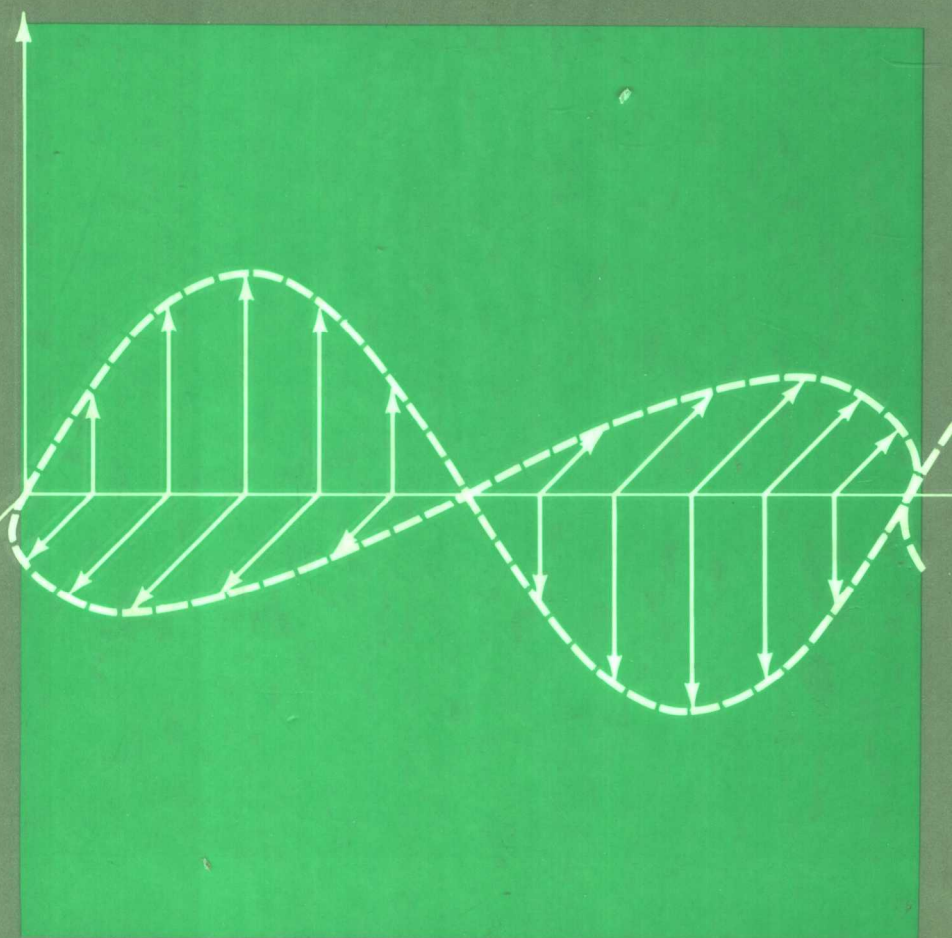


INTRODUCTION TO ELECTROMAGNETIC FIELDS

SECOND EDITION

CLAYTON R. PAUL
SYED A. NASAR



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Syed A. Nasar

Department of Electrical Engineering
University of Kentucky

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To
the memory of my father,
Oscar Paul,
November 8, 1896–November 27, 1974.
By the conduct of his life,
he taught compassion and fairness.

Clayton R. Paul

To
Professor Robert M. Saunders,
who made me aware
of the immensity that lies
beyond Maxwell's equations,
with gratitude and best regards.

Syed A. Nasar

PREFACE

This text is intended for use as an introduction to the subject of electromagnetic fields at the undergraduate level of an electrical engineering curriculum. As a prerequisite, we assume that the reader has the standard background in calculus, differential equations, and elementary physics.

The subject of electromagnetic fields is perhaps the most fundamentally important topic in electrical engineering. Therefore, the student should be motivated by the material and its presentation to engage in a serious study of the topic. Since the quantities of interest are functions of not only time but also spatial parameters, the material is inherently more difficult for the beginning student than, for example, electric circuit theory. In order to solve most electromagnetic fields problems, the student must be able to visualize and understand the meaning of the governing equations. Consequently, an important aspect of the presentation of this material is a clear explanation of the fundamental principles and concepts. Our intent in writing this text is to make the subject matter interesting and motivating and to present the important concepts with a minimum of unnecessary detail so that the reader can distinguish the “forest from the trees.”

This text grew out of a need for an introductory electromagnetic fields text which bridges the gap between the existing texts which cover static fields in considerable detail but do not give sufficient coverage of time-varying fields and those that cover both topics but with considerably more detail and sophistication than is required at this level of instruction. We presume that the student has been introduced to the basic static field concepts such as Coulomb’s law and Gauss’ law through the standard elementary physics courses. Consequently, the discussion of static field concepts is minimized so that the more important topics of time-varying fields and the engineering applications (uniform plane waves, transmission lines, waveguides, and antennas) can be covered in sufficient depth.

Chapter 1 provides an introduction and motivational survey. In revising Chapter 1 we have included more illustrative examples of applications of electromagnetic field theory. Chapter 2 presents all of the necessary vector algebra and vector calculus tools and concepts. Much of the material in Chapter 2 is review material, which can be covered rapidly. In this edition we have included a discussion of generalized, orthogonal coordinate systems in order to provide a more unified basis for discussion of the specific coordinate systems. Chapter 3 contains the static electric field concepts (Coulomb's law, electric field, Gauss' law, potential, energy, capacitance, resistance, and mechanical forces). A section on the concept and calculation of resistance for arbitrary structures has been added in this second edition. This follows the section on the calculation of capacitance for arbitrary structures present in the first edition. A section on power dissipation has also been included in this edition. Chapter 4 presents the static magnetic field concepts (the Biot-Savart law, Ampère's law, energy, inductance, mechanical forces, and magnetic circuits). This chapter has been substantially rewritten to emphasize the duality between the electric and magnetic fields. Relatively brief discussions of material properties are included in Chapters 3 and 4. In many institutions, these static field topics are covered to some degree in the elementary physics courses. In this case, Chapters 3 and 4 may serve as review material.

Chapter 5 begins the discussion of time-varying field concepts by introducing and discussing Maxwell's equations, the boundary conditions, Poynting vector, and the important sinusoidal, steady-state solution technique. This chapter is essentially unchanged from the first edition. Chapter 6 begins the discussion of the applications and implications of Maxwell's equations from an engineering standpoint. The concept of electromagnetic waves is discussed in considerable detail so that this fundamental concept will be firmly understood. Reflection and transmission of uniform plane waves as well as polarization of these waves are also discussed. We have added a section on group velocity which was not present in the first edition. A major addition in this second edition is the discussion of oblique incidence of uniform plane waves on plane material boundaries. The topics of plane wave propagation in arbitrary space directions, Snell's laws, Brewster angle, and the critical angle have been included along with the general developments on oblique incidence.

Chapter 7 contains a discussion of wave propagation on transmission lines. Both transient and sinusoidal, steady-state behavior are discussed. The emphasis is on fundamental principles, and the Smith chart and its applications are discussed in detail in Appendix C. We have included in this second edition a discussion of the per-unit-length resistance calculation for the standard transmission line structures which was not present in the first edition. Chapter 8 presents a discussion of rectangular waveguides, which, although brief, covers the essential points. In this second edition, we have added sections on attenuation in waveguides, and cavity resonators. The intent again was to highlight the essential concepts and keep the discussion brief.

The topic of antennas is covered in Chapter 9 in somewhat more detail than is customary in a text aimed at this level. The elemental, electric (Hertzian dipole),

the elemental, magnetic (loop) dipole, and the long, linear dipole are discussed in a somewhat standard manner as is the topic of linear arrays. Antenna directivity and gain are also discussed. Coupling between two antennas is considered, and the important concepts of reciprocity with regard to impedance and pattern for an antenna in either a transmitting or receiving mode are derived. The Friis transmission equation is also derived. A section on the effect of reflections from an imperfect ground has been added to this second edition.

Chapter 10 contains the traditional techniques for solution of static field problems for which simple, closed-form solutions are not obtainable. Discussions of Laplace's equation in cylindrical and spherical coordinates are added in this revised version, and the section on image methods has been expanded. Solution techniques for Laplace's and Poisson's equations, as well as numerical methods (finite-difference and method of moments) and analog and graphical methods, are discussed. Ordinarily, in other texts, this chapter is placed after the material covered in Chapter 4. We have chosen to include this chapter as the last one in the text so that the reader is led to an early consideration of the important topics of time-varying fields after a brief review of static field concepts in Chapters 3 and 4. With this organization, the reader has a proper appreciation of the hierarchy of importance of the material in each chapter.

Appendix A summarizes various vector identities, vector calculus operations, and transformations between coordinate systems. Appendix B contains a discussion of Faraday's law for moving contours which is unchanged from the first edition. In Appendix C, the discussion of the Smith chart is essentially the same as in the first edition. We have, however, added sections on double stub tuners, quarter-wave transformers, broadband matching and pads and use of the chart for lossy lines.

Most of the additions to the first edition were suggested by those who have used the text. We are grateful to those who have provided their comments for improvement.

Over half the end-of-chapter problems are new in this edition. Answers to selected problems are given at the end of the text, as was requested by most of the users of the first edition.

The text should be suitable for either a one-semester or a two-semester sequence in electromagnetic fields. In a one-semester course, it would be appropriate and possible to review Chapter 2 (vectors) and cover Chapters 5, 6, 7, 8, and 9, and Appendix C (Smith charts). A two-semester sequence would be a more leisurely coverage of the entire text. Chapters 2, 3, 4, 10, 5, and possibly a portion of Chapter 6 may be covered in detail in the first semester, and the remainder of Chapter 6 and Chapters 7, 8, 9 and Appendixes B and C may be covered in the second semester.

The question of what is the most appropriate and effective way of presenting electromagnetic fields at this level is difficult to answer. We have chosen to blend the more traditional approach of discussing static fields first with the attitude of minimizing that discussion in order to get to the topic of time-varying fields as soon as possible. This approach seems to have the pedagogical advantage of discussing the more easily understood static field concepts first before delving

into the inherently more difficult time-varying field concepts. In line with the attitude that it is essential that the reader visualize and understand the basic concepts in electromagnetic fields in order to begin to master the subject matter, we have tried to simplify the notation and minimize the mathematical details where possible. Elective and graduate courses will be able to delve more deeply into the details once these basic concepts are firmly understood.

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Clayton R. Paul
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CHAPTER 1

□ Introduction

1.1 Brief Historical Development

The basic concepts of modern electromagnetic field theory have evolved over many years. In fact, an awareness of magnetism appears to be as old as recorded history. The discovery of the polarities of lodestone by Pierre de Maricourt dates to around 1269, and from that time through the early seventeenth century the progress in the study of magnetism was rather slow.¹ During the seventeenth century, however, there was a considerable revival of interest and there were several notable contributions by several scientists toward understanding magnetism. A. Kirchner demonstrated that the two poles of a magnet have equal strength, and Newton attempted to formulate the law for a bar magnet. But the correct inverse square law was postulated by John Michell in 1750 and reconfirmed by Coulomb at a later date. In 1785 Coulomb also demonstrated the law of electric force between charged bodies.† Coulomb's inverse square law (or simply Coulomb's law) may be said to be the starting point of modern electromagnetic field theory. Subsequent landmarks in the development of electromagnetic field theory include the derivations of Laplace's equation in 1782, Poisson's equation in 1813, and Gauss' divergence theorem in the same year. These developments essentially belong to the general topic of electrostatics.

We know that motion of charges constitutes electric current. Experiments with electric current could be performed only after the invention of the battery by Volta in 1800. Having a source of continuous current available, Oersted, in 1820, was able to demonstrate the production of magnetic fields by electric currents. Oersted's discovery prompted others to investigate the relationships

† It is claimed that the inverse square law of electric force was established in 1773 by Cavendish, who did not publicize his findings.¹

between electric currents and magnetic fields. In 1820, Ampère announced a discovery relating to forces between electric current-carrying conductors and magnets and the mutual attraction (repulsion) of two electric currents. These experiments led to the formulation of Ampère's law. During 1820, Biot and Savart repeated Oersted's experiment to determine a law of force governing the forces between current-carrying conductors and gave us the Biot-Savart law. These developments belong to the general topic of magnetostatics.

During the period of Oersted and Ampère, Faraday was also experimenting on the interaction between current-carrying conductors and magnetic fields, and he developed an electric motor in 1821. Furthermore, Faraday's experiments on developing induced currents by changing the magnetism (or magnetic field) led to the law of electromagnetic induction in 1831.[†] Faraday also proposed the concept of magnetic lines of force. Thus, the foundation of all electromagnetic phenomena was laid. In 1864, Maxwell proposed "A Dynamical Theory of the Electromagnetic Field" and thus unified the experimental researches of over a century through a set of equations known as Maxwell's equations.² These equations were later verified experimentally by Hertz in 1887. It is generally accepted that all macroscopic electromagnetic phenomena are governed by Maxwell's equations.

In the following chapters we will study the basic laws governing electromagnetic fields. Before we take up the details of the analyses, we will look briefly at the range of applications of electromagnetic field theory.

1.2 Some Applications of Electromagnetic Field Theory

It was mentioned previously that Maxwell's equations govern all macroscopic electromagnetic phenomena. Therefore, it is not practicable to list here a large number of applications of electromagnetic field theory. Rather, the general nature of problems that are best handled by field theory will be pointed out, and the broad range of applications of electromagnetic field theory will be identified.

The term *field* is a concept used to describe a distribution of some quantity throughout a region of space. For instance, the electric field is measured by the force on a unit charge of electricity, and the magnetic field is measured by the force on a magnetic dipole. (Electric and magnetic fields will be defined precisely in Chaps. 3 and 4, respectively.) Thus, we notice that fields are three-dimensional spatial phenomena, and the mathematical formulation of field phenomena is always in terms of distributed parameters (in contrast to the lumped-parameter description of electric circuits). We might say that distributed-parameter field phenomena are given by partial differential equations and that lumped-parameter circuit behavior is expressed by ordinary differential equations.

[†] Joseph Henry of Albany, New York, is said to have made this discovery earlier than Faraday.

Indeed, Maxwell's equations are a set of partial differential equations, as we shall see in Chap. 5. Strictly speaking circuits are approximate analogs for field relationships, and are used for the sake of convenience. A simple example of this approximation is shown in Fig. 1.1. Figure 1.1a shows a solid cylinder of length l , area of cross section A , and made of a conducting material of conductivity σ . If we assume a uniform current-density distribution over the conductor cross section, its resistance is $R = l/\sigma A$, and the circuit analog then becomes as shown in Fig. 1.1b. The Ohm's law relationship for the solid conductor and its circuit analog are also shown in Fig. 1.1. As we shall see in later chapters, the other two common circuit elements, inductance and capacitance, are also approximate analogs corresponding respectively to magnetic and electric field in a given region.

From the preceding discussion, it is clear that the formulation in terms of fields implies an exact description of the electromagnetic phenomenon occurring in a specified region. There is a danger in using this approach to solve every problem in electrical engineering. Circuit concepts and other valid approximations are convenient to use in numerous situations. Electromagnetic theory must be applied where the approximations leading to the analogs break down. For instance, the radiation of electromagnetic waves from an antenna can only be described by Maxwell's equations. In Fig. 1.1a, if the distribution of the current is nonuniform over the conductor cross section, as would be the case for an alternating current of high frequency, the determination of the resistance involves an application of the field equations. The criteria for the use of the field equations are: (1) that the problem at hand is a distributed-parameter problem, (2) that the resulting equations can be solved without unreasonable difficulty,

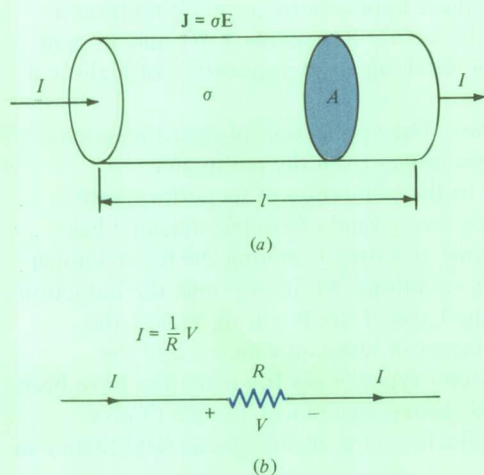


FIGURE 1.1

(a) A solid conductor showing the field relationship $\mathbf{J} = \sigma \mathbf{E}$, known as Ohm's law. (b) A circuit analog.