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Non-local Structural Mechanics

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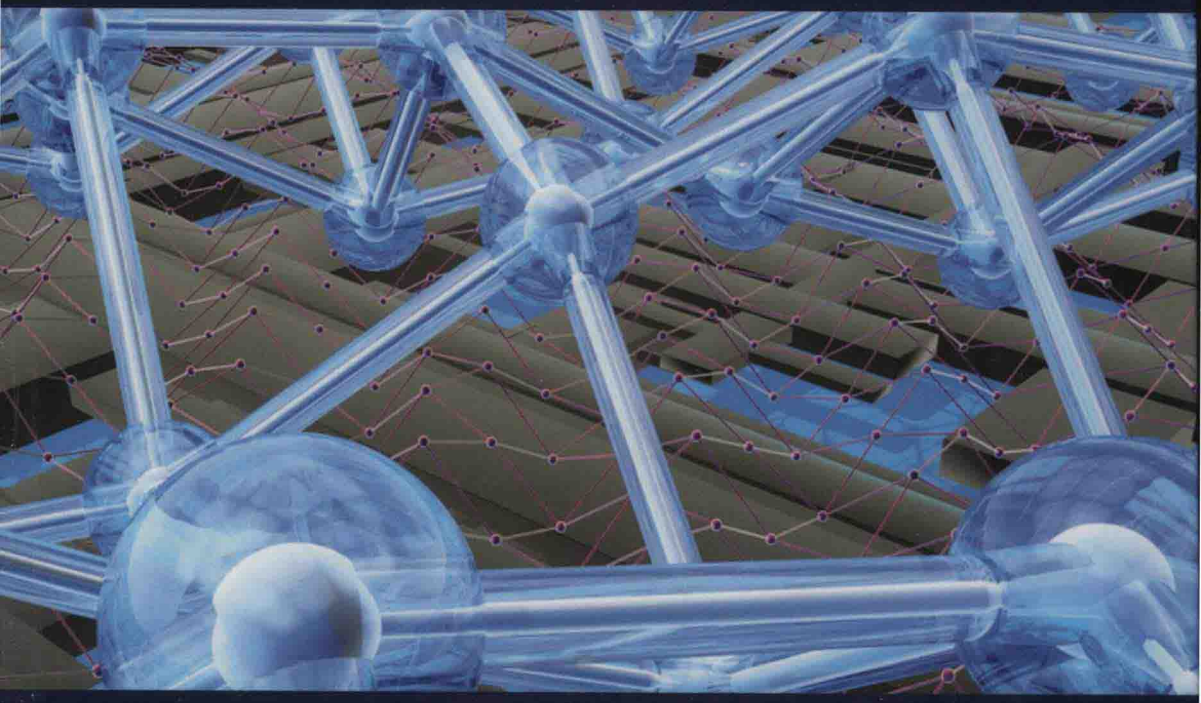

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Non-local Structural Mechanics

Preface

Nanoscale experiments demonstrate that the mechanical properties of nanodimensional materials are much influenced by "size effects" or "scale effects". Over the past decade, the non-local elasticity theory (non-local mechanics) has emerged as one widely promising size-dependant continuum theory. Significant progress has been made in fundamental and applied computational research in this area. The robust conventional local elasticity theory underpins the bulk of application of continuum mechanics in applied science and engineering since its inception in the early 19th Century. The utilization of the local elasticity theory in the context of nanoscale objects (such as carbon nanotube and graphene structures, etc.) has been questioned repeatedly in various research articles over the past decade. Non-local elasticity theory, pioneered from 1970s, can be applied over all scales and is considered to be more suitable for analyzing popular nanoscale objects such as carbon nanotube and graphene sheets.

This book is an initial comprehensive text to cover non-local elasticity theory for static, dynamic and stability analysis of a wide-ranging nanostructures. The authors have drawn on their own research experience to write this book. The text is written from a mechanics standpoint, comprising fundamental and complex theories that are relevant across a wide range of nanomechanical systems. The book introduces the reader to the fundamentals, as well as more in-depth aspects, of non-local mechanics and the associated latest research applications. The book brings together the vast research work for non-local mechanics in the context of nanoscale structures such as nanotubes and graphene sheets. The aim of this book is to systematically present the latest developments in the modeling and

analysis of popular nanostructures. The authors have chosen to focus on the mathematical and computational aspects. This book will be relevant to aerospace, mechanical and civil engineering disciplines and various subdisciplines within them. The intended readers of this book include senior undergraduate students and graduate students doing projects or doctoral research in the field of small-scale structures. Researchers, professors and practicing engineers working in the field of small-scale structures will find this book useful.

There are very few books which are dedicated to non-local continuum mechanics, one of which is the classic book by Eringen from 1980s. Eringen's book explains the fundamental and origin of non-local theory. The book by Gopalakrishnan and Narendar [GOP 13] is based on wave propagations in nanostructures where the non-local continuum mechanics is presented. The fundamentals of wave propagation in nanotubes and topics such as rotating nanotubes, coupled nanotubes and nanotubes are addressed in this book. A recent book by Elishakoff [ELI 12] discusses in some detail the fundamental aspects of non-local beam mechanics for nanostructures applications. These books represent the state-of-the art at the time of their publications. And the contents of non-local mechanics theory are limited. The aim of this present book is to devote all its chapters on applications of non-local mechanics to nanoscale structures. As a significant amount of work has recently gone into the research of non-local mechanics and many recent publications have been achieved, this book also covers some of these latest developments with an introduction to fundamentals in a concise way, focusing on theoretical and computational aspects, although some references to experimental works are given. This book aims to give science and engineering graduate students and researchers a detailed understanding of the methods of non-local analysis necessary for nanoscale structures.

Our book covers the essential fundamental applications and important references related to non-local mechanics theory. Chapter 1 gives an introduction to non-local elasticity mechanics. Vibration analysis of the simplest non-local elasticity theory which is the non-local rod theory is considered in Chapter 2. Chapter 3 considers non-local elastic beam theories in details. Important theories such as non-local Euler–Bernoulli beam theory, non-local Timoshenko beam theory and non-local Reddy beam theory are presented in the context of vibration and buckling. Chapter 4 gives an

introduction of non-local mechanics to two-dimensional small-scale structures via non-local plate theories. Non-local mechanics applied to simple double-nanobeam system is considered in Chapter 5. Chapter 6 considers double-nanoplate-system based on non-local elasticity theory. Chapter 7 describes the applications of non-local mechanics to multiple nanostructures. Cases related to multiple-nanorod, multiple-nanobeam and multiple-nanoplates are addressed in the context of vibration and buckling. Chapter 8 takes up the topic of use of computational method such as finite element method under the umbrella of non-local mechanics. Finite element methods for dynamics of non-local systems are concisely addressed in this chapter. Examples such as axial vibration of nanorod, bending vibration of nanobeams and transverse vibration of nanoplates are presented. How the non-local finite element is applied to nanodimension structure such as single-walled carbon nanotube, double-walled carbon nanotube and single layer graphene sheets is illustrated. Chapter 9 gives a detailed description of dynamic finite element analysis of axially vibrating non-local rods. Cases of mechanical damping are addressed in this chapter. Chapter 10 describes an important application of non-local mechanics to graphene structures such as in the field of vibration-based mass nanosensors. As non-local mechanics theories are recently validated with the molecular dynamics simulations, in Chapter 11 we give an introduction to molecular dynamics for small-scale structures.

This book is a result of 7 years of research in the area of non-local mechanics theory. The book's initial chapters began taking shape when Professor Adhikari and Dr Murmu were working on project of "scale dependent theory for nanomechanical systems" in Civil and Computational Engineering Center, University of Swansea, Wales, UK. Later chapters originated from research works with numerous colleagues, students, collaborators and mentors. We are deeply indebted to all of them for numerous stimulating scientific discussions, exchanges of ideas and in many occasions' direct contributions toward the intellectual content of the book. The authors particularly like to thank Dr S. C. Pradhan (IIT Kharagpur), Professor P. Kozić (University of Niš), Professor M.I. Friswell (Swansea University), Dr Y. Lei (Chansha), Professor F. Scarpa (University of Bristol), Dr C. Wang (Swansea University), Professor W.A. Curtin (École Polytechnique Fédérale de Lausanne) and Dr M. Cajić (Serbian Academy of Sciences, Belgrade).

Besides the names mentioned here, I am thankful to many colleagues, fellow researchers and students working in this field of research around the world, whose names cannot be listed here. The lack of explicit mentions by no means implies that their contributions are any less important. The opinions presented in this book are entirely of the authors, and none of our colleagues, students, collaborators and mentors has any responsibility for any shortcomings.

Tony MURMU
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1

Introduction to Non-local Elasticity

Recently, interest in nanotechnology is growing rapidly. The inventions of carbon nanotubes (CNTs) by Iijima [IIJ 91, IIJ 93] and successful extraction of graphene sheets [GEI 07] have motivated this interest. Because of its novel potential applications, recently nanomaterials have gained considerable attention among experimental, computational and theoretical research communities. As compared to more conventional materials, these nanomaterials possess superior mechanical, thermal, electrical and electronic properties [MOO 11]. Now, it is possible to arrange atoms into nanostructures that are only a few nanometers in size. For utilization and engineering of these nanoelements, proper experimental, computational and continuum mechanics-based methodologies are needed for future analysis in nanoengineering. One of the updated continuum mechanics methods for analysis of nanostructures is the non-local elasticity theory. In this chapter, we introduce some fundamental aspects to illustrate why nanostructures and non-local elasticity theory are important.

1.1. Why the non-local elasticity method for nanostructures?

The understanding of the mechanical response of nanoscale structures (small-scale structures of nanometer dimension), such as bending, vibration and buckling, is indispensable for the development and accurate design of nanostructures such as carbon nanotubes (CNTs) and graphene-based nanodevices. Figure 1.1 shows a single-walled CNT and single-walled graphene sheets. The dots in the figure represent carbon atoms. So far,

experimentation on the study of actions of structures at the nanoscale is achievable, but quite difficult. Handling each and every parameter at the nanometer scale is a complicated task. Furthermore, computer simulation methods such as molecular dynamics (MD) modeling and simulation of nanostructures is computationally very expensive and time-consuming for macroscale material systems. Also, from an engineer's perspective, we may not be expertise in MD as it involves more of a chemistry dealing with atoms, molecules, bonds and interatomic forces.

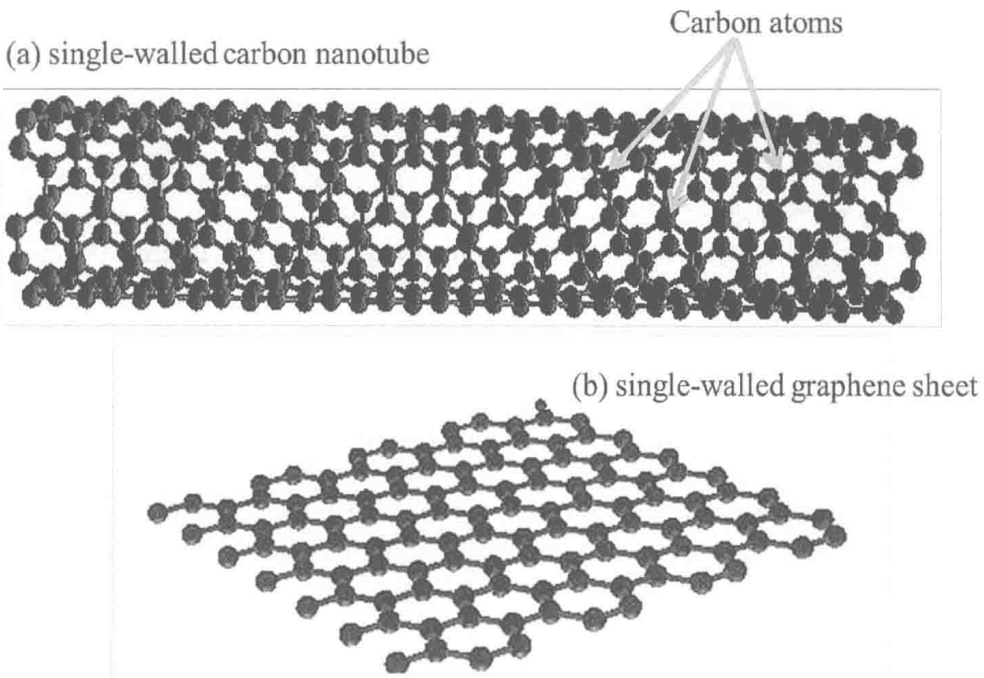

Figure 1.1. *Schematic diagrams of a) single-walled carbon nanotubes and b) single-walled graphene sheets. The mechanical behavior of these nanoscale structures can be analyzed by non-local elasticity theory along with molecular dynamics and experimental work. For a color version of the figure, see www.iste.co.uk/murmu/non-local.zip*

The experimentation and MD simulation for CNTs graphene and graphene-based systems are not always straightforward. So, how can these potential material nanostructures be effortlessly predicted in terms of bending, vibration, buckling and other studies for designing nanodevices (say in nanoelectromechanical systems, NEMS)? One approach is to utilize