Micro-scale Plasticity Mechanics

微尺度塑性力学

By

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编著

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当代科学技术基础理论与前沿问题研究丛书

中国科学技术大学

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内 容 简 介

本书系统地介绍了材料微尺度力学行为的尺寸效应实验现象，重点介绍了几种具有代表性的微尺度应变梯度塑性理论及对微尺度实验现象的解释，以及对裂纹尖端微尺度范围内解理断裂的应用。此外，还融会贯通地介绍了国内外学者的原创性工作和创新性学术思想。

全书共8章。第1章介绍了应变梯度塑性理论的应用背景及经典微极理论；第2章介绍了金属材料典型的微尺度力学实验现象；第3至7章介绍了几种典型的应变梯度理论及其应用；第8章介绍了应变梯度理论在微观断裂力学中的应用。

本书适合从事固体微尺度力学、先进材料的微结构设计与力学性能优化、微机电和微电子元件力学行为研究的科技工作者及工程师使用和参考，也可供力学专业及材料专业的高年级本科生和研究生阅读参考。

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Published by University of Science and Technology of China Press

96 Jinzhai Road, Hefei 230026, P. R. China

图书在版编目(CIP)数据

微尺度塑性力学 = Micro-scale Plasticity Mechanics：英文/陈少华，王自强编著.—合肥：中国科学技术大学出版社，2009.4

（当代科学技术基础理论与前沿问题研究丛书：中国科学技术大学校友文库）

“十一五”国家重点图书

ISBN 978-7-312-02268-5

Ⅰ.微… Ⅱ.①陈…②王… Ⅲ.塑性力学—英文 Ⅳ.O344

中国版本图书馆 CIP 数据核字(2009)第 046780 号

中国科学技术大学出版社出版发行

地址：安徽省合肥市金寨路 96 号，230026

网址：http://press.ustc.edu.cn

合肥晓星印刷有限责任公司印刷

全国新华书店经销

开本：710×1000 1/16 印张：18.25 字数：290 千

2009 年 5 月第 1 版 2009 年 5 月第 1 次印刷

印数：1—1 500 册

定价：58.00 元

总　　序

侯建国
（中国科学技术大学校长、中国科学院院士、第三世界科学院院士）

大学最重要的功能是向社会输送人才。大学对于一个国家、民族乃至世界的重要性和贡献度，很大程度上是通过毕业生在社会各领域所取得的成就来体现的。

中国科学技术大学建校只有短短的50年，之所以迅速成为享有较高国际声誉的著名大学之一，主要就是因为她培养出了一大批德才兼备的优秀毕业生。他们志向高远、基础扎实、综合素质高、创新能力强，在国内外科技、经济、教育等领域做出了杰出的贡献，为中国科大赢得了"科技英才的摇篮"的美誉。

2008年9月，胡锦涛总书记为中国科大建校五十周年发来贺信，信中称赞说：半个世纪以来，中国科学技术大学依托中国科学院，按照全院办校、所系结合的方针，弘扬红专并进、理实交融的校风，努力推进教学和科研工作的改革创新，为党和国家培养了一大批科技人才，取得了一系列具有世界先进水平的原创性科技成果，为推动我国科教事业发展和社会主义现代化建设做出了重要贡献。

据统计，中国科大迄今已毕业的5万人中，已有42人当选中国科学院和中国工程院院士，是同期（自1963年以来）毕业生中当选院士数最多的高校之一。其中，本科毕业生中平均每1,000人就产生1名院士和700多名硕士、博士，比例位居全国高校之首。还有众多的中青年才俊成为我国科技、企业、教育等领域的领军人物和骨干。在历年评选的"中国青年五四奖章"获得者中，作为科技界、科技创新型企业界青年才俊代表，科大毕业生已连续多年榜上有名，获奖总人数位居全国高校前列。鲜为人知的是，有数千名优秀毕业生踏上国防战线，为科技强军做出了

重要贡献，涌现出20多名科技将军和一大批国防科技中坚。

为反映中国科大五十年来人才培养成果，展示毕业生在科学研究中的最新进展，学校决定在建校五十周年之际，编辑出版《中国科学技术大学校友文库》，于2008年9月起陆续出书，校庆年内集中出版50种。该《文库》选题经过多轮严格的评审和论证，入选书稿学术水平高，已列为国家"十一五"重点图书出版规划。

入选作者中，有北京初创时期的毕业生，也有意气风发的少年班毕业生；有"两院"院士，也有IEEE Fellow；有海内外科研院所、大专院校的教授，也有金融、IT行业的英才；有默默奉献、矢志报国的科技将军，也有在国际前沿奋力拼搏的科研将才；有"文革"后留美学者中第一位担任美国大学系主任的青年教授，也有首批获得新中国博士学位的中年学者；……在母校五十周年华诞之际，他们通过著书立说的独特方式，向母校献礼，其深情厚意，令人感佩！

近年来，学校组织了一系列关于中国科大办学成就、经验、理念和优良传统的总结与讨论。通过总结与讨论，使我们更清醒地认识到，中国科大这所新中国亲手创办的新型理工科大学所肩负的历史使命和责任。我想，中国科大的创办与发展，首要的目标就是围绕国家战略需求，培养造就世界一流科学家和科技领军人才。五十年来，我们一直遵循这一目标定位，有效地探索了科教紧密结合、培养创新人才的成功之路，取得了令人瞩目的成就，也受到社会各界的广泛赞誉。

成绩属于过去，辉煌须待开创。在未来的发展中，我们依然要牢牢把握"育人是大学第一要务"的宗旨，在坚守优良传统的基础上，不断改革创新，提高教育教学质量，早日实现胡锦涛总书记对中国科大的期待：瞄准世界科技前沿，服务国家发展战略，创造性地做好教学和科研工作，努力办成世界一流的研究型大学，培养造就更多更好的创新人才，为夺取全面建设小康社会新胜利、开创中国特色社会主义事业新局面贡献更大力量。

是为序。

2008年9月

Preface

Micro-scale plasticity mechanics was developed in 1990s due to the developments in micro-design, micro-manufacturing and microelectronic packaging. It is a new field that attracts many researchers' interests in the world.

Many experiments have found that materials display strong size effects when the characteristic length scale associated with non-uniform plastic deformation is on the order of microns. The classical plasticity theories can not predict the size effects of material behavior at the micron scale since there is no length scales including in their constitutive relations. Apparently, some microscopic understanding of plasticity is necessary in order to accurately describe deformation at small scales. These considerations have motivated Fleck and Hutchinson to develop a phenomenological theory of strain gradient plasticity intended for applications to materials and structures whose dimension controlling plastic deformation falls roughly within a range from a tenth of a micron to ten microns. After that, a lot of scholars make further contributions to this area with the considerations to propose theories with more clearly physical backgrounds and more simple frameworks.

In this book, we introduce the experimental backgrounds of the micro-scale mechanics with the help of many typical micro-scale experiments. After that, we systematically introduce several typical micro-scale plasticity theories and their applications in explaining the experimental results. Lastly, micro-scale plasticity theories are applied in the fracture mechanics field to explain the cleavage fracture in the scope of micro-meters near the crack tip. This book includes not only the achievements of many foreign scholars, but also those of the authors themselves.

Many scientists in China contribute to this area, such as Prof. K. C. Hwang of Tsinghua University, Prof. G. K. Hu of Beijing Institute of Technology, etc. Due to the limitations of the length of this book, we did not focus on their achievements. The readers can consult them face to face if it is necessary.

Chapters 1~5 and 8 are written by Shaohua Chen; Chapters 6~7 are written by Tzuchiang Wang.

SC would give his gratitude to his wife, Miss Wen-Ling, for her collections and scanning of the electronic photos in Chapters 1~5 and 8.

The work of the two authors is supported by NSFC.

Shaohua Chen & Tzuchiang Wang

March 29, 2009 in Beijing

Contents

1 Introduction

1.1 Brief introduction of experimental observations

Many experiments have found that materials display strong size effects when the characteristic length scale associated with non-uniform plastic deformation is on the order of microns. For example, Fleck et al. (1994) did torsion experiments of thin copper wires with different micrometer diameters and found that the non-dimensional torque increases by a factor of 3 as the wire diameter decreases from 170 to 12 microns, while no increase of work-hardening in simple tension is observed. In ultra thin beams bending experiments, Stolken and Evans (1998) observed a significant increase in the non-dimensional bending hardening moments when the beam thickness decreases from 100 to 12.5 microns, while the results for simple tension experiments display no size effects. For an aluminum-silicon matrix reinforced by silicon carbide particles, Lloyd (1994) observed that the flow strength increases when the particle diameter was reduced from 16 to 7.5 microns with the volume fraction of particles fixed at 15%. More convincing experimental evidence of the size dependence of material behavior at the micro level is from the micro or nano-indentation hardness tests. The measured indentation hardness of metallic materials increases by a factor of two when the depth of indentation decreases from 10 microns to 1 micron (Nix, 1989; Stelmeshenko et al., 1993; Ma and Clarke, 1995; Poole et al., 1996; McElhaney et al., 1998).

The classical plasticity theories can not predict the size effects of material behavior at the micron scale since there is no length scales including in their constitutive relations. The predictions based on the classical plasticity theories for non-uniform deformation do not show a size effect after normalization. However, there is an impending need to deal with design and manufacturing issues at the micron level, such as in thin films whose thickness is on the order of 1 micron or less, actuators and micro-electro-mechanical systems (MEMS) where the entire system size is less than 10 microns; microelectronic packaging where features are smaller than 10 microns; advanced composites where particle or fiber size is on the order of 10 microns; as well as in micromachining. The current design tools, such as finite element method (FEM) and computer aided design (CAD), are based on classical continuum theories, which may not be suitable at such a small length scale. On the other hand, it is still not possible to perform quantum and atomistic simulations on realistic time and length scales required for the micro level structures. A continuum theory for micro level applications is thus needed to bridge the gap between conventional continuum theories and atomistic simulations.

Another objective that needs the development of a micron level continuum theory is to link macroscopic fracture behavior to atomistic fracture processes in ductile materials. In a remarkable series of experiments, Elssner et al. (1994) measured both the macroscopic fracture toughness and atomic work of separation of an interface between a single crystal of niobium and a sapphire single crystal. The macroscopic work of fracture was measured using a four-point bend specimen designed for the determination of interfacial toughness, while the atomic value was inferred from the equilibrium shapes of microscopic pores on the interface. The interface between the two materials remained atomistically sharp, i.e., the crack tip was not blunted even though niobium is ductile and has a large number of dislocations. The stress level needed to produce atomic decohesion of a lattice or a strong interface is typically on the order of 0.03 times Young's modulus, or 10 times the tensile yield stress. Hutchinson (1997) pointed out that the maximum stress level that can be achieved near a crack tip is not larger than 4 or 5 times the tensile yield stress of metals, according to models based on conventional plasticity theories. This clearly falls short of triggering the atomic decohesion observed in Elssner et al.'s (1994) experiments. Attempts to link macroscopic cracking to atomic fracture are frustrated by the inability of conventional plasticity theories to model stress-strain behavior adequately at the small scales involved in crack tip deformation.

Apparently, some microscopic understanding of plasticity is necessary in order to accurately describe deformation at small scales.

1.2 An overview of strain gradient plasticity theory

When a material is deformed, dislocations are generated, moved, and stored, and the storage causes the material to work harden. Dislocations become stored for one of two reasons: they accumulate by trapping each other in a random way, or they are required for compatible deformation of various parts of the material. In the former case the dislocations are referred to as statistically stored dislocations (Ashby, 1970), while in the latter case they are called geometrically necessary dislocations and are related to the gradients of plastic shear in a material

(Nye, 1953; Cottrell, 1964; Ashby, 1970). Plastic strain gradients appear either because of the geometry of loading or because of inhomogeneous deformation in the material, as in the aforementioned experiments. As examples: in the plastic twisting of a cylinder or bending of a beam, the strain is finite at the surface but zero along the axis of twist or of bending (Figures 1.1(a) and 1.1(b)); in the hardness test the strain is large immediately beneath the indenter but zero far from it; and in the plastic zone at the tip of a crack in an otherwise elastic medium steep gradients of plastic strain appear (Figures 1.1(c) and 1.1(d)); in the deformation of plastic crystals containing hard, non-deforming particles, local strain gradients are generated between particles; and in the plastic deformation of polycrystals, the mismatch of slip at the boundaries of the grains can induce gradients of plastic strain there (Figures 1.1(e) and 1.1(f)).

These considerations have motivated Fleck and Hutchinson (1993, 1997) and Fleck et al. (1994) to develop a phenomenological theory of strain gradient plasticity intended for applications to materials and structures whose dimension controlling plastic deformation falls roughly within a range from a tenth of a micron to ten microns. This theory has been applied to many problems where strain gradient effects are expected to be important, including analyses of crack tip fields (Huang et al., 1995, 1997; Xia and Hutchinson, 1996). The Fleck-Hutchinson theory fits the mathematical framework of higher order continuum theories of elasticity (Toupin, 1962; Koiter, 1964; Mindlin, 1963, 1964), with the strain gradients represented either in terms of the gradients of rotation in the couple-stress theory of strain gradient plasticity (Fleck and Hutchinson, 1993; Fleck et al., 1994) or in terms of both rotation and stretch gradients in a more general isotropic-hardening theory based on all the quadratic invariants of the strain gradient tensor (Fleck and Hutchinson, 1997). The couple stress theory used by Fleck and Hutchinson (1993) also bears some resemblance to the early work of

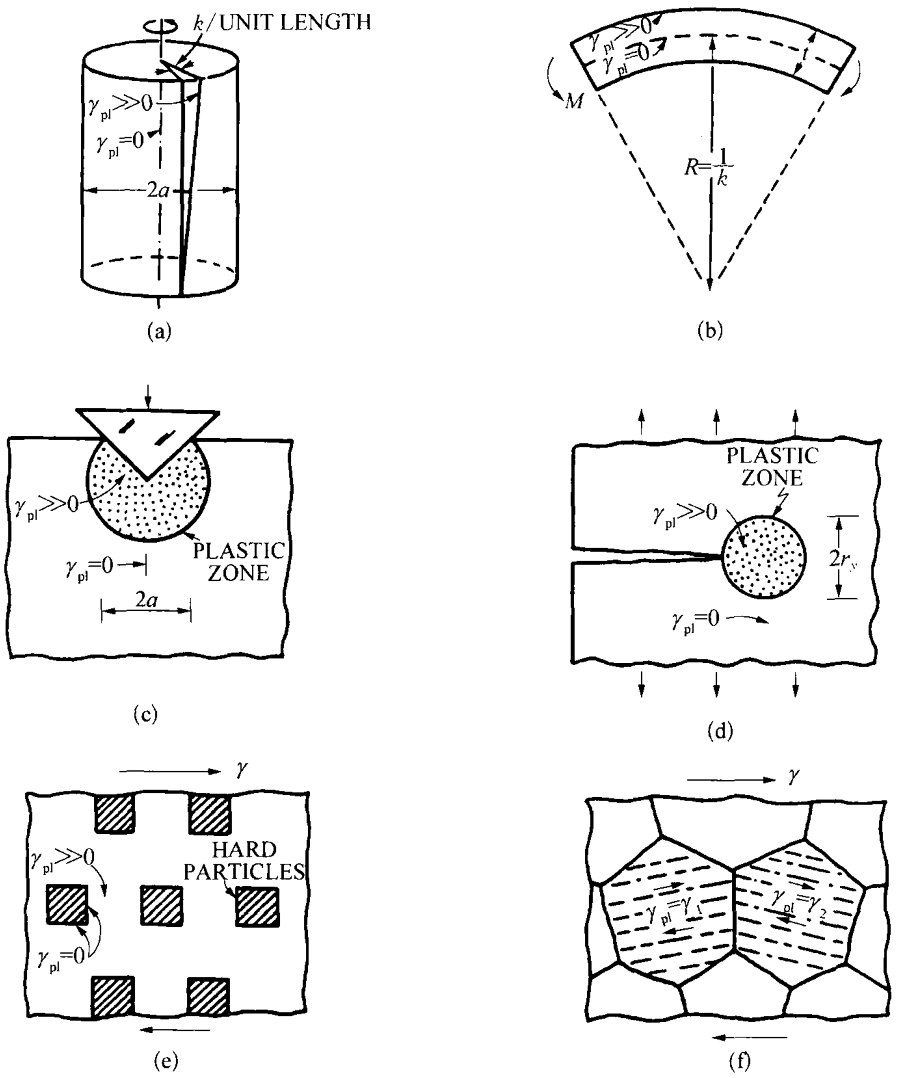

Figure 1. 1 Plastic strain gradient are caused by the geometry of deformation (a, b), by local boundary conditions (c, d) or by the microstructure itself (e, f). (Fleck et al., 1994)

Kroener (1962) who studied the connection between lattice curvature associated with dislocations and couple stresses and developed a non-local continuum theory based on that connection. The work-conjugate of the rotation and/or stretch gradient of deformation defines the higher order stress which is required for this class of strain gradient theory to satisfy the Clausius-Duhem thermodynamic restrictions on the constitutive model for second deformation gradients (Gurtin, 1965a, b; Acharya and Shawki, 1995). From a dimensional consideration, an internal constitutive length parameter, l, was introduced to scale the rotational gradient terms in the couple-stress theory of strain gradient plasticity (Fleck and Hutchinson, 1993; Fleck et al., 1994). This length scale is thought of as an internal material length related to the storage of geometrically necessary dislocations, and is found to be approximately 4 microns for copper from Fleck et al.'s (1994) twisting of thin wire experiments, and 6 microns for nickel from Stolken and Evans' (1997) bending of ultra-thin beam experiments. The contribution of the strain gradient could be symbolically represented as $l\mathrm{d}\varepsilon/\mathrm{d}x \sim \varepsilon(l/D)$ where D represents the characteristic length of the deformation field usually corresponding to the smallest dimension of geometry (e.g., thickness of a beam, radius of a void, depth of indentation). When D is much larger than the material length, l, the strain gradient terms become negligible in comparison with strains, and strain gradient plasticity then degenerates to the conventional plasticity theory. However, as D becomes comparable to l as in the aforementioned experiments, strain gradient effects begin to play a dominating role. The couple-stress theory of strain gradient plasticity has had some success in estimating the size dependence observed in the aforementioned torsion of thin wires (Fleck et al., 1994) and bending of thin beams (Stolken and Evans, 1998). However, its prediction of indentation hardness (Shu and Fleck, 1998) falls short of agreement with the significant increase of 200% or even 300% observed in micro-indentation or nano-indentation tests (Nix, 1989; De Huzman et al., 1993; Stelmashenko et al., 1993; Ma and Clarke, 1995; Poole et al., 1996; McElhaney et al., 1998). For this reason, Fleck and Hutchinson (1997) proposed an extended theory of strain gradient plasticity theory which includes both rotation gradient and stretch gradient of the deformation in the constitutive model. The workconjugates of rotation and stretch gradients of deformation are couple stress and higher order stress, respectively Accordingly, two more internal material lengths are introduced in addition to l

Fleck and Hutchinson (1993, 1997) used the dislocation theory to motivate their formulation of strain gradient plasticity However, the actual theory was formulated by replacing effective stresses and strains in conventional plasticity with higher order effective stresses and strains which contain strain gradient terms scaled by a phenomenological material length to be determined from experiments. In other words, the Fleck-Hutchinson theory is developed primarily based on the macroscopically measured uniaxial stress-strain behavior. Micromechanical experiments such as micro-indentation, micro-torsion and micro-bending were not used at the stage of theory construction, but rather were used to fit the material length l. The remarkable agreement between the strain gradient law proposed by Nix and Gao (1998) and the micro-indentation data for various materials indicates that the linear relation between the square of indentation hardness and the inverse of indent depth represents a fundamental, intrinsic nature of deformation at the microscale This provides a strong motivation to develop an alterative formulation in which the strain gradient law in Nix and Gao (1998) is incorporated as a fundamental postulate. Gao et al. (1999) proposed a multiscale, hierarchical framework to facilitate such a marriage between plasticity and dislocation theory. A mesoscale cell with linear variation of strain field is considered. Each point within the cell is considered as a microscale sub-cell within which dislocation interaction is assumed to (approximately) obey the Taylor relation so that the strain gradient law proposed by Nix and Gao (1998) applies On the microscale, the effective strain gradient η is to be treated as a measure of the density of geometrically necessary dislocations whose accumulation increases the flow stress strictly following the Taylor model. In the other words, microscale plastic law is assumed to occur as slip of statistically stored