

# **Fundamentals of Automatic Control**

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FUNDAMENTALS OF AUTOMATIC CONTROL

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# Preface

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Automatic control is an essential part of modern technology. A machine tool, a chemical process, a space vehicle—virtually all segments of industry rely on automatic control systems to meet high performance requirements.

Today's control systems received much of their impetus from the military demands of World War II, and a well-established body of linear control theory evolved during the 1940s and 1950s. Automatic control methods based on this theory have proven themselves in practice and have been widely applied. Although recent developments have centered on the control of complex non-linear systems, the classical linear methods remain the foundation of automatic control, and it is the purpose of this text to present the fundamentals of these methods of analysis and design.

Engineering technicians are increasingly involved with control systems, not only from the standpoint of testing, calibration, and maintenance but also in the more standardized aspects of design. For this reason many engineering technology programs include an introduction to automatic control. Since the mathematics content of these programs generally includes an introduction to calculus, the subject of automatic control can be presented to technicians at a level that combines a mathematical basis with intuitive reasoning.

The various books on automatic control that have been published number well into the hundreds. Therefore, the question may be raised, "Why another book on this subject?" A cursory examination will reveal that most available texts are written at an engineering level. They emphasize the theoretical aspects of control systems and assume a thorough working knowledge of calculus and differential equations. Only a limited number of control books have been directed toward the needs of the technician, and these books tend to be descriptive in nature and to make minimal use of mathematics. This book attempts to bridge the gap between the elementary works and advanced works.

Although the theory and methods in the book are intended for a general controls application, the servomechanism is used throughout as a representative control system. Not only are servomechanisms extensively employed in industry but they are also well suited to the laboratory experience which is highly recommended in the study of automatic control.

In regard to prerequisites, a basic knowledge of physics, including mechanics and electricity and magnetism, is assumed. Some background in

electronics would be helpful in understanding amplifier topics; however, this is not vital to the overall subject. The book does not require an extensive mathematics background but does suppose a sound knowledge of algebra, trigonometry, and complex numbers. An introductory knowledge of calculus will be helpful in studying control theory from a mathematical point of view; however, this need not be extensive. A familiarity with the concepts of differentiation and integration should prove adequate. Differential equations are used in the text; however, they are introduced and explained as required for an understanding of the subject matter.

The first chapter of the book introduces the basic concept of negative feedback in automatic control and the block-diagram representation of control systems. Chapter 2 is concerned with the representation of physical systems by means of mathematical models. Chapter 3 describes a number of the devices used in automatic control categorized as transducers, power actuators, and amplifiers.

Chapter 4 describes the transient performance of simple control systems with a minimum of mathematical development. This discussion is continued in Chap. 5, which deals with the widely used Laplace transform method of time-domain analysis. This method is not covered rigorously but is presented as a working tool with emphasis on its application to control systems.

The frequency-response method of analysis and design is extensively covered in Chaps. 6 and 7. This method is largely graphical in nature and therefore should be readily comprehended and applied by the engineering technology student.

Chapter 8 discusses the testing of control systems from a hands-on point of view.

Nonlinearities and several other topics of an advanced nature are briefly introduced in the concluding chapter. Although a thorough treatment of nonlinearities is beyond the scope of this book, the technician should be aware of their presence and the limitations that they impose on linear analysis.

The book is appropriate for a one-term course in automatic control. Depending on the level of the students and the scope of the course, it would not be necessary to include all chapters. For a less analytical approach, Chaps. 2 and 5 may be largely deleted without loss of continuity. Chapter 3 could be covered selectively, omitting the sections covering devices not common to servomechanisms.

Although the book is specifically intended for the engineering technology student, it should also be useful to the practicing engineer or technician who wishes an introduction to automatic control.

The uniqueness of a basic text is not in the contents as such but in the presentation in a manner appropriate to a certain group of readers. For the subject matter of this book, the author is generally indebted to the many individuals who have contributed to the field of automatic control. In particular I would like to single out my former associates at the Goodyear Aerospace

Corporation with whom I worked on numerous control projects ranging from missile guidance to industrial process control.

I must also express sincere appreciation to my aunt, Miss Helen Hassler, for her indispensable help in converting the manuscript to typed form. Finally I am most grateful to my wife, Helen, for her continuous encouragement and understanding during the seemingly endless task of preparing the manuscript.

*Robert C. Vequik*

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# Chapter One

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## introduction

### 1-1 GENERAL INTRODUCTION

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Automatic control is playing an ever-increasing role in almost all phases of our modern way of life. From familiar household uses such as the automatic bread toaster and thermostatically controlled furnace to sophisticated control systems for power generation and space exploration, automatic control influences our daily living.

In a sense, the advent of automatic control represents a second industrial revolution. The industrial revolution of the nineteenth century made greater quantities of power available for man's use. Wind power and water power were largely displaced by the introduction of steam power. Fuels including coal, gas, and petroleum and its byproducts became the primary sources of energy for manufacturing and transportation. The form of energy that has played the greatest part in industrial development is electricity, and this is likely to continue with the conversion of nuclear energy to electrical form.

To utilize power effectively, it has been necessary to learn to control and regulate it. The industrial revolution of the twentieth century has been marked by man acquiring this ability. With power and the means of control of it, much of the physical and mental work can be done by machines with a higher degree of performance than would otherwise be possible.

Many persons engaged in industrial activities need to work with control systems, if only to use them. Consequently, to the engineer and technician a knowledge of automatic control is useful for its own sake. However, several other benefits are also to be acquired. Automatic control systems are physical systems which have dynamic behavior. Hence the study of control theory provides a basis for the general understanding of dynamic systems. Such understanding is vital since a system must be understood in order for it to be controlled effectively.

## 2 Fundamentals of automatic control

Control systems frequently employ components of many types, including mechanical, electrical, electronic, hydraulic, pneumatic, and combinations thereof. Hence, one who works in the control field must be familiar with the principles and characteristics of a broad range of devices.

Automatic control is an interdisciplinary subject since it draws on knowledge from many differing fields. Its study can be beneficial in bringing together a number of subjects that have been considered separately and applying them to a common problem.

The philosophy of automatic control also has other far-reaching implications. Although we will be entirely concerned with engineering applications of automatic control, its concept is being extended to other areas. Economic, business, and political systems are being increasingly regarded in terms associated with automatic control. Although the variabilities of human behavior diffuse the analysis of such systems, as the understanding of their dynamics increases, the ability to control them will increase also.

In addition the human body itself includes numerous complex control systems among its functions. The control of body temperature, chemical composition, and physical actions such as driving an automobile are examples of such body systems.

### 1-2 HISTORY OF AUTOMATIC CONTROL

Although earlier applications may have existed, the first control system not involving a human being is considered to be the flyball governor developed by James Watt in 1788. Watt concluded that a man opening and closing a steam valve was not the best way of keeping the speed of a steam engine constant. His governor, shown in Fig. 1-1,

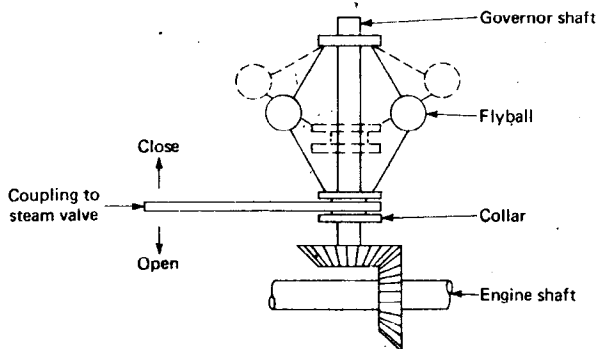


Figure 1-1 Flyball governor

is a completely mechanical device that connects to the shaft of a steam engine. As the speed of the governor shaft increases, the balls rotate faster and move outward. The coupling to the steam control valve then moves up the shaft, closing the valve and slowing the engine. A decrease in speed results in the opposite motion of the flyballs and a corresponding increase in engine speed.

It soon became apparent that an unfortunate byproduct of attempts to improve the accuracy of automatic control is instability. This is characterized by a tendency for the system to oscillate or hunt about a specified operating condition. In the case of governor control of the steam engine, the speed would tend to rise and fall about the desired value. Early efforts to eliminate this problem were largely on an intuitive or trial-and-error basis.

The first theoretical analysis of automatic control was a study of the stability of the flyball governor by James Clerk Maxwell in 1868.<sup>1</sup> Shortly thereafter other significant contributions to the theory of automatic control were made by Hurwitz in 1875, Routh in 1884, and Liapounov in 1892. Further applications of governors to other engines soon appeared; and, in the early 1900s, their application in industrial control began. About the same time, automatic control devices were studied for such uses as ship steering.

Modern interest in control is generally regarded as beginning in 1934 when Hazen,<sup>2</sup> drawing on the work of earlier originators, published the first precise theory of automatic control systems. About this time work by Nyquist,<sup>3</sup> Black,<sup>4</sup> and Bode<sup>5</sup> in the design of stable amplifiers for telephone systems gave added impetus to the development of feedback theory.

The demands of World War II greatly accelerated work in the field of automatic control. In particular the need for precise and rapid control of ships, aircraft, and radar antenna systems led to significant advances in theory and practice. With the lifting of wartime security restrictions in 1945, rapid progress has taken place in the control field. Since that time many books and innumerable articles and papers have been written and the application of control systems in the industrial and military fields has been almost without limit.

1 J. C. Maxwell, "On Governors," *Proc. R. Soc. (Lond.)* vol. 16, 1868, p. 270.

2 H. L. Hazen, "Theory of Servomechanisms," *J. Franklin Inst.*, vol. 218, September 1934, p. 279.

3 H. Nyquist, "Regeneration Theory," *Bell Syst. Tech. J.*, vol. 11, January 1932, pp. 126-147.

4 H. S. Black, "Stabilized Feedback Amplifiers," *Bell Syst. Tech. J.*, vol. 13, January 1934, pp. 1-18.

5 H. W. Bode, "Amplifiers," U.S. Patent 2,123 178 (1938).

## 4 Fundamentals of automatic control

During the 1940s and 1950s two distinct yet interrelated approaches to the analysis and design of control systems evolved. Development in the United States centered on frequency-response methods, while much significant work in the U.S.S.R. used transient analysis or time-response techniques. In recent years these two approaches have become melded, and much contemporary analysis is based on simultaneous application of both methods.

Much in the way of modern automatic control has been made possible by two parallel developments: semiconductor devices and electronic computers. The successive use of electron tubes, transistors, and integrated circuits has provided control equipment and measuring instruments offering precision, reliability, and economy. The advent of the computer age has had a marked influence on control systems. High-speed digital computers have extended the realm of control to large and complex systems involving a number of interrelated quantities. Digital computers are finding extensive application in industrial process control systems in which many variables are measured and controlled simultaneously to achieve optimum performance.

### 1-3 OPEN-LOOP AND CLOSED-LOOP CONTROL

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If we examine the word *control*, we find several meanings given in the dictionary, e.g., command, direct, govern, and regulate. Thus, a control system may be regarded as a group of physical components arranged to direct the flow of energy to a machine or process in such a manner as to achieve the desired performance.

The word *automatic* means self-moving or self-acting; thus an automatic control system is a *self-acting* control system.

An important distinction applied to control systems, whether automatic or otherwise, is that between *open-loop* and *closed-loop* operation. Automatic control, including this distinction, can perhaps be best introduced by means of a simple example.

Assume that the oven shown in Fig. 1-2 is heated by an electric heater controlled by a switch that provides several levels of current to the heating element. The setting of the switch represents the *input quantity* since it activates the system to produce the output. The temperature of the oven is the *output* or *controlled quantity*. Since the current to the heater is the quantity being altered, we can think of it as the *manipulated quantity*. ✓

For a given setting of the control switch the oven temperature will reach a value related to the heater current and heat losses through the oven walls. If the temperature is unsatisfactory, this fact by itself can

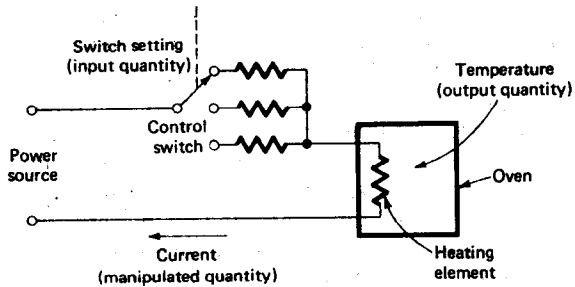


Figure 1-2 Oven-temperature control, open-loop

in no way alter the input to the oven control. Thus it can be said that the output quantity has no effect on the input quantity. The control in this case is identified as an *open-loop control system*. If some condition such as the ambient temperature surrounding the oven should change, the oven temperature will also change. Thus the open-loop control system cannot correct for changes that disturb the controlled quantity.

A human being can be added to the system for the purpose of maintaining the oven temperature at a desired value. This is shown in Fig. 1-3. By observing a thermometer within the oven, the person could alter the position of the control switch to more nearly achieve the required temperature. It is important to note that the addition of the human operator has provided a means by which the output is *fed back* and compared with the desired value. Actual control of the heating element depends on the error or difference between the desired and actual temperatures. Any necessary change is made in the direction

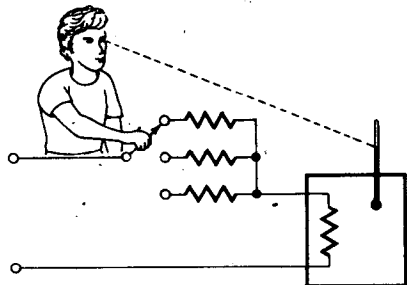


Figure 1-3 Oven-temperature control, manual closed-loop

## 6 Fundamentals of automatic control

of reducing this error. For example, if the temperature were low, the heater current would be increased to provide more heat.

Systems in which the output has an effect upon the input are called *closed-loop control systems* or, more commonly, *feedback control systems*. The feedback nature of the system in Fig. 1-3 may be seen by tracing the closed loop. The function of comparing the actual temperature with the desired value is performed in the mind of the operator, and the command is executed by the response of his muscles. Even though closed-loop control of the oven is established by the operator, the results may be less than satisfactory. The temperature will cycle above and below the desired value as the operator adjusts the switch even though the excursions would tend to decrease with operator experience. A change in the level of the power source or in the ambient temperature surrounding the oven would adversely affect the system's output and cause the operator to "relearn" a switching pattern. In addition, watching a thermometer and operating a switch would be a dull and time-consuming task. This arrangement could be described as a *manual closed-loop control system*.

To further improve performance and obtain more precise control, the human may be replaced by a mechanical, electrical, or other form of comparison and control unit. Figure 1-4 shows the oven control system with closed-loop control implemented on an automatic basis, that is, without human intervention. The temperature is now measured by a thermocouple, a device that generates an electrical voltage proportional to temperature (see Sec. 3-10). This voltage is fed back and compared with a reference voltage that represents the desired temperature. The difference between the two voltages is amplified electronically and controls the current to the heating element. Since the voltage feedback is subtracted from the reference voltage, it is known as a *negative-feedback signal*.

The example has illustrated how automatic closed-loop control is

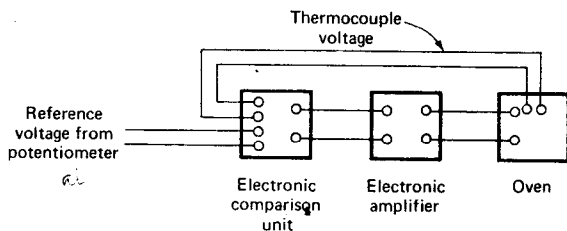


Figure 1-4 Oven-temperature control, automatic closed-loop



employed to perform a task more rapidly and consistently than can be accomplished by a human being. It also frees an individual from a rather menial task. The use of negative feedback in a closed-loop control system tends to maintain a desired value of a quantity or condition by measuring the existing value, comparing it to the desired value, and using the difference as a means of initiating action to reduce the difference. The negative-feedback concept has become the foundation for automatic-control-system design.

#### 1-4 CONTINUOUS AND DISCONTINUOUS CONTROL

Within the distinction between open-loop and closed-loop control, a further classification may be made. This is based on the manner in which the actuating signal is used to control the transfer of energy from source to load. Two categories of control that are possible are *discontinuous* or *digital* control and *continuous* or *analog* control.

A discontinuous or digital signal varies in a discrete manner and may take only certain discrete values between its limits. The simplest form of discontinuous control is *on-off* control in which the function of the input signal is to turn the power flow off or on. Once this switching has been carried out, no further control is possible other than reversing the operation. Since only two discrete states are available, the operation may be regarded as discontinuous. The lamp circuit shown in Fig. 1-5a is an example of on-off control.

Another form of discontinuous control is *step* control in which energy transfer is possible at one of several possible rates. For example, a transformer with a tapped secondary and a multiposition switch may be connected to the lamp as shown in Fig. 1-5b. In this case a number of voltages are available which provide discrete increments of power to the lamp. In addition the oven-temperature control shown in Figs. 1-2 and 1-3 represents a step control system.

With analog control the signals throughout the system vary in a continuous manner, and any value between the available maximum and minimum can be obtained. In Fig. 1-5c a variable resistor is placed

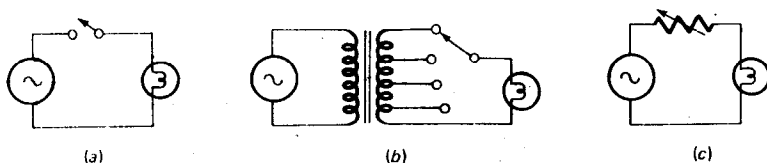


Figure 1-5 (a) On-off, (b) step, and (c) continuous control of a lamp