between force, mass, and acceleration. We utilize a lumped-system approach, and, therefore, the mathematical model consists of ordinary differential equations (ODEs). Mechanical systems with translational motion and rotational motion about a fixed axis are treated in this chapter.

The reader should keep in mind that the overall goal of this chapter is to derive the mathematical models that govern the behavior of mechanical systems. We do not (yet) attempt to obtain the mechanical system's response to force or motion inputs. Obtaining the system's response to known inputs is discussed in Chapters 6–9.

2.2 MECHANICAL ELEMENT LAWS

A mechanical system is composed of inertia, stiffness, and energy-dissipation elements. In addition, it may possess mechanical transformers, such as gears or levers. This section presents brief descriptions of the fundamental laws that govern these mechanical elements.

Inertia Elements

Inertia elements are either lumped masses (translational mechanical systems) or moments of inertia (rotational mechanical systems). They are easily identified in Newton's second law

$$\text{Force} = \text{mass} \times \text{acceleration} \qquad \text{(translational system)}$$

$$\text{Torque} = \text{moment of inertia} \times \text{angular acceleration} \qquad \text{(rotational system)}$$

Therefore, the inertia element is the ratio of force and acceleration (or torque and angular acceleration). A rigid body that has translational ("straight-line") motion has all of its mass lumped into a single element, m, with units of kg. A rigid body with purely rotational motion about an axis has all of its mass lumped into a moment of inertia, J, which is defined as

$$J = \int r^2 dm \tag{2.1}$$

where dm is an incremental mass with radial distance r from the axis of rotation. Equation (2.1) shows that J has units of kg-m^2. Equations for moments of inertia can be derived for homogeneous, rigid bodies with standard shapes. One example is a cylindrical disk with radius R and a uniform mass distribution with total mass M. The moment of inertia about the axis of symmetry for a uniform disk is

$$J = \frac{1}{2}MR^2 \tag{2.2}$$

Inertia elements can store potential energy due to position in a gravitational field, or kinetic energy due to motion. Potential energy ξ_P of a mass m in a uniform field with gravitational constant g is

$$\xi_P = mgh \tag{2.3}$$

where h is the vertical position of the mass measured from a reference height. Equation (2.3) shows that potential energy has the dimensions of force (mg) and length (h), or units of N-m or joule (J). Kinetic energy ξ_K of mass m moving with velocity $\dot{x} = dx/dt$ is

$$\xi_K = \frac{1}{2}m\dot{x}^2 \tag{2.4}$$

Kinetic energy of moment of inertia J rotating with angular velocity $\dot{\theta}$ is

$$\xi_K = \frac{1}{2}J\dot{\theta}^2 \tag{2.5}$$

As stated in Chapter 1, we adopt the over-dot convention throughout this book to indicate the derivative with respect to time; hence $\dot{\theta} = d\theta/dt$ and $\ddot{x} = d^2x/dt^2$. Equation (2.4) shows that translational kinetic energy has dimensions of mass (m) and velocity squared ($\dot{x}^2$), or units of kg-m^2/s^2, which is equivalent to N-m or joules. Equation (2.5) shows that rotational kinetic energy has dimensions of moment of inertia (J) and angular velocity squared ($\dot{\theta}^2$), or units of kg-m^2 rad^2/s^2, which is equivalent to N-m or joules. Clearly, all energy equation expressions must have the same units of N-m or joules.

Stiffness Elements

When a mechanical element stores energy due to a deformation or change in shape, it can be modeled as a stiffness element. In such cases, a fundamental relationship between force and the resulting deformation is required to model stiffness. The simplest force–deformation relationship is Hooke's law, which states that the force required to stretch or compress a spring is proportional to the displacement. Figure 2.1 shows a spring that is fixed at its left end, but free at the right end. Suppose a tensile force F is applied at the right (free) end and x is the corresponding displacement of the free end from its equilibrium (unstretched) position. The force required to produce displacement x is

$$F = kx \tag{2.6}$$

where k is called the *spring constant* and has units of N/m. Clearly, Eq. (2.6) is a *linear* relationship between force and displacement. Figure 2.1 shows that the positive convention for displacement x is to the right and,

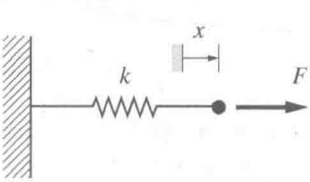

Figure 2.1 Force stretching the free end of a spring.