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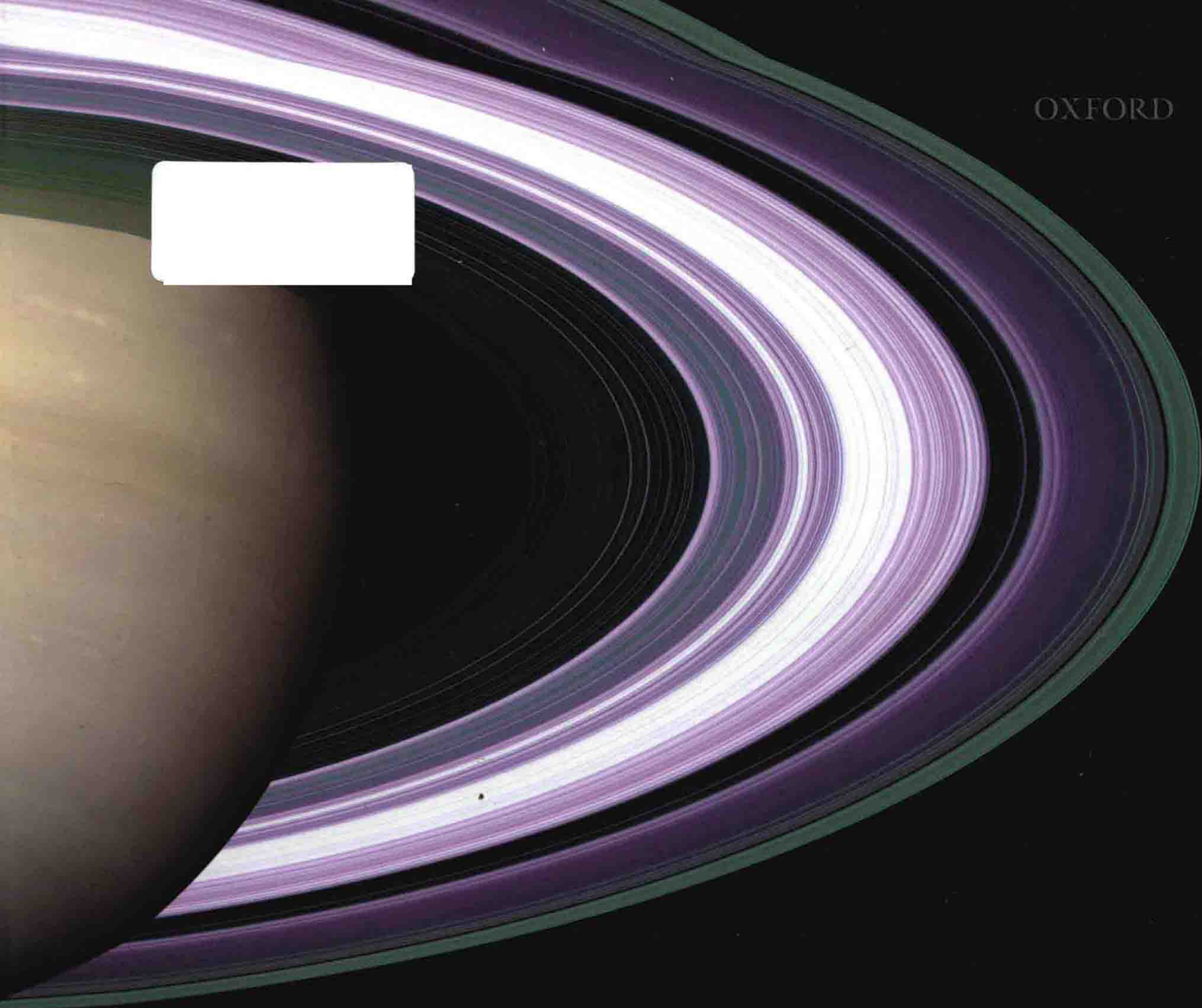

Introduction to Modern Dynamics

Chaos, Networks, Space and Time

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UNIVERSITY PRESS

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Great Clarendon Street, Oxford, OX2 6DP,
United Kingdom

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It furthers the University's objective of excellence in research, scholarship,
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Impression: 1

Published in the United States of America by Oxford University Press
198 Madison Avenue, New York, NY 10016, United States of America

British Library Cataloguing in Publication Data
Data available

Library of Congress Control Number: 2014939331

ISBN 978–0–19–965703–2 (hbk.)
ISBN 978–0–19–965704–9 (pbk.)

Printed and bound by
CPI Group (UK) Ltd, Croydon, CR0 4YY

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Preface
The Best Parts of Physics

The best parts of physics are the last topics that our students ever see. These are the exciting new frontiers of nonlinear and complex systems that are at the forefront of university research and are the basis of many of our high-tech businesses. Topics such as traffic on the World Wide Web, the spread of epidemics through globally-mobile populations, or the synchronization of global economies are governed by universal principles just as profound as Newton's laws. Nonetheless, the conventional university physics curriculum reserves most of these topics for advanced graduate study. Two justifications are given for this situation: first, that the mathematical tools needed to understand these topics are beyond the skill set of undergraduate students, and second, that these are specialty topics with no common theme and little overlap.

Introduction to Modern Dynamics: Chaos, Networks, Space and Time dispels these myths. The structure of this book combines the three main topics of modern dynamics—chaos theory, dynamics on complex networks, and the geometry of dynamical spaces—into a coherent framework. By taking a geometric view of physics, concentrating on the time evolution of physical systems as trajectories through abstract spaces, these topics share a common and simple mathematical language with which any student can gain a unified physical intuition. Given the growing importance of complex dynamical systems in many areas of science and technology, this text provides students with an up-to-date foundation for their future careers.

While pursuing this aim, *Introduction to Modern Dynamics* embeds the topics of modern dynamics—chaos, synchronization, network theory, neural networks, evolutionary change, econophysics, and relativity—within the context of traditional approaches to physics founded on the stationarity principles of variational calculus and Lagrangian and Hamiltonian physics. As the physics student explores the wide range of modern dynamics in this text, the fundamental tools that are needed for a physicist's career in quantitative science are provided as well, including topics the student needs to know for the graduate record examination (GRE). The goal of this textbook is to modernize the teaching of junior-level dynamics, responsive to a changing employment landscape, while retaining the core traditions and common language of dynamics texts.

A unifying concept: geometry and dynamics

Instructors or students may wonder how an introductory textbook can contain topics, under the same book cover, on econophysics and evolution as well as the physics of black holes. However, it is not the physics of black holes that matters, rather it is the description of general dynamical spaces that is important, and the understanding that can be gained of the geometric aspects of trajectories governed by the properties of these spaces. All changing systems, whether in biology or economics or computer science or photons in orbit around a black hole, are understood as trajectories in abstract dynamical spaces.

Newton takes a back seat in this text. He will always be at the heart of dynamics, but the modern emphasis has shifted away from $F = ma$ to a newer perspective where Newton's laws are special cases of broader concepts. There are economic forces and forces of natural selection that are just as real as the force of gravity on point particles. For that matter, even the force of gravity recedes into the background as force-free motions in curved space–time takes the fore.

Unlike Newton, Hamilton and Lagrange retain their positions here. The variational principle and the extrema of dynamical quantities are core concepts in dynamics. Maxima or minima of action integrals and metric distances provide trajectories—geodesics—in dynamical spaces. Conservation laws arise naturally from Lagrangians, and energy conservation enables simplifications using Hamiltonian dynamics. Space and geometry are almost synonymous in this context. Defining the space of a dynamical system takes first importance, and the geometry of the dynamical space then determines the set of all trajectories that can exist in it.

A common tool: dynamical flows and the ODE solver

A mathematical flow is a set of first-order differential equations that are solved using as many initial values as there are variables, which defines the dimensionality of the dynamical space. Mathematical flows are one of the foundation stones that appears continually throughout this textbook. Nearly all of the subjects explored here—from evolving viruses to orbital dynamics—can be captured as a flow. Therefore, a common tool used throughout this text is the numerical solution to the ordinary differential equation (ODE). Computers can be both a boon and a bane to the modern physics student. On the one hand, the easy availability of ODE solvers makes even the most obscure equations easy to simulate numerically, enabling any student to plot a phase plane portrait that contains all manner of behavior. On the other hand, physical insight and analytical understanding of complex behavior tend to suffer from the computer-game nature of simulators. Therefore, this textbook places a strong emphasis on analysis, and behavior in limits, with the goal to reduce a problem to a few simple principles, while making use of computer simulations to capture both the whole picture as well as the details of system behavior.

Traditional junior-level physics: how to use this book

All the traditional topics of junior-level physics are here. From the simplest description of the harmonic oscillator, through Lagrangian and Hamiltonian physics, to rigid body motion and orbital dynamics—the core topics of advanced undergraduate physics are retained and are found interspersed through this textbook. The teacher and student can plan a path through these topics here:

- Newton's laws (Section 1.1)
- Harmonic oscillators (Section 1.1)
- Coordinate systems and inertial frames (Sections 1.2 and 1.3)
- Rotating frames and fictitious forces (Section 1.5)
- Rigid body motion (Section 1.6)
- Lagrangian physics (Section 2.1)
- Conservation laws and collisions (Section 2.2)
- The Hamiltonian function (Section 2.3)
- Orbital dynamics and planetary motion (Section 2.4)
- Phase space (Section 2.5)
- Nonlinear oscillators (Section 3.4)
- Coupled oscillators (Section 4.1)
- Special relativity: covered in comprehensive detail (Chapter 10)

What's simple in complex systems?

The traditional topics of mechanics are integrated into the broader view of modern dynamics that draws from the theory of complex systems. The range of subject matter encompassed by complex systems is immense, and a comprehensive coverage of this topic is outside the scope of this book. However, there is still a surprisingly wide range of complex behavior that can be captured using the simple concept that the geometry of a dynamic space dictates the set of all possible trajectories in that space. Therefore, simple analysis of the associated flows provides many intuitive insights into the origins of complex behavior. The special topics covered in this textbook are:

- Chaos theory (Chapter 3)

Much of nonlinear dynamics can be understood through *linearization* of the *flow* equations (equations of motion) around special *fixed points*. Visualizing the dynamics of multi-parameter systems within multidimensional spaces is made simpler by concepts such as the *Poincaré section*, *strange attractors* that have *fractal geometry*, and *iterative maps*.

- Synchronization (Chapter 4)

The nonlinear *synchronization* of two or more oscillators is a starting point for understanding more complex systems. As the whole can be greater than the sum of the parts, the global properties often emerge from the local interactions of the parts. Synchronization of oscillators is surprisingly common and robust, leading to *frequency-entrainment*, *phase-locking*, and *fractional resonance* that allow small perturbations to control large networks of interacting systems.

- Network theory (Chapter 5)

Everywhere we look today we see networks. The ones we interact with daily are social networks and related networks on the World Wide Web. In this chapter, individual nodes are joined into networks of various geometries, such as *small-world networks* and *scale-free networks*. The *diffusion* of disease across these networks is explored, and the synchronization of *Poincaré phase oscillators* can induce a *Kuramoto transition* to complete synchronicity.

- Neural networks (Chapter 6)

Perhaps the most complex of all networks is the brain. This chapter starts with the single neuron, which is a *limit-cycle oscillator* that can show interesting *bistability* and *bifurcations*. When neurons are placed into simple neural networks, such as *perceptrons* or *feed-forward networks*, they can do simple tasks after training by *error back-propagation*. The complexity of the tasks increases with the complexity of the networks, and *recurrent networks*, like the *Hopfield neural net*, can perform associated memory operations that challenge even the human mind.

- Evolutionary dynamics (Chapter 7)

Some of the earliest explorations into nonlinear dynamics came from studies of *population dynamics*. In a modern context, populations are governed by evolutionary pressures and by genetics. Topics such as viral mutation and spread, as well as the evolution of species within a *fitness landscape*, are understood as simple balances within *quasi-species* equations.

- Econophysics (Chapter 8)

A most baffling complex system that influences our daily activities, as well as the trajectory of our careers, is the economy in the large and the small. The dynamics of *microeconomics* determines what and why we buy, while the dynamics of *macroeconomics* drives entire nations up and down economic swings. These forces can be (partially) understood in terms of nonlinear dynamics and flows in economic spaces. *Business cycles* and the diffusion of prices on the *stock market* are no less understandable than evolutionary dynamics (Chapter 7) or network dynamics (Chapter 5), and indeed draw closely from those topics.

- Geodesic motion (Chapter 9)

This chapter is the bridge between the preceding chapters on complex systems, and the succeeding chapters on relativity theory (both special and general). This is where the geometry of space is first fully defined in terms of a *metric tensor*, and where trajectories through a *dynamical space* are discovered to be paths of *force-free motion*. The *geodesic equation* (a geodesic flow) supersedes Newton's second law as the fundamental equation of motion that can be used to define the path of masses through potential landscapes and of light through space–time.

- Special relativity (Chapter 10)

In addition to traditional topics of *Lorentz transformations* and *mass–energy* equivalence, this chapter presents the broader view of trajectories through Minkowski *space–time* whose geometric properties are defined by the *Minkowski metric*. Relativistic forces and non-inertial (accelerating) frames connect to the next chapter that generalizes all relativistic behavior.

- General relativity (Chapter 11)

The physics of *gravitation*, more than any other topic, benefits from the over-arching theme developed throughout this book—that the geometry of a space defines the properties of all trajectories within that space. Indeed, in this geometric view of physics, Newton's force of gravity disappears and is replaced by force-free geodesics through *warped* space–time. Mercury's orbit around the Sun, and trajectories of light past *black holes*, are simply geodesic flows whose properties are easily understood using the tools developed in Chapter 3 and expanded upon throughout this textbook.

- Book Website:

The website for this book is www.works.bepress.com/ddnolte where students and instructors can find the Appendix and additional support materials. The Appendix provides details for some of the mathematical techniques used throughout the textbook, and provides a list of Matlab programming examples for specific Homework problems among the computational projects. The Matlab programming examples are located in a single pdf file on the website.

Acknowledgments

I gratefully acknowledge the many helpful discussions with my colleagues Ephraim Fischbach, Andrew Hirsch, Sherwin Love, and Hisao Nakanishi during the preparation of this book, and for the help of Bradley C. Abel and Su-Ju Wang who carefully checked all of the equations. Special thanks to my family, Laura and Nicholas, for putting up with my "hobby" for so many years, and also for their encouragement and moral support. I also thank the editors at Oxford, Jessica White for her help in preparing the manuscript, and especially Sonke Adlung for helping me realize my vision.

Contents

Part 1

Geometric Mechanics

A unifying viewpoint of physics has emerged, over the past century, that studying the geometric properties of special points and curves within dynamical spaces makes it possible to gain a global view of the dynamical behavior, rather than focusing on individual trajectories. Dynamical spaces can have many different dimensions and many different symmetries. They carry with them names like *configuration space*, *state space*, *phase space*, and *space–time*. This section introduces the mathematical tools necessary to study the geometry of dynamical spaces and the resulting dynamical behavior within those spaces. Central to geometric mechanics is Hamilton's principle, which states that the dynamical path taken through a space, among all possible paths, has the special property that the action integrated along that path is independent of small deviations. The *principle of stationary action* is the origin of all extrema principles that lie at the heart of dynamics, ultimately leading to the geodesic equation of general relativity, in which matter warps space, and trajectories execute force-free motion through that space.