Representation Theory I
Finite Dimensional Algebras

Edited by V. Dlab, P. Gabriel and G. Michler

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Finite Dimensional Algebras

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Edited by V. Dlab, P. Gabriel and G. Michler

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PREFACE

The fourth international meeting on representations of algebras took place amidst a period of a number of important developments in the subject. The meeting was held at Carleton University, Ottawa, on August 16-26, 1984; following a previously established format (see Proceedings of the previous meetings, Springer Lecture Notes # 488 , 831 , 832, 903 and 944), the meeting consisted of a WORKSHOP (August 16-18, 1984) and a CONFERENCE (August 20-25, 1984). One of the particular objectives of the meeting was an attempt to bridge the area of general representation theory and the representation theory of finite groups. The Organizing Committee consisted of J. Alperin, M. Auslander, R. Bautista, S. Brenner, M.C.R. Butler, V. Dlab, W. Feit, P. Gabriel, E.L. Green, G. Michler, L.A. Nazarova, I. Reiner, I. Reiten, C.M. Ringel, A.V. Rojter, K.W. Roggenkamp and H. Tachikawa. A preliminary version of the Proceedings has appeared in two volumes of the Carleton-Ottawa Mathematical Lecture Notes Series. We should like to acknowledge financial assistance from the Natural Sciences and Engineering Research Council of Canada and Carleton University for support of the meeting.

Seven short series of lectures were read in the Workshop by M. Artin ("Singularity Theory"), D. Benson ("Modules for Finite Groups"), E.L. Green ("Auslander's Theory for Lattices"), G. Michler ("Brauer Conjectures in Simple Groups"), A.V. Rojter ("Representations of Completed Partially Ordered Sets"), R. Salmeron ("Multiplicative Bases of Algebras") and H. Tachikawa ("Selfinjective Algebras"). An informal Problem Session was held as part of the Workshop with a panel consisting of M. Auslander, M.R.C. Butler, J. Alperin (Chairman), W. Feit, G. Michler and H. Tachikawa.

Recent advances in the representation theory were reported in 64 lectures; these are listed on page IX. Not all contributions to the Conference appear in these Proceedings; in fact, some of the authors have chosen to publish their contributions which appeared in the preliminary version, elsewhere. On the other hand, some of the papers which were not reported, are included. At this point, we should like to express our gratitude to all referees for their most helpful assistance.

The Proceedings are printed in two Volumes; the first volume contains the papers related to the general representation theory,the second one to the representation theory of groups and orders. In addition, an updated bibliography of the field (1979-1985) has been attached to the second volume; we wish to express our thanks to Drs. I. Assem and S. Menzie for their assitance in compiling the references.

We wish also to extend our thanks to the Secretary, Ms. Suzanne Drahotsky, for her generous help, efficiency and much appreciated secretarial assistance.

Ottawa-Zürich-Essen, September 1985

Vlastimil Dlab, Peter Gabriel, Gerhard Michler

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* denotes the speaker

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GROUPS

ORDERS

HOMOLOGICAL PROPERTIES OF WILD HEREDITARY ARTIN ALGEBRAS

Dagmar Baer

0. Introduction

Tame hereditary Artin algebras have a lot of nice homological properties: For instance, the categories $\mathcal{P}$, $\mathcal{R}$ and $\mathcal{I}$ consisting of all preprojective, regular and preinjective indecomposables, respectively, are noetherian on both sides [18]. Moreover, the Gabriel dimensions of the ringoids $\mathcal{P}$, $\mathcal{R}$ and $\mathcal{I}$ are two, one and zero, respectively [12]. Here, according with Gordon and Robson ([16], see also [15]), we call the Gabriel dimension of a ringoid $\mathcal{A}$ the Krull dimension (in the sense of [9]) of the category of all additive functors from $\mathcal{A}$ into the abelian groups.

The main purpose of this paper is the study of wild hereditary algebras. It turns out that their homological behaviour is as bad as possible: The Gabriel dimensions of $\mathcal{P}$ and $\mathcal{R}$ do not exist (Theorem 3.9 and Theorem 4.3). In particular, $\mathcal{P}$ and $\mathcal{R}$ cannot be left noetherian [15]. Even the Ore condition for the preprojective algebra (see [14], [8], [4] for a definition), a rather weak condition, is not satisfied in the wild case (Cor. 3.2). Consequently, the preprojective algebra, being prime by Prop. 3.4, cannot have a polynomial identity (Cor. 3.5).

The previous results are proved with the aid of linear methods based on the definition of weak defect functions. On the one hand, these functions share some properties with the well-known tame defect([13], [7]), on the other hand, they behave quite different. Chapter 2 gives a short survey of the weak defect theory needed in this paper, pointing out both aspects (see Prop. 2.2 and Prop. 2.3).

Fundamental for the definition of the weak defect is the fact that, in the wild case, the rational Grothendieck group has a basis consisting of dimension vectors of regular modules (Cor. 1.4). This follows immediately from Theorem 1.3 stating that every preprojective module occurs as a kernel of an epimorphism between regular modules.

1. Wild hereditary Artin algebras and their Grothendieck group

Throughout this paper, let A be an hereditary connected basic Artin algebra of infinite representation type. We can assume that A is a finite dimensional k-algebra for some field k [1].

Let us consider the category mod-A of all finitely presented right modules and the full subcategories $\mathcal{P}$, $\mathcal{R}$ and $\mathcal{I}$ consisting of all preprojective, regular and preinjective indecomposables, respectively. If A is tame, it is well-known that the category add($\mathcal{R}$) of all finite direct sums of modules in $\mathcal{R}$ is closed under the formation of kernels. Moreover, the tame representation type is characterized by this fact:

1.1. Lemma. Let A be of wild representation type. Then there exist a quasi-simple module $S \in \mathcal{R}$, a positive integer n and an exact sequence

$$0 \to K \to (\mathrm{TrD})^n(S) \to S \to 0$$

such that K has no preprojective direct summand.

Here, TrD denotes the Auslander-Reiten functor "transpose of the dual" [2].

Proof: We choose a quasi-simple module S (see [21] for a definition) of minimal A-length. Following an idea of C.M.Ringel, we consider $T:=\bigoplus_{i=0}^{m} (\mathrm{TrD})^{2i}(S)$ - m denoting the number of simple A-modules - and get $0 \neq \mathrm{Ext}_A^1(T,T) \cong D(\mathrm{Hom}(T,(\mathrm{DTr})(T))$ by ([17], Lemma 4.3). Consequently there exist a nonzero integer n and a morphism $0 \neq \nu\colon (\mathrm{TrD})^n(S) \to S$ being surjective by our assumption on the length of S.

Let $K = \ker(\nu)$, then we get the formula $\underline{\dim}((\mathrm{TrD})^n(S)) = \underline{\dim}(K) + \underline{\dim}(S)$ for the dimension vectors and hence, by applying the Coxeter transformation c, $\underline{\dim}(S) = c^n(\underline{\dim}(K)) + \underline{\dim}((\mathrm{DTr})^n(S))$. Since A is wild, $c^n(\underline{\dim}(K))$ cannot be zero by a criterion of Berman, Moody and Wonenberger [5]. Consequently, following again our assumption on the length, there exists a negative coordinate of $c^n(\underline{\dim}(K))$. K has no preinjective direct summand, thus n must be positive and we get a preprojective summand of K. o

1.2. Corollary A is wild if and only if $\mathrm{add}(\mathcal{R})$ is not closed under the formation of kernels. o

The next theorem yields that even every preprojective module occurs as a kernel of an epimorphism in $\mathrm{add}(\mathcal{R})$. This result is fundamental for wide parts of the paper.

1.3. Theorem. Let A be of wild representation type and $P \in \mathcal{P}$. Then there exist $R, R' \in \mathrm{add}(\mathcal{R})$ and an exact sequence

$$0 \to P \to R \to R' \to 0.$$

Proof: $\mathrm{add}(\mathcal{R})$ is not closed under the formation of kernels, hence it is easy to get modules $\bar{P} \in \mathcal{P}$, $\bar{R}, \bar{R}' \in \mathrm{add}(\mathcal{R})$ and an exact sequence

$$\eta\colon 0 \to \bar{P} \overset{\alpha}{\to} \bar{R} \overset{\beta}{\to} \bar{R}' \to 0.$$