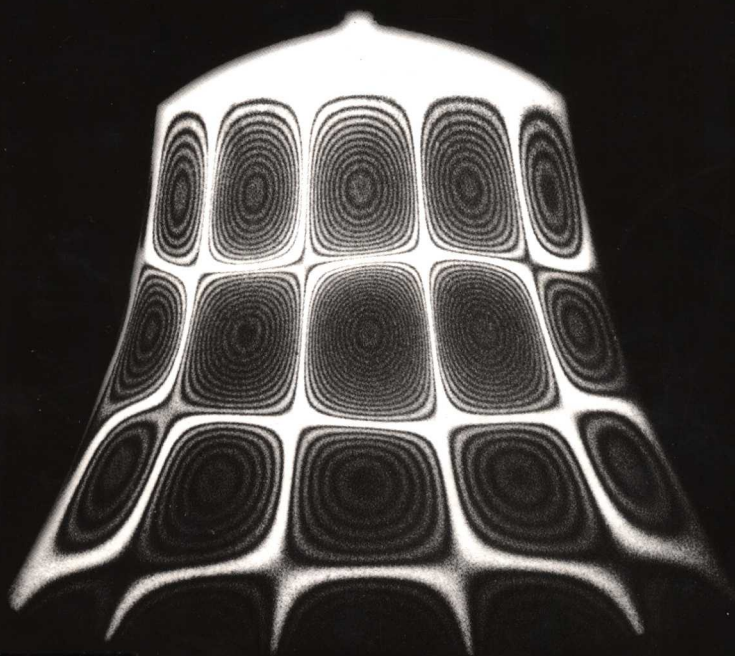


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PERCUSSION  
INSTRUMENTS



THOMAS D. ROSSING

World Scientific

Instruments

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Series in Popular Science – Vol. 3

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S C I E N C E   O F

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**PERCUSSION  
INSTRUMENTS**

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藏书章

**THOMAS D. ROSSING**

*Northern Illinois University*



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## ***Foreword***

Percussion instruments are amongst the oldest instruments in the world. They are also, undoubtedly, the most universal. The New Grove Dictionary of Musical Instruments has 1,523 different entries for drums alone. Yet, ironically, percussion has been slow to develop and be utilized in Western art music; hence, the lack of Western scientific interest in these instruments. While extensive research and discovery has been done for the voice, strings and wind instruments, very little attention has been paid to percussion instruments. In the music of the 20th century, however, everything has changed. Percussion has become a dominant player in both contemporary classical music, as well as pop and world music.

Percussion is incredibly versatile with the ability to produce a full range of timbre from noise to nearly pure sound and with a dynamic range that is probably only surpassed by electronic music. Percussion is also constantly evolving. And while the standard instruments will be here for a long time to come, newer instruments are being built and applied in various musical settings. The steel pan of Trinidad and Tobago is an example of a highly evolved instrument that was only conceived of in the middle of the 20th century but now holds an important place in musics of the world. The tuning refinement that has been done on steel pans is phenomenal and on par with many classical instruments which are many times older.

Fortunately, Thomas Rossing has chosen to devote a great deal of his research time exploring the vast array of percussion instruments and discovering the multitude of modes of vibration they produce. I had the privilege of working with Dr. Rossing at Northern Illinois University in the 1970s while teaching and pursuing my Master's degree there. I was able to apply the information and techniques that I learned from him in building and tuning my own percussion instruments. This eventually led to the creation of my own company, Woodstock Percussion, which manufactures Woodstock Chimes and other percussion instruments and distributes them throughout the world.

While the information found in Dr. Rossing's *Science of Percussion Instruments* is certainly important to anyone studying the science of sound, it is absolutely invaluable to the performer and instrument builder. Many percussionists intuitively understand the various instruments which they are called upon to play and use that knowledge to produce many different sounds on each and every one. So, in some cases, Dr. Rossing is merely confirming what percussionists already know intuitively. However, between the covers of his book are a number of wonderful surprises which undoubtedly will affect the way we approach our instruments. *Science of Percussion Instruments* can be used as a reference tool or read in its entirety. How you apply the wealth of information it contains is up to you. Enjoy, and best of luck in your journey.

*Garry Kvistad*  
*CEO, Woodstock Percussion, Inc.*  
*Shokan, New York*  
*January 2000*

## Preface

Although percussion instruments may be oldest musical instruments (with the exception of the human voice), relatively little has been written about scientific research on these instruments. By way of contrast, string and wind instruments have been the subjects of several good scientific books in recent years. Because the sounds of percussion instruments change so rapidly with time, their study and analysis require equipment that wasn't widely available until quite recently.

I began to study the science of percussion instruments some 25 years ago when Garry Kvistad, who was teaching percussion at Northern Illinois University at that time, asked me some interesting questions about their behavior. Garry and I did some experiments together, and we had many interesting discussions, some of which he carried with him, I believe, when he started his own company. Meanwhile we have studied the acoustics of a wide variety of percussion instruments. Many of them are discussed in *The Physics of Musical Instruments* (Springer-Verlag, New York, 1991, 1998).

Studying the science of percussion instruments has taken me all over the world and has put me in touch with a large number of interesting people: performers, teachers, instrument builders, and other scientists. Besides Garry Kvistad, I would especially like to mention Jacob Malta and André Lehr. Jake Malta, founder of Malmark, Inc., has been a friend for many years. André Lehr, who I consider to be the world's foremost authority on bells, has retired from the Royal Eijsbouts foundry but still devotes much time and effort to the National Carillon Museum in Asten in The Netherlands.

Is it necessary for a musician or a musical instrument builder to understand the science of their instrument? I would argue that it is if they want to compete with the best of their trade. Most builders of fine instruments have mastered the science of their instruments, although in many cases they have done it rather painfully by trial and error. Likewise, skilled performers have learned the science of their instruments by experience. I often remind my students that Stradivari knew all about the physics of violins but it took 300 years to learn it! It is my hope that studying this book will shorten the learning curve for both instrument builders and performers.

This book is written primarily for musicians, but it should be of interest to students of science as well. I have kept the mathematics as simple as possible by translating ideas from the language of physics (mathematics) to non-mathematical language. Readers who wish to go beyond the simple ideas in this book can easily follow the references to more scientific books and to the original scientific literature. Where some principles of physics or perception are necessary to understanding the concepts, these principles are briefly presented in "interludes."

Needless to say, I welcome comments from readers. Who knows, some of these comments may lead to further research. Happy reading.

Thomas D. Rossing  
DeKalb, Illinois, 1999

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## Chapter 1

### The Percussion Family

Percussion instruments may be our oldest musical instruments (with the exception of the human voice), but recently they have experienced a new surge in interest and popularity. Many novel percussion instruments have been developed recently and more are in the experimental stage. What is often termed “contemporary sound” makes extensive use of percussion instruments. Yet, relatively little material has been published on the acoustics of percussion instruments.

So reads the introduction to the chapter on percussion instruments in a textbook *The Science of Sound*, the first edition of which I wrote some twenty years ago. In the meantime we have studied the acoustics of many percussion instruments in our laboratory: timpani, snare drums, handbells, gongs, tamtams, cymbals, steel pans, and other instruments. Nevertheless, these words are still true; relatively little material has been published on the acoustics of percussion instruments.

#### 1.1. The Percussion Family

The term percussion means “struck” and strictly speaking percussion instruments are those in which sound is produced by striking. However, the percussion section in a modern orchestra employs many instruments that do not depend upon striking a blow. Indeed, the percussion section is expected to create any unusual sound effect that a composer has in mind. New instruments are constantly being invented and added to the percussionist’s repertoire.

There are several ways that have been used to classify percussion instruments. Sometimes they are classified into four groups: *idiophones* (xylophone, marimba, chimes, cymbals, gongs, etc.); *membranophones* (drums); *aerophones* (whistles, sirens, etc.); and *chordophones* (piano, harpsichord). There may be differences of opinion as to whether aerophones and chordophones properly belong in the percussion family. Whistles and sirens are generally played by percussionists in the orchestra; the piano and harpsichord are not. At any rate, this book deals mainly with idiophones and membranophones.

Another way of classifying percussion instruments is by whether or not they convey a definite sense of pitch. Idiophones that convey a definite pitch include bells, chimes, xylophones, marimbas, gongs, and steel pans. Membranophones that convey a definite pitch include timpani, tabla, and mrdanga. Sometimes we described percussive sounds as having a “high” or “low” pitch even if they do not convey an identifiable pitch, but it would be more correct to describe this as high or low range or tessitura.

Percussionists in a modern orchestra or band may have hundreds of instruments to play. Generally the timpanist plays only the timpani, but the other percussionists divide the remaining instruments depending upon the demands of the music. Some works require as many as ten or more percussionists; Schoenberg’s *Gurrelieder*, for example, calls for two timpanists and ten other percussionists.

Percussion instruments generally use one or more of the following basic types of vibrators: strings, bars, membranes, plates, air columns, or air chambers. The first four are mechanical; the latter two are pneumatic. Two of them (the string and the air column) tend to produce harmonic overtones; bars, plates, and membranes, in general, do not. The inharmonic overtones of complex vibrators give percussion instruments their distinctive timbres.

This book on the science of percussion instruments considers a large number of instruments: how they vibrate and how they produce sound. In order to understand this, we must consider some basic physics of vibrating systems as well as some psychoacoustics of hearing and perception. This will be done by inserting, when needed, sections or chapters dealing with fundamental principles. Often these are labeled as “interludes.” The musician without much previous study of the scientific principles may wish to refer back to these interludes from time to time as the book is being read.

## 1.2. Historical Notes

Most natural systems follow some type of rhythm: beating hearts, the motion of the planets, ocean waves, phases of the moon, the seasons, the list is long. It is only natural that primitive humans would begin striking sticks or stones together rhythmically. Rhythm is one of the key ingredients in music, and percussion instruments often establish and maintain the rhythm in the performance of music.

One of the best histories of percussion instruments is *Percussion Instruments and Their History* by renowned percussionist James Blades.[1] This book traces percussion instruments from their primitive origins to composers’ use of modern percussion.

Blades points out that the earliest instruments were probably idiophones, instruments made of naturally sonorous material which can produce sound without the addition of stretched skin or column of air. These are of five types: shaken idiophones (rattles), stamped idiophones (pits, boards, hollow tubes); scraped idiophones (notched sticks or rasps); concussion idiophones (pairs of similar items such as sticks); and struck idiophones (one or more pieces of sonorous material struck with a stick or bone).

Early in our lives we learned to play with rattles. It is interesting that rattles are among the earliest of percussion instruments. The gourd rattle, a seed pod in which the dried seeds remain, was widely used in primitive societies, especially in Africa. Rattles are still popular in orchestras and ensembles, especially for the performance of Latin American music.

Scraped instruments are found as far back as the early Stone Age. A stick could be drawn across a notched stone, bone, shell, or gourd to produce a raspy sound. The bone scraper has been closely associated with the hunt, erotic rituals, and funeral ceremonies. Scrapers were found among Indian tribes in North and South America, and also in Africa.

The earliest drums were probably log drums of various types. Later, it was discovered that by stretching an animal skin across the cavity in the log, a louder sound could be made. Eventually the membrane drum came to be the most important percussion instrument. The earliest drums were probably struck with the hands, but the use of sticks as

beaters was found to increase the loudness of the sound. Later a second membrane was added. Today, there are thousands of different types of drums found throughout the world. Throughout the years, drums have been used for signaling, for sending messages, and for marshaling troops to battle as well as for performing music.

### 1.3. Percussion Ensembles

Although percussion instruments have most often been used in ensemble with string and wind instruments, a number of successful ensembles have relied on percussion instruments alone. Sometimes these ensembles use one type of instrument, such as steel bands (see Chapter 10) and marimba orchestras (see Chapter 6), more often they employ a variety of percussion instruments, such as the Black Earth ensemble, shown in Fig. 1.1, whose members were once artists-in-residence at our university. A steel band is shown in Fig. 1.2.



Fig. 1.1. The Black Earth Percussion Ensemble at Northern Illinois University used a large variety of percussion instruments, as can be seen in this photo.



Fig. 1.2. The Northern Illinois University Steel Band is an example of a percussion ensemble using one type of percussion instrument.

## References

1. J. Blades, *Percussion Instruments and their History* (Faber and Faber, London, 1974).

## Chapter 2

### Drums with Definite Pitch

Drums have played an important role in nearly all musical cultures. They have been used to transmit messages, convey the time of day, send soldiers into battle, and warn of impending danger. Drums are practically as old as the human race.

The earliest drums were probably chunks of wood or stone placed over holes in the earth. Then it was discovered that more sound could be obtained from hollow tree trunks, the ancestors of our contemporary log drums. The most familiar type of drum consists of a membrane of animal skin or synthetic material stretched over some type of air enclosure.

In this chapter, we will discuss drums that convey a strong sense of pitch, including kettledrums, tabla, roto-toms, and boobams. We will learn that the modes of vibration of these drums have frequencies which are nearly harmonics of a fundamental, and that is why we hear a definite pitch.

#### 2.1. Vibrations of Strings: A Little Bit of Physics

A guitar string is probably the simplest of all musical vibrators. Yet its vibrations can be deceptively complex. When drawn to one side and released, the string vibrates in a rather complex way that can be described as a combination of normal modes of vibration. For example, if it is plucked at its center, the nearly triangular shape it assumes, as it vibrates to and fro, can be thought of as being made up of simple modes having frequencies that correspond to the fundamental plus the odd-numbered harmonics, as listed in Fig. 2.1.

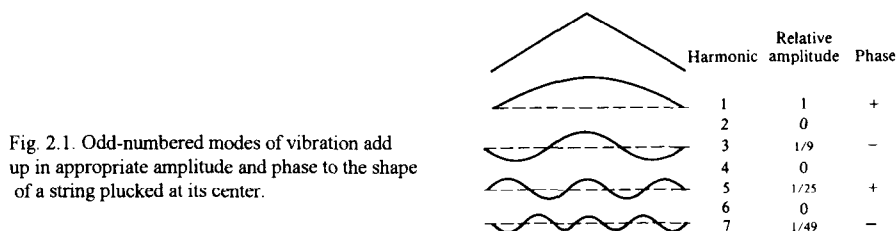


Fig. 2.1. Odd-numbered modes of vibration add up in appropriate amplitude and phase to the shape of a string plucked at its center.

Figure 2.1 illustrates how the modes associated with the odd-numbered harmonics, when each is present in the right proportion, can add up at one instant to give the initial shape of the string. Modes 3, 7, 11, etc. must be opposite in phase from modes 1, 5, and 9 in order to give a maximum in the center. Also, the relative amplitudes are in the ratios  $1/n^2$ , where  $n$  is the number of the mode.

The force necessary to restore the string to its center (equilibrium) position when it is displaced comes from the force or tension applied to the string. To tune the string to a higher frequency, the guitarist increases the tension by means of the tuning machines or pegs. Increasing the tension increases the frequencies of all the various modes of vibration but maintains their harmonic ratios.

## 2.2. Vibrations of Membranes: Key to Understanding Drums

A membrane can be thought of as a two-dimensional string, in that its restoring force is due to tension applied from the edge. A membrane, like a string, can be tuned by changing the tension. Membranes, being two-dimensional, can vibrate in many modes that are not normally harmonic; that is, the frequencies of the higher modes are not simple integers times the fundamental frequency.

Four modes of a circular membrane are shown in Fig. 2.2. In the first mode (the fundamental), the entire membrane moves in the same direction, although the center moves through the greatest amplitude. In the other modes, there are one or more nodal circles or nodal diameters that act as boundaries or pivot lines. The parts of the membrane on either side of a nodal line move in opposite directions.

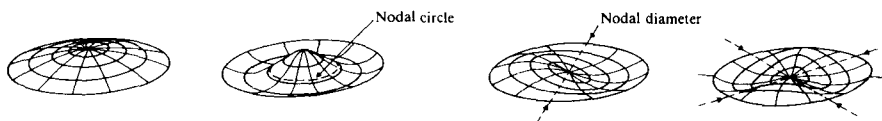


Fig. 2.2. Modes of a circular membrane. The first two modes have circular symmetry; the second two do not.

A membrane, like a string, can be tuned by changing its tension. A major difference between vibrations in a membrane and in a string, however, is that while the mode frequencies in a string are harmonics of the fundamental, in a two-dimensional membrane they are not. Another difference is that in a membrane, nodal lines (circles and diameters) replace nodal points along the string.

Nodal circles and nodal diameters in the first 12 modes of a membrane are shown in Fig. 2.3. A nodal circle always occurs at the edge of the membrane where it is supported. Above each diagram are given two numbers that designate the number of nodal diameters and circles, respectively. For example, the (21) mode has two nodal diameters and one nodal circle (at the edge).

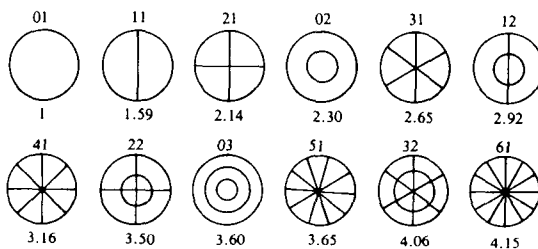


Fig. 2.3. Vibrational modes of a membrane, showing radial and circular nodes and the customary mode designation (the first number gives the number of radial modes, and the second the number of circular nodes, including the one at the edge). The number below each mode diagram gives the frequency of that mode compared to the fundamental (01) mode.

In either a string or a membrane, the modal frequencies vary as the square root of the tension. Thus to double the frequency, the tension would have to be quadrupled, which is quite impractical. A more practical example would be that to raise the frequency by 6 % (corresponding to a semitone on the musical scale) the tension would have to increase by about 12%.

Actually the frequency of a membrane is determined by the ratio of tension to mass per unit area, so increasing the thickness of a drumhead by 12% lowers the mode frequencies by 6% just as increasing the tension by 12% raises it by the same amount. Increasing the radius by 12%, on the other hand, decreases the mode frequencies by a full 12%.

### 2.3. Timpani

The timpani or kettledrums are the most frequently used drums in the orchestra, one member of the percussion section usually devoting attention exclusively to them. During the last century, various mechanisms were developed for changing the tension to tune the drumheads rapidly. Most modern timpani have a pedal-operated tensioning mechanism in addition to six or eight tensioning screws around the rim of the kettle. The pedal typically allows the player to vary the tension over a range of 2:1, which corresponds to a tuning range of about a musical fourth. A modern pedal-equipped kettledrum is shown in Fig. 2.4.

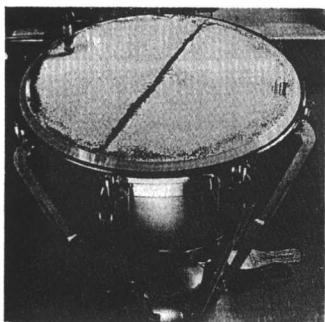


Fig. 2.4. Kettledrum.

At one time all timpani heads were calfskin, but this material has gradually given way to Mylar (polyethylene terephthalate). Calfskin heads require a great deal of hand labor to prepare and great skill to tune properly. Some orchestral timpanists prefer them for concert work under conditions of controlled humidity, but use Mylar when touring. Mylar is insensitive to humidity and easier to tune, due to its homogeneity. A thickness of 0.19 mm (0.0075 inch) is considered standard for Mylar timpani heads. Timpani kettles are roughly hemispherical; copper is the preferred material, although fiberglass and other materials are also used.

Although the modes of vibration of an ordinary membrane are not harmonic in frequency, a carefully tuned kettledrum sounds a strong fundamental plus two or more harmonic overtones. Lord Rayleigh [1] recognized the principal note as coming from the (11) mode and identified overtones about a perfect fifth (3:2 frequency ratio), a major seventh (15:8 frequency ratio), and an octave (2:1 frequency ratio) above the principal tone. Timpanist H. W. Taylor [2] identified a tenth (octave plus a third, 5:2 frequency ratio) by humming near the drumhead, a technique some timpanists use to fine-tune their instruments.

How are the inharmonic modes of an ordinary membrane coaxed into a harmonic relationship in the timpani? Three effects contribute:

(1) The membrane vibrates in a "sea of air," and the mass of this air lowers the frequency of the vibrational modes, especially those of low frequency;



- (2) The air enclosed by the kettle has resonances of its own that interact with the modes of the membrane that have similar shapes;
- (3) The stiffness of the membrane raises the frequencies of the higher overtones.
- Our studies show that the first effect (air loading) is mainly responsible for establishing the harmonic relationship of kettledrum modes; the other two effects only “fine tune” the frequencies but may have considerable effect on the decay rate of the sound [3].

#### 2.4. Timpani Sound

Vibration frequencies of a kettledrum, a drumhead with the kettle, and an “ideal” (unloaded) membrane are given in Table 2.1 along with ratios to the principal (11) mode. Note that the enclosed air in the kettle raises the frequencies of the (01), (02), and (03) modes which are circularly symmetrical. The (11), (21), (31), and (41) modes in the kettledrum have frequencies in the ratios 1 : 1.5 : 1.99 : 2.44, which is sufficiently close to the harmonic ratios 1 : 1.5 : 2 : 2.5 to give the kettledrum a strong sense of pitch. To preferentially excite these modes, the timpanist generally strikes the head about one-fourth of the way from the edge to the center. Striking the head at its center preferentially excites the rapidly-decaying (01) mode (along with the (02) and (03) modes, of course), producing a “thud.”

Table 2.1. Vibration frequencies of a 65-cm (26-inch) kettledrum, a drumhead without the kettle, and an ideal membrane (perfectly flexible and unloaded by air)

Mode	Kettledrum		Drumhead alone		Ideal membrane
	$f$	$f/f_{11}$	$f$	$f/f_{11}$	$f/f_{11}$
01	127 Hz	0.85	82 Hz	0.53	0.63
11	150	1.00	155	1.00	1.00
21	227	1.51	229	1.48	1.34
02	252	1.68	241	1.55	1.44
31	298	1.99	297	1.92	1.66
12	314	2.09	323	2.08	1.83
41	366	2.44	366	2.36	1.98
22	401	2.67	402	2.59	2.20
03	418	2.79	407	2.63	2.26
51	434	2.89	431	2.78	2.29
32	448	2.99	479	3.09	2.55
61	462	3.08	484	3.12	2.61
13	478	3.19	497	3.21	2.66
42			515	3.32	2.89

Sound spectra obtained by striking a 65-cm (26-inch) diameter kettledrum in its normal place (about one-fourth of the way from the edge to the center) and at the center are shown in Fig. 2.5. Note that the (0,1) mode appears much stronger when the drum is struck at the center, as do the other symmetrical modes [(0,2) and (0,3)]. These modes die out rather quickly, however, so they do not produce a very clear sound. In fact, striking the drum at the center produces quite a dull, thumping sound.