University Civil Engineering Major
Recommended Teaching Material (English Version)
高等学校土建类专业英文版推荐教学用书

ELASTICITY AND PLASTICITY

（弹性与塑性力学）

〔美〕陈惠发　A.F.萨里普　著
余天庆　王勋文　刘再华　编

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弹性力学和塑性力学是固体力学中的两个重要基础理论，本书第一篇讲述弹性力学理论，第二篇讲述塑性力学理论。

矢量和张量分析是弹性力学和塑性力学的重要教学工具，矢量和张量的指标记法及运算方法首先在第一章中阐述。第二至四章讲述弹性的基本概念和理论，第五至七章讲述塑性的基本概念和理论，第八章是关于金属的塑性理论，第九章简要地介绍求解弹性和弹塑性问题的有限元方法。

本书是土木工程专业研究生系列教材之一，也是为了适应大学进行“双语教学”的需要而编写的。《弹性与塑性力学》是本书的中文版，已由中国建筑工业出版社出版。这两本书是另一套“双语教材”《混凝土与土的本构方程》和《Constitutive Equations for Concrete and Soil》的姐妹篇。本书适用于机械、土木、航空航天、交通、材料等专业的大学本科和研究生的教学用书，也可作为工程师的提高和研究参考书。

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责任编辑：郭　栋

责任设计：孙　梅

责任校对：关　健　王雪竹

Foreword

These dual-language four-volume textbooks grew out of my two-volume monograph entitled **"Constitutive Equations for Engineering Materials"** with Volume 1 subtitled **"Elasticity and Modeling"** published by john Wiley inter-Science, New York, in 1982. Volume 2 subtitled **"Plasticity and Modeling"** together with an updated version of Volume 1 were published more than a decade later by Elsevier, Amsterdam, in 1994. The first parts of Volumes 1 and 2 of the original books are intended as textbooks for elasticity and plasticity respectively, while the later parts of each volume is intended as reference books for advanced materials as required for many civil engineering applications.

The materials in my original two-volume monograph are now reorganized in two separate books published in both Chinese version and English version, respectively: one book for the fundamentals of theories of elasticity and plasticity, and the other book for their implementations to soil and concrete materials and their applications. This book entitled **"Elasticity and Plasticity"** is intended for engineers with a basic background in mechanics, strength of materials, calculus, material behavior of metals, and some basic concept of finite element methods. The companion book entitled **"Constitutive Equations for Concrete and Soil"** provides a compact and convenient summary of the mathematical modeling techniques for material behavior in nonlinear finite element analysis for civil engineers working in the areas of reinforced concrete and soil mechanics.

Elasticity and Plasticity is organized into three parts consisting of nine chapters, with Part One (Chapters 1-4) presenting the foundation of elasticity theory. Chapter 1 introduces the basic concepts and notations of vector and tensor analysis. A novel approach to the analysis of stress and strain is employed in Chapters 2 and 3, which not only elucidate these principles but provide a clear physical understanding as well, Chapter 4 explains the general assumptions used in the formulation of elasticity-based constitutive models. Here, mathematical and physical reasoning are used to derive the general forms of the equations.

Part Two (Chapters 5-7) extends the elasticity-based stress-strain models to the plastic range and develops plasticity-based models for engineering applications. Here, as in Part One, Chapters 5 and 6 provide the necessary foundations of the theory of plasticity by first describing the characteristics and modeling of uniaxial behavior of materials, and then followed by the extension of the yield point in simple tension into the yield criteria in multi-dimensional stress space. Chapter 7 explains the general assumptions used in the formulation of plasticity-based constitutive models. Here, as in elasticity and modeling, mathematical and physicsl reasoning are used to derive the general forms of the equations.

Part Three (Chapters 8-9) develops the constitutive models for metals, shows the necessary numerical procedures for computer solutions, and presents some typical problems in structural engineering applications. These two chapters serve as a transition to the more complicated problems involving soil and concrete materials described in the companion book entitled **"Constitutive Equations for Concrete and Soil"**.

The development of these four textbooks in dual language on constitutive modeling of engineering materials was strongly influenced by the following two factors:

(1) The future direction of research and education in solid mechanics and structural engineering is in the area of modeling, simulation and validation. Modeling is mechanics and material science, simulation is computing and software development, and validation is experimentation and field measurements. Constitutive modeling of engineering materials is a critical element for the future advances in civil engineering applications.

(2) The realization of the importance of English language in the globalization of the world economy. The dual-language publication of these textbooks provides an excellent tool for those who wish to learn about constitutive modeling and to use computer simulation in the standard terminology of mechanics, materials, and computing in both English and Chinese.

I express my sincere thanks to Professor T. Q. Yu (Hubei University of Technology) and Dr. X. W. Wang (China Academy of Railway Sciences) for suggesting and carrying out the reorganization of my original two-volume monograph into the present dual language version of the four books. Student, researcher, or practitioner, novice and expert alike, will profit much from reading these books, either in English or Chinese, and having them for references in the years to come.

W. F. Chen
Honolulu, Hawaii
August 2003

NOTATION

Given below is a list of the principal symbols and notations used in the book. All notations and symbols are defined in the text when they first appear. Symbols which have more than one meaning are defined clearly when used to avoid confusion, and usually the correct meaning will be obvious from the context.

Stresses and Strains

$\sigma_1, \sigma_2, \sigma_3$	Principal stresses
σ_{ij}	Stress tensor
s_{ij}	Stress deviator tensor
σ	Normal stress
τ	Shear stress
$\sigma_{oct} = \frac{1}{3}I_1$	Octahedral normal stress
$\tau_{oct} = \sqrt{\frac{2}{3}J_2}$	Octahedral shear stress
$\sigma_m = \sigma_{oct}$	Mean normal (hydrostatic) stress
$\tau_m = \sqrt{\frac{2}{5}J_2}$	Mean shear stress
s_1, s_2, s_3	Principal stress deviators
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Principal strains
ε_{ij}	Strain tensor
e_{ij}	Strain deviator tensor
ε	Normal strain
γ	Engineering shear strain
$\varepsilon_v = I_1'$	Volumetric strain
$\varepsilon_{oct} = \frac{1}{3}I'_1$	Octahedral normal strain
$\gamma_{oct} = 2\sqrt{\frac{2}{3}J_2'}$	Octahedral engineering shear strain
e_1, e_2, e_3	Principal strain deviators

Invariants

$I_1 = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_{ii} =$ first invariant of stress tensor

$J_2 = \frac{1}{2}s_{ij}s_{ij}$

$= \frac{1}{6}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2] + \tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2$

$=$ second invariant of stress deviator tensor

$J_3 = \frac{1}{3}s_{ij}s_{jk}s_{ki} =$ third invariant of stress deviator tensor

$\cos 3\theta = \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}}$ where θ is the angle of similarity defined in Figure1.13●

$I_1' = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \varepsilon_v$ = first invariant of strain tensor

$\rho = \sqrt{2J_2}$ = deviatoric length defined in Figure1.12●

$\xi = \frac{1}{\sqrt{3}} I_1$ = hydrostatic length defined in Figure1.12●

$J_2' = \frac{1}{2} e_{ij} e_{ij}$

$= \frac{1}{6}[(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2] + \varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{zx}^2$

= second invariant of strain deviator tensor

Material Parameters

f_c'	Uniaxial compressive cylinder strength ($f_c' > 0$)
f_t'	Uniaxial tensile strength
f_{bc}'	Equal biaxial compressive strength ($f_{bc}' > 0$)
E	Young's modulus
ν	Poisson's ratio
$K =$	$\frac{E}{3(1-2\nu)}$ = Bulk modulus
$G =$	$\frac{E}{2(1+\nu)}$ = Shear modulus
c, ϕ	Cohesion and friction angle in Mohr–Coulomb criterion
α, k	Constants in Drucker–Prager criterion
k	Yield (failure) stress in pure shear

Miscellaneous

{ }	Vector
[ ]	Matrix
C_{ijkl}	Material stiffness tensor
D_{ijkl}	Material compliance tensor
$f(\ )$	Failure criterion or yield function
x, y, z or x_1, x_2, x_3	Cartesian coordinates
δ_{ij}	Kronecker delta
$W(\varepsilon_{ij})$	Strain energy density
$\Omega(\sigma_{ij})$	Complementary energy density
$l_{ij} =$	$\cos(x_i', x_j)$ = The cosines of the angles between x_i' and x_j axes (see Section 1.11)
ε_{ijk}	Alternating tensor defined in Section 1.10

● [美]陈惠发. Constitutive Equations for Concrete and Soil. 北京:中国建筑工业出版社, 2005.

CONTENTS

PART ONE

BASIC CONCEPTS IN ELASTICITY

CHAPTER ONE

VECTORS AND TENSORS

1.1 INTRODUCTION

The use of vector and tensor notations is commonplace in the current literature when stress, strain and constitutive equations are discussed. A basic knowledge of these notations is therefore essential to an intelligent assessment of the material presented. The preference of such notations over expanded forms for the physical quantities arises mainly from the important advantage of conciseness or brevity with which the various relationships can be expressed in mathematical terms, thereby allowing greater attention to be paid to physical principles rather than to the equations themselves.

The material included here in the review of vectors and tensors covers only those areas that have applications in the main subject concerning stresses, strains, and their relationships in the elastic and inelastic ranges.

1.2 COORDINATE SYSTEM

For the present, we restrict ourselves to Cartesian coordinate systems. In a three-dimensional space, a Cartesian coordinate system is pictured as a set of three mutually orthogonal axes denoted as the x-, y-, and z-axes. For future convenience, the axes are more conveniently designated as x_1-, x_2-, and x_3-axes, rather than the more familiar notation x, y, and z. The sketch shown in Fig. 1.1 assumes the use of the right-hand notation where the x_2- and x_3-axes lie in the plane of the paper and the x_1-axis is directed toward the reader.

In this notation, the axes are parallel, respectively, to the (right-hand) middle finger pointing toward the viewer, the thumb extending to the right, and the index finger vertically up. The positive directions are as indicated. If we imagine a right-hand screw, the rotation of the x_1-axis toward the x_2-axis

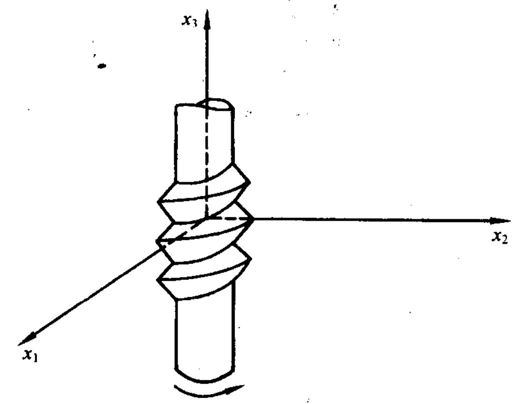

FIGURE 1.1 Right-handed screw notation.

causes a travel of the screw in the positive direction of the x_3-axis. Similar travels along positive directions can be checked by taking the indices 1, 2, and 3 in cyclic order. Because of this, the coordinate system shown in Fig. 1.1 is said to be *right-handed*. A system that is not right-handed is called *left-handed*. If the left hand were chosen, the positive x_3-axis in Fig. 1.1 would be pointed downward. Note that any two right-handed systems, arranged to have a common origin, may be rotated into one another so that their axes coincide. This holds also for any two left-handed systems, but not for one of each. In this book, we restrict ourselves to the use of *right-handed coordinate systems*.

1.3 VECTOR ALGEBRA

A *vector* is a quantity that possesses both magnitude and direction, as contrasted to a scalar, which possesses magnitude alone. For example, velocity is a vector, and temperature is a scalar. A vector is usually represented by an arrow, drawn in the direction of the vector whose length is made proportional to the magnitude of the vector.

Unit vectors $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, and $\boldsymbol{e}_3$ are shown in Fig. 1.2 along the three mutually perpendicular axes. The unit vector $\boldsymbol{e}_1$, for example, is of unit length (measured from the origin) and lies along the x_1-axis. Thus it is necessarily perpendicular to the other two axes, x_2 and x_3.

Next, an arbitrary point P in space with the coordinates v_1, v_2, v_3 may be represented by the vector OP or $\boldsymbol{V}$. This vector $\boldsymbol{V}$ may be visualized as a

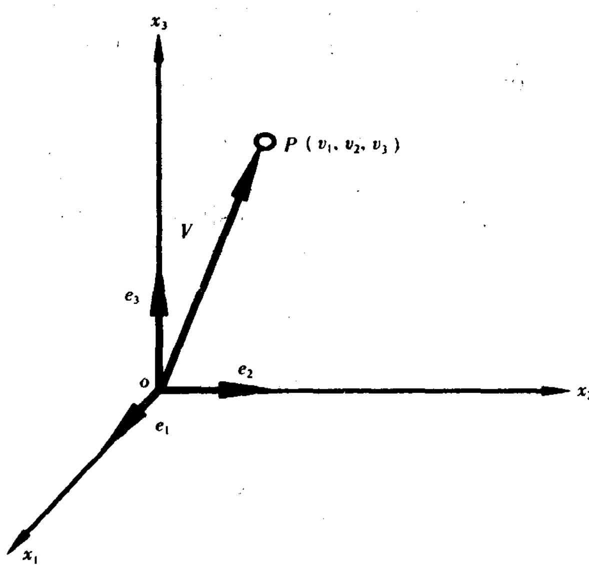

FIGURE 1.2 Position and unit vectors in right-handed Cartesian coordinate system. x_1, x_2, x_3 = Cartesian coordinate axes; O = origin.

combination of vectors V_1, V_2, and V_3. Thus

$$V = V_1 + V_2 + V_3 \tag{1.1}$$

or, in terms of unit vectors,

$$V = v_1 e_1 + v_2 e_2 + v_3 e_3 \tag{1.2}$$

where v_1, v_2, and v_3 are now scalar quantities. More simply, the expression may be abbreviated as

$$V = (v_1, v_2, v_3) \tag{1.3}$$

The ordering of the scalar multipliers in this form is obviously of great importance. This may be seen from the extremely close similarity between the vector notation for V and the Cartesian coordinates of point P.

It is usual to consider V_1, V_2, and V_3 as components of V or, conversely, the vector V as being resolved into its components. The particular point at which a vector acts is usually understood from the context and need not be specified separately. In the sketch of Fig. 1.2, the vector V happens to act at the origin of the coordinate system.

Two vectors V and U are defined to be equal when their respective components are equal; that is, the condition for equality is given by

$$v_1 = u_1, \qquad v_2 = u_2, \qquad v_3 = u_3 \tag{1.4}$$

or, more compactly,

$$v_i = u_i, \qquad i = 1, 2, 3 \tag{1.5}$$

In general, equality is indicated by merely writing

$$v_i = u_i \tag{1.6}$$

and taking for granted that, since the subscript i is unspecified, the equation must hold for each of the three possible values of this subscript.

If a vector V is multiplied by a positive scalar α, the result αV is defined to be a new vector coinciding with V in direction but of magnitude α times as great. If α is negative, the effect of the negative sign is defined to be a reversal of direction.

The sum of two vectors U and V is defined according to the parallelogram law as shown in Fig. 1.3. Obviously, the addition and subtraction of vectors are defined when these operations are performed on the components of the vectors:

$$W = U \pm V = (u_1 \pm v_1)e_1 + (u_2 \pm v_2)e_2 + (u_3 \pm v_3)e_3 \tag{1.7a}$$