

RICHARD E. TRUEMAN

*an introduction to*

# QUANTITATIVE METHODS FOR DECISION MAKING

second edition

# **An Introduction to Quantitative Methods for Decision Making**

**SECOND EDITION**

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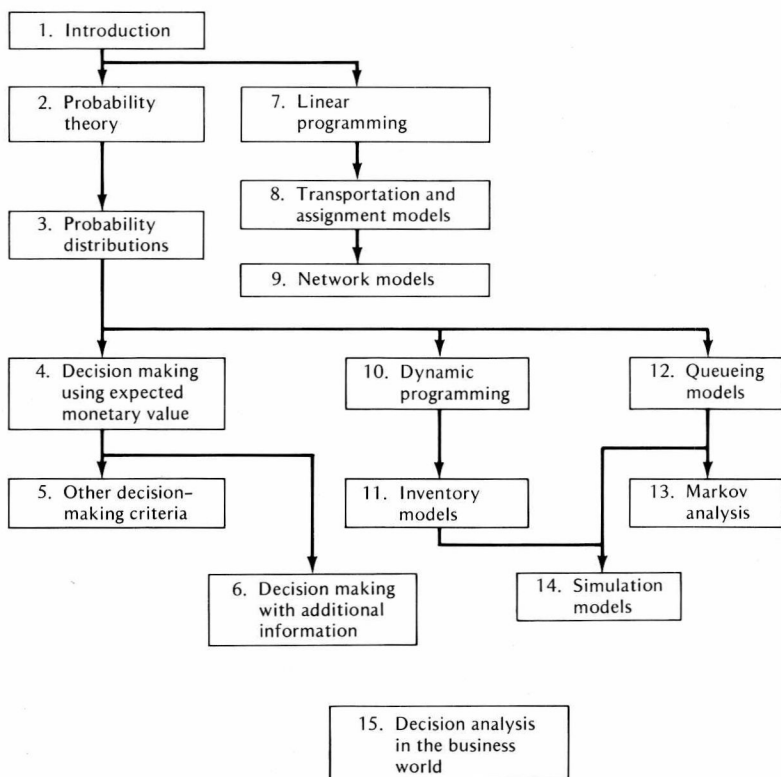
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*To my mother and the memory of my father*



## TEXT ORGANIZATION

## PREFACE

In the second edition of this text, an overall effort has been made to further clarify a number of highly detailed topics, in many cases by incorporating simpler introductory examples. The introductory chapter has been considerably expanded, presenting a short history of decision analysis and an introduction to the primary emphasis in this text, the development and analysis of mathematical models for decision making. Two new chapters have been added, the first on Markov analysis and the second a final wrapup chapter presenting an overview of decision analysis in the business world. One chapter, on the very specialized topic of Bayesian decision analysis using continuous probability distributions, has been deleted, although an appreciable portion of this material was placed in the chapter introducing Bayesian decision analysis. In a number of the chapters, a section on practical applications has been added, generally detailing the successful implementation of a particular type of mathematical model in situations involving important business problems.

This book's primary objective is to introduce the reader to a number of the more important mathematical approaches and specialized techniques which can be used in the analysis of basic business problems requiring quantitative decisions. The aim is to develop an understanding of problem-solving methods, based upon a careful discussion of problem formulation, mathematical analysis, and solution procedures, utilizing numerous examples involving relatively believable business situations. Graphical presentations are frequently employed to clarify mathematical concepts and to simplify problem solving. For a number of the topics covered, sensitivity analysis is employed to determine the range over which a given solution is optimal and to explore the effect of errors or changes in input data values.

The first part of the text starts with the development of probability concepts and then covers in detail, discrete and continuous probability distributions, including subjective probability distributions. The next two chapters develop the approaches to decision making under uncertainty using expected value analysis, considering both monetary and utility values. This material lays the foundation for Bayesian statistical decision theory, the study of the economics of decision making when additional information is available.

The development and analysis of operations research models is the subject of the second part of the text. Topical coverage related to decision making under certainty includes linear programming, with problem formulation and post-optimality analysis of computer solutions stressed, transportation and assignment problems, network models, and deterministic dynamic programming and inventory models. Topics involving decision making under

uncertainty include project scheduling, probabilistic dynamic programming and inventory models, queueing theory, Markov analysis, and simulation models.

The text contains enough material for a two-semester course in Quantitative Methods in Business at the upper-division undergraduate level or a one-semester MBA course, with some topics omitted. It could also be used for a one-quarter or one-semester course in Decision Theory or Decision Analysis (perhaps including linear programming), or a one-semester course in Operations Research. This text is designed to be essentially self-contained. Mathematical and probability concepts are introduced as needed, and, although mathematical rigor is retained, calculus is not required. A basic knowledge of algebra is assumed, and an introductory statistics course would undoubtedly be helpful, but it is by no means a prerequisite. Appendices at the end of the text include tables of the normal, cumulative binomial, and cumulative Poisson probability distributions, as well as a table of the normal loss integral and tables of random numbers and random variates.

I would like to express my appreciation to editor Rosalind Sackoff for her dedicated direction of this project, to consulting editor Robert Winkler for valued advice, and to reviewers of the second edition, in particular Professors Donald Aucamp, Southern Illinois University; Christopher Barry, University of Florida; Philip Carter, Michigan State University; Robert Clickner, Temple University; Mal Golden, University of Miami, Fla.; Paul Lerman, Fairleigh Dickinson University; Joe Moder, University of Miami, Fla.; William Roach, University of Kansas; Lloyd Rosenberg, Baruch College; Bruce Skalbeck, University of Wisconsin-Oshkosh; Richard Withycombe, University of Montana; Bruce Woodworth, Oregon State University. Thanks also go to my colleague Abe Feinberg, who contributed a substantial number of new chapter exercises for the second edition. I am most grateful for the encouragement, patience, and cooperation of my wife, Margaret, over the six-year period of this project.

R. E. T.

*Woodland Hills, California*  
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# CHAPTER 1

## INTRODUCTION

### 1.1 NEED FOR QUANTITATIVE APPROACHES TO DECISION MAKING

The complexity of today's business operations, the high costs of technology, materials, and labor, as well as competitive pressures and the shortened time frame in which many important decisions must be made, all contribute to the difficulty of making effective decisions. For all these reasons, the need of the decision maker for every possible aid in the decision process has never been more apparent. The question as to what constitutes an "effective" decision is a difficult one to answer, because it depends on so many different economic, social, and political factors and viewpoints. Very few business decisions are made, however, which are not primarily based on quantitative measures of some nature. For example, a decision on the location of a new manufacturing plant would be primarily based on such economic factors as construction costs, prevailing labor rates, taxes, energy and pollution control costs, marketing transportation costs, and related factors. On the other hand, a decision as to the location of a new federal regional hospital may be heavily influenced by national, state, and local politics. Even in this case, quantitative measures, such as the number of voters expected to favor a given location (and who can thus be expected to be favorably inclined toward a politician promoting that location), may well play an important part in the final decision. An understanding of the applicability of quantitative methods to business decision making is, therefore, of fundamental importance to the business student.

In a very real sense, quantitative methods, along with their almost indispensable companion, the high-speed digital computer, represent a new form of applied science