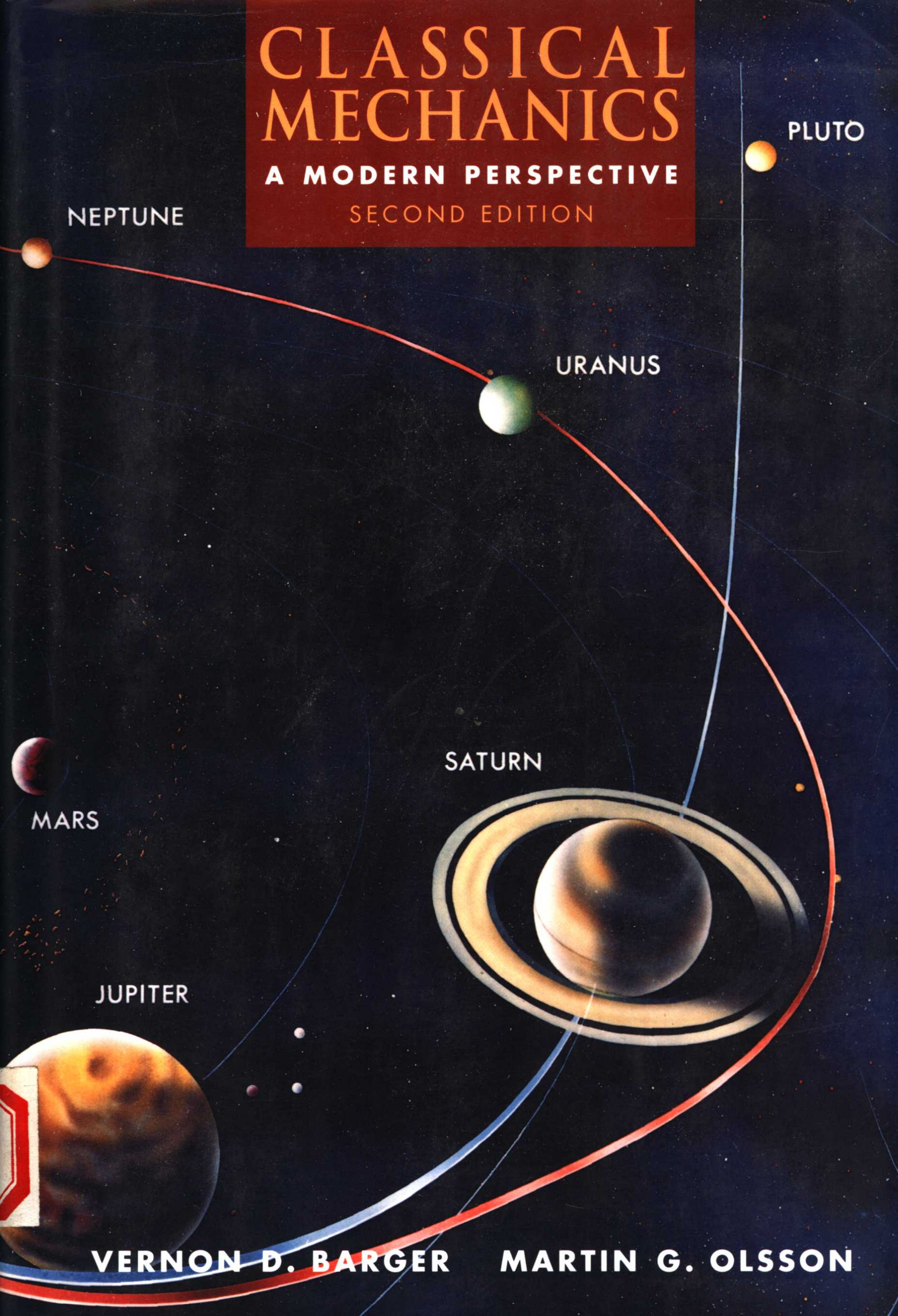
CLASSICAL MECHANICS

A MODERN PERSPECTIVE

SECOND EDITION



VERNON D. BARGER MARTIN G. OLSSON

CLASSICAL MECHANICS:
A Modern Perspective

Second Edition

Vernon Barger
University of Wisconsin, Madison

Martin Olsson
University of Wisconsin, Madison

McGRAW-HILL, INC.

New York St. Louis San Francisco Auckland Bogotá Caracas Lisbon
London Madrid Mexico City Milan Montreal New Delhi
San Juan Singapore Sydney Tokyo Toronto

CLASSICAL MECHANICS:
A Modern Perspective

This book is printed on acid-free paper.

4567890 DOC DOC 9098765

ISBN 0-07-003734-5

The editor was Jack Shira;
the production supervisor was Annette Mayeski.
The photo editor was Anne Manning.
R. R. Donnelley & Sons Company was printer and binder.

Library of Congress Catalog Card Number: 94-72897

Photo Credits

Front, 2-page spread, endpaper: Skydiving over Sydney, Australia (Associated Press photo). Wide World Photos.

Front, reverse side of right endpaper: Lunar module over the moon with earth in the background (NASA photo). Courtesy NASA.

Back, reverse of left endpaper: Jupiter and its Galilean moons (NASA Voyager photo). Courtesy Jet Propulsion Laboratory.

Back, left endpaper: Einstein ring due to gravitational lensing of a distant quasar (National Radio Astronomy Observatory photo).

Back, right endpaper: Rings of glowing gas encircling the 1987A supernova. (Hubble Space Telescope photo). Courtesy Space Telescope Science Institute.

Fig. 7.17: Richard Wainscoat, University of Hawaii, Institute for Astronomy.

Fig. 8.15: National Radio Astronomy Observatory.

INTERNATIONAL SERIES IN PURE AND APPLIED PHYSICS

Barger and Olsson: Classical Mechanics: A Modern Perspective
Bjorken and Drell: Relativistic Quantum Fields
Bjorken and Drell: Relativistic Quantum Mechanics
Fetter and Walecka: Quantum Theory of Many-Particle Systems
Fetter and Walecka: Theoretical Mechanics of Particles and Continua
Feynmann and Hibbs: Quantum Mechanics and Path Integrals
Itzykson and Zuber: Quantum Field Theory
Morse and Feshbach: Methods of Theoretical Physics
Park: Introduction to the Quantum Theory
Schiff: Quantum Mechanics
Stratton: Electromagnetic Theory
Tinkham: Group Theory and Quantum Mechanics
Townsend: A Modern Approach to Quantum Mechanics
Wang: Solid-State Electronics

About the Authors

Vernon D. Barger earned his B.S. and Ph.D. degrees at the Pennsylvania State University and joined the faculty of the University of Wisconsin-Madison in 1963, where he continues to teach and do research. He is currently Vilas Professor and Director of the Institute for Elementary Particle Physics Research. He has been a Visiting Professor at the University of Hawaii and the University of Durham, and a Visiting Scientist at the SLAC, CERN and Rutherford Laboratories. Dr. Barger's fellowships include Fellow of the American Physical Society, John Simon Guggenheim Memorial Foundation Fellow, and Senior Visiting Fellow of the British Science-Engineering Research Council. Barger has co-authored three other textbooks and has published more than 300 scientific articles.

Martin G. Olsson earned his B.S. degree at the California Institute of Technology and his Ph.D. degree at the University of Maryland. A member of the University of Wisconsin faculty since 1964, he has published over 100 research papers. With Barger he also coauthored the textbook *Classical Electricity and Magnetism: A Contemporary Perspective*. Olsson was the recipient of a University distinguised teaching award and has served as Chair of the Physics Department. He has held visiting positions at the University of Durham and CERN, Los Alamos and Rutherford Laboratories.

PREFACE

In the twenty-one years that have elapsed since the original edition was published, we have collected many ideas for improvements. In deciding which changes to make, we have continued with our original philosophy of a reasonably concise presentation that includes numerous applications of interest in the real world. By incorporating feedback from students in our classes, we have tried to make the textbook even more student friendly.

The original edition was designed for an intensive one semester course of 45 lectures and the present text preserves that option with the basic material contained in the first 8 chapters. A one-semester course may include Chapters 1, 2, 3.1–3.3, 3.7, 4, 5.1–5.4, 5.6, 6.1–6.5, 6.7–6.9, 7.1–7.7, 7.10, 8.1–8.2. Several new chapters are included to accommodate longer courses of two quarters or two semesters and to provide enrichment for students taking a one-semester course. Numerous new exercises have been added. Short answers to most exercises are given in an Appendix.

The major changes include the following:

• One of the salient features of the first edition was the introduction of Lagrangian methods at an early stage. In the new edition more Lagrangian material and examples are included which made it natural to devote a single chapter to an introduction to the Lagrangian approach. We have integrated a parallel track development of Lagrangian and Newtonian methods throughout the text.

• We updated the section on the Grand Tour of the outer planets in view of the spectacular success of the Voyager space mission. In the first edition, more than five years before the launch, we did not anticipate how truly revolutionary this odyssey would be.

• In the treatment of tops we now use the Euler angles and the Lagrangian to obtain the equations of motion.

• We have expanded the gravitation chapter to introduce the physical ideas that underlie general relativity and qualitatively explore its consequences.

• An area of exploding interest today is cosmology and we devote a new chapter to the Newtonian description of the universe as a whole. First we classify the possible universes consistent with Hubble's law and Newtonian dynamics; then we use the virial theorem together with astronomical observations to discuss the evidence that most of the matter in the universe is in the form of dark matter.

• A chapter on special relativity is added for curricula where relativity is taught in the mechanics course. A description is given of an experimental test of time dilation with round-the-world flights with atomic clocks.

• The years since the original edition saw the emergence of non-linear dynamics as a major area in physics. We give an introduction to this area by describing solutions to the Duffing equation for a damped and driven anharmonic oscillator. After considering approximate analytic solutions, we explore numerical solutions including the period-doubling route to chaos. This chapter may provide a convenient starting point for students who want to do an undergraduate thesis involving numerical studies of nonlinear systems: it is at a somewhat higher level than the other chapters.

• We have deleted a few sections from the original edition in the interest of keeping a reasonable length. Numerous sections have been rewritten to make the derivations more understandable. Throughout the text we have made improvements in notation.

Many colleagues and students contributed greatly to the development of this new edition and we wish to thank them for their help and encouragement. In particular, we would like to express our appreciation to the following people. Throughout many drafts of the manuscript, Professor Charles Goebel generously gave us excellent advice and made substantial contributions to the contents. Amy Barger and Andrew Barger gave valuable student input on the manuscript and solved many of the exercises. Professor Micheal Berger provided input from his classroom experience with the book. Professors Art Code and Jacqueline Hewett were very helpful in providing photos. Collin Olson, James Ireland and Andrew Barger made computer-generated figures. Ed Stoeffhaas skillfully typeset the manuscript using TEX and created many of the new illustrations. Jack Shira, as editor of this series, was extremely helpful and supportive of our efforts to produce an improved textbook.

We have found classical mechanics to be an extremely interesting course to teach since it offers the opportunity for students to develop an appreciation for the physical explanation of diverse phenomena. We sincerely hope that students will enjoy using the book as much as we have enjoyed creating it!

Vernon Barger
Martin Olsson

Contents

Chapter 1

ONE-DIMENSIONAL MOTION

The formulation of classical mechanics represents a giant milestone in our intellectual and technological history, as the first mathematical abstraction of physical theory from empirical observation. This crowning achievement is rightly accorded to Isaac Newton (1642–1727), who modestly acknowledged that if he had seen further than others, "it is by standing upon the shoulders of Giants." However, the great physicist Pièrre Simon Laplace characterized Newton's work as the supreme exhibition of individual intellectual effort in the history of the human race.

Newton translated the interpretation of various physical observations into a compact mathematical theory. Three centuries of experience indicate that mechanical behavior in the everyday domain can be understood from Newton's theory. His simple hypotheses are now elevated to the exalted status of laws, and these are our point of embarkation into the subject.

1.1 Newtonian Theory

The Newtonian theory of mechanics is customarily stated in three laws. According to the first law, a particle continues in uniform motion (*i.e.*, in a straight line at constant velocity) unless a force acts on it. The first law is a fundamental observation that physics is simpler when viewed from a certain kind of coordinate system, called an *inertial frame*. One cannot define an inertial frame except by saying that it is a frame in which Newton's laws hold. However, once one finds (or imagines) such a frame, all other frames which move with respect to it at constant velocity, with no rotation, are also inertial frames. A coordinate system fixed on the surface of the earth is not an inertial frame because of the acceleration due to the rotation of the earth and the earth's motion around the sun. Nevertheless, for many purposes it is an adequate approximation to regard a coordinate frame fixed on the earth's surface as an inertial frame. Indeed, Newton himself discovered nature's true laws while riding on the earth!

The essence of Newton's theory is the second law, which states that *the time rate of change of momentum of a body is equal to the force acting on the particle.* For motion in one dimension, the second law is

$$F = \frac{dp}{dt} \tag{1.1}$$

where the momentum p is given by the product of (mass) $\times$ (velocity) for the particle

$$p = mv \tag{1.2}$$

The second law provides a definition of force. It is useful because experience has shown that the force on a body is related in a quantitative way to the presence of other bodies in its vicinity. Further, in many circumstances it is found that the force on a body can be expressed as a function of x, v, and t, and so (1.1) becomes

$$F(x,v,t) = \frac{dp}{dt} = m\frac{d^2x}{dt^2} \tag{1.3}$$

This differential equation is called the *equation of motion.* Here m is assumed to be constant. For the remainder of this book we use Newton's notation $\dot{x} = dx/dt$; $\ddot{x} = d^2x/dt^2$. Newton's second law is then

$$F(x,\dot{x},t) = m\ddot{x} = ma \tag{1.4}$$

where $a = \ddot{x}$ is the acceleration. In the special case $F = 0$, integration of (1.1) gives $p =$ constant in accordance with the first law.

While Newton's laws apply to any situation in which one can specify the force, very few interesting physical problems lead to force laws amenable to simple mathematical solution. The fundamental force laws of gravitation and electromagnetism do have simple forms for which the second law of motion can often be solved exactly. The use of approximate empirical forms to approximate the true force laws of physical situations involving frictional and drag forces is one of the arts that will be taught in this book. However, in this modern age of computers, one can handle arbitrary force laws by the brute-force method of numerical integration.

The third law states that if body A experiences a force due to body B, then B experiences an equal but opposite force due to A. (One speaks of this as the force between the two bodies.) As a consequence, the rates of change of the momenta of particles A and B are equal but opposite, and therefore the total $p_A + p_B$ is constant. This law is extremely useful, for instance in the treatment of rigid-body motion, but its range of applicability is not as universal as the first two laws. The third law breaks down when the interaction between the particles is electromagnetic, because the electromagnetic field carries momentum.

It is a remarkable fact that macroscopic phenomena can be explained by such a simple set of mathematical laws. As we shall see, the mathematical solutions to some problems can be complex; nevertheless, the physical basis is just (1.1). Of course, there is still a great deal of physics to put into (1.1), namely, the laws of force for specific kinds of interactions.

1.2 Interactions

Using the planetary orbit data analysis by Kepler, Newton was able to show that all known planetary orbits could be accounted for by the following force law

$$F = -\frac{GM_1M_2}{r^2} \tag{1.5}$$

This states that force between masses M_1 and M_2 is proportional to the masses and inversely proportional to the square of the distance between them. The negative sign in (1.5) denotes an attractive force between the masses. The force acts along the line between the two masses and thus for non-rotational motion the problem is effectively one-dimensional. Newton proposed that this gravitational law was universal, the same force law applying between us and the earth as between celestial bodies (and more generally between any two masses). The universality of the gravitational law can be verified, and the proportionality constant G determined, by delicate experimental measurements of the force between masses in the laboratory. The value of G is

$$G = 6.672 \times 10^{-11}\,\mathrm{m}^3/(\mathrm{kg\,s})^2 \tag{1.6}$$

The dominant gravitational force on an object located on the surface of the earth is the attraction to the earth. The gravitational force between two spherically symmetric bodies is as if all the mass of each body were concentrated at its center, as Newton proved. We will give a proof of this assertion in Chapter 8. The earth is very nearly spherical so we can use the force law of (1.5). Thus for an object of mass m on the surface of earth, the force is

$$F = -m \frac{M_E G}{R_E^2} = -mg \tag{1.7}$$

where g is the gravitational acceleration,

$$g \simeq 9.8 \text{ m/s}^2 \tag{1.8}$$

Using the measured value of $R_E = 6{,}371\,\text{km}$ along with the measured values of g and G as given above, we may use (1.7) to deduce the mass of the earth to be

$$M_E = 5.97 \times 10^{24} \text{ kg} \tag{1.9}$$

Since the earth's radius is large, the gravitational force of an object anywhere in the biosphere is given to good accuracy by (1.7); even at the top of the atmosphere ($\approx$ 200 km up) the force has decreased by less than 10% from its value at the surface of the earth. Consequently, in many applications on earth, we can neglect the variation of the gravitational force with position.

The static Coulomb force between two charges e_1 and e_2 is similar in form to the gravitational-force law of (1.5),

$$F = k\frac{e_1 e_2}{r^2} \tag{1.10}$$

This force is attractive if the charges are of opposite sign and repulsive if the charges are of the same sign. The constant k depends on the system of electrical units; in SI units, $k = (4\pi\epsilon_0)^{-1} \simeq 9 \times 10^9 \text{N-m}^2/\text{C}^2$.

Another force with a wide range of application is the spring force or Hooke's law, which is expressed as

$$F = -kx \tag{1.11}$$

with $k > 0$. Here k is a spring constant which is dependent on the properties of the spring and x is the extension of the spring from its relaxed position. This particular force law is a very good approximation in many physical situations (*e.g.*, the stretching or bending of materials) which are initially in equilibrium.

Frictional forces prevent or damp motions. The static frictional force between two solid surfaces is

$$|F| \leq \mu_s N \tag{1.12}$$

The force F acts to prevent sliding motion. N is the perpendicular force (normal force) holding the surfaces together, and μ_s is a material-dependent coefficient. Equation (1.12) is an *approximate* formula for frictional forces which has been deduced from empirical observations. The frictional force which retards the motion of sliding objects is given by

$$F = \mu_k N \tag{1.13}$$

It is observed that this force is nearly independent of the velocity of the motion for velocities which are neither too small (where there is molecular adhesion) nor too large (where frictional heating becomes important). For a given pair of surfaces, the coefficient of kinetic friction μ_k is less than the coefficient of static friction μ_s.

Frictional laws to describe the motion of a solid through a fluid or a gas are often complicated by such effects as turbulence. However, for sufficiently small velocities, the approximate form

$$F = -bv \tag{1.14}$$

where b is a constant, holds. The drag coefficient b in (1.14) is proportional to the fluid viscosity. For a sphere of radius a moving slowly through a fluid of viscosity η the Stokes law of resistance is calculated to be

$$b_{\text{sphere}} = 6\pi a\eta \tag{1.15}$$

At higher, but still subsonic velocities, the drag law is

$$F = -cv^2 \tag{1.16}$$

For instance, the drag force on an airplane is remarkably well represented by a constant times the square of the velocity. The drag coefficient c for a body of cross-sectional area S moving through a fluid of density ρ is given by

$$c = \tfrac{1}{2} C_D S \rho \tag{1.17}$$

where C_D is a dimensionless factor related to the geometry of the body (about 0.4 for a sphere).

Externally imposed forces can take on a variety of forms. Of those depending explicitly on time, sinusoidally oscillating forces like

$$F = F_0 \cos \omega t \tag{1.18}$$

are frequently encountered in physical situations.

In a general case the forces can be position-, velocity-, and time-dependent,

$$F = F(x, v, t) \tag{1.19}$$

Among the most interesting and easily solved examples are those in which the forces depend on only one of the above three variables, as illustrated by the examples in the following three sections.

1.3 The Drag Racer: Frictional Force

A number of interesting engineering-type problems can be solved from straightforward application of Newton's laws. As an illustration, suppose we consider a drag racer that can achieve maximum possible acceleration when starting from rest. The external forces on the racer which must be taken into account are (1) gravity, (2) the normal forces supporting the racer at the wheels, and (3) the frictional forces which oppose the rotation of the powered rear wheels. A sketch indicating the various external forces is given in Fig. 1-1.

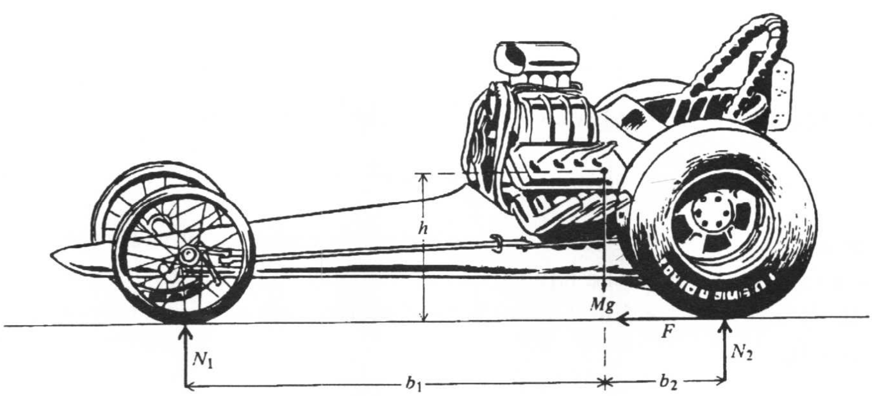


FIGURE 1-1. Forces on a drag racer.

Since the racer is in vertical equilibrium, the sum of the external vertical forces must vanish,

$$N_1 + N_2 - Mg = 0 \tag{1.20}$$

Both N_1 and N_2 must be positive. For the horizontal motion we apply Newton's second law,

$$F = Ma \tag{1.21}$$

The frictional force F is bounded by

$$F \leq \mu N_2 \tag{1.22}$$

The maximum friction force occurs just as the racer tires begin to slip relative to the drag strip, because the coefficient of kinetic friction is smaller than the coefficient of static friction. For maximal initial acceleration we must have the maximum friction force $F = \mu N_2$. Referring back to (1.20), a maximal $N_2 = Mg$ is obtained when $N_1 = 0$, that is, when the back wheels completely support the racer. The greatest possible acceleration is then

$$a_{\mathbf{max}} = \frac{\mu (N_2)_{\mathbf{max}}}{M} = \mu g \tag{1.23}$$

We see that the optimum acceleration is independent of the racer's mass. Under normal conditions the coefficient of friction μ between rubber and concrete is about unity. Thus a racer can achieve an acceleration of about 9.8 $\mathrm{m/s^2}$. In actual design a small normal force N_1 on the front wheels is allowed for steering purposes.

The standard drag strip is $\approx$ 400 m (1/4 mi) in length. If we assume that the racer can maintain the maximum acceleration for the duration of a race and that the coefficient of friction is constant, we can calculate the final velocity and the elapsed time. The differential form of the second law is

$$F = Ma = M\frac{dv}{dt} = M\ddot{x} \tag{1.24}$$

When the acceleration a is constant, a single integration

$$\int_{v_0}^{v} dv = a\int_0^t dt \tag{1.25}$$