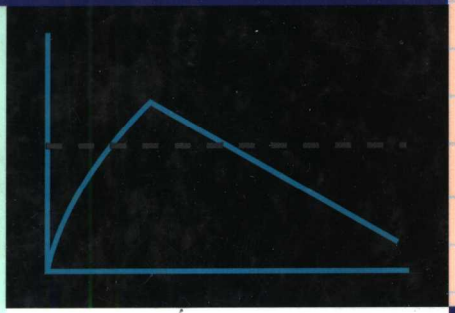
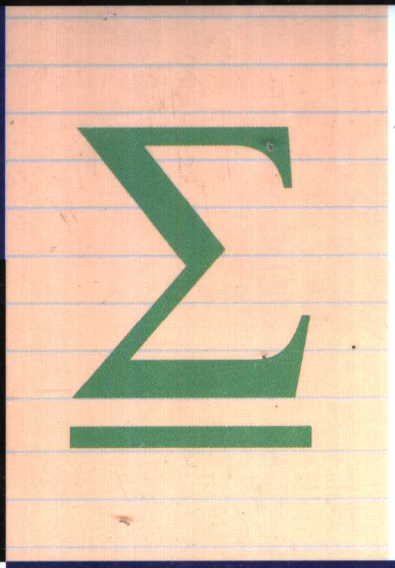
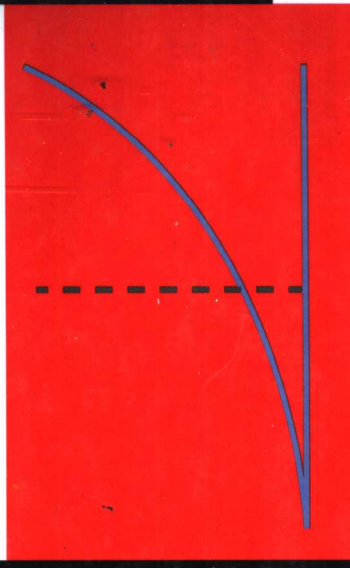
BRIEF APPLIED CALCULUS

ecology population
economics nutrition
ecology population
economics nutrition







PRODUCTIVITY
PRODUCTIVITY
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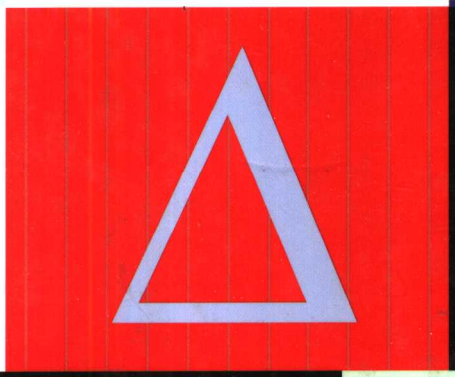
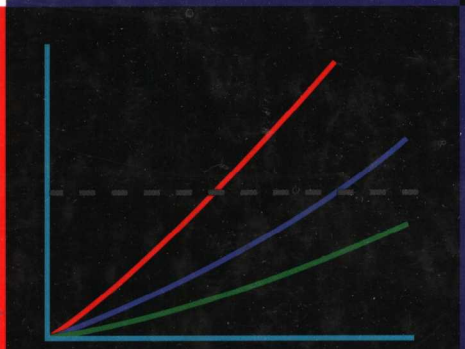
PRODUCTIVITY

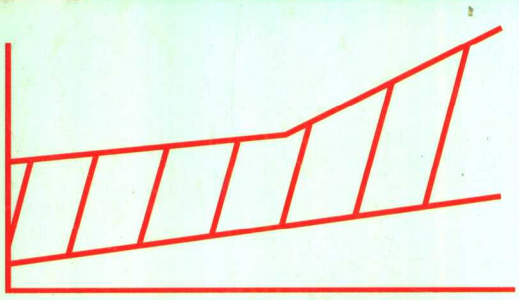
$f''(x)$





income profit
income profit
income profit

VELOCITY
REVENUE
VELOCITY
REVENUE
VELOCITY



BERRESFORD

BRIEF APPLIED CALCULUS

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PREFACE

A recent scientific study of yawning found that more yawns occurred in calculus class than anywhere else.* This book hopes to remedy that situation. Rather than being another dry recitation of standard results, it endeavors to exhibit some of the many fascinating and useful applications of calculus in business, the sciences, and everyday life. Even beyond its utility, however, there is a beauty to calculus, and it is hoped that this book will convey some of its elegance and simplicity.

This book is an introduction to calculus and its applications for students in the management, social, behavioral, and biomedical sciences, and other fields. The seven-chapter "brief" edition contains more than enough material for a one-semester course, and the eleven-chapter edition contains additional chapters on trigonometry, differential equations, sequences and series, and probability for a two-semester course.

Features

Realistic Applications The basic nature of such a course should be very "applied," so this book contains an unusually large number of applications, many appearing in no other textbook. We will use calculus to predict the national debt, maximize longevity, estimate the dangers of cigarette smoking, study global warming, judge the authenticity of the Shroud of Turin, and evaluate strategies for controlling heroin, marijuana, and liquor sales. These applications show that calculus is not just the manipulation of abstract symbols but is deeply connected to everyday life.

Graphing Calculators (Optional) Calculators with capabilities previously available only on computers have changed the way calculus can be taught. Reading this book does not require a graphing calculator, but having one will simplify the calculations in many problems, and may at the same time deepen understanding of calculus by permitting students to concentrate on *concepts*. Throughout the book are **Graphing**

* Baenninger, Ronald. Some Comparative Aspects of Yawning in Betta splendens, Homo sapiens, Panthera leo, and Papoi spinx. Journal of Comparative Psychology, 1987, vol. 101, No. 4, 349–354.

Calculator Explorations and **Exercises,** which explore new topics, carry out otherwise "messy" calculations, or show the limitations and pitfalls of technology. While any graphing calculator (or a computer) may be used, the displays shown in the text are from the Texas Instruments TI-82. For those not using a graphing calculator, the Graphing Calculator Explorations are boxed so that they can be omitted easily. A discussion of the essentials of graphing calculators follows this preface. Brief TI-82 and BASIC programs for Riemann sums and numerical integration (and in the longer edition, Euler's method, Newton's method, and normal distributions) are included in the text. More extensive TI-82 programs that display graphs appropriate to the calculations are available from the author upon request. Please send a blank, formatted disk (specify IBM or Mac) and a self-addressed envelope to the author, Mathematics Department, Long Island University, Brookville, NY 11548.

Enhanced Readability An elegant four-color design was used to increase the appeal and readability of this book. In particular, color was frequently used for pedagogical purposes. For the sake of continuity, references to earlier material have been minimized by restating results whenever they are used. Where references are necessary, explicit page numbers are given.

Application Previews Many sections begin with an Application Preview that presents an interesting application of the mathematics developed in the section. They are self-contained (although some exercises are based on them), and serve to motivate interest in the section. Topics include world records in the mile run, Stevens' law of psychophysics, inflation, AIDS, and predicting personal wealth.

Practice Problems Learning calculus requires a student's active participation—"mathematics is not a spectator sport." Throughout the reading are short pencil and paper "Practice Problems" designed to consolidate understanding of one topic before another is introduced. Complete solutions to all practice problems are at the end of the book.

Annotations Notes to the right of mathematics state the results in words, demonstrating the important skill of "reading mathematics." They also provide explanation and justification for the steps in a calculation, and interpretation of the results.

Extensive Exercises Anyone who has ever learned calculus did so by solving many problems, and the exercises are the most essential part of the learning process. The exercises are graded from routine drill to significant applications. **Applied Exercises** have both general and specific titles, such as "Environmental Science: Pollution Control." Exer-

cises marked with the symbol  require a business or scientific calculator with keys like  and  for natural logarithms and powers. Exercises marked by  require a graphing calculator. At the end of the book are answers to the odd-numbered exercises, and answers to *all* Chapter Review and Cumulative Review exercises.

Levels of Reinforcement Because of the many new ideas and techniques in calculus, frequent summaries and reviews are necessary, and this book contains three different levels of summaries: at the end of sections, at the end of chapters, and cumulative review problems at the end of *groups* of chapters. The **Chapter Summaries** review the major developments of the chapter and are keyed to particular review exercises. Also at the end of each chapter are **Hints and Suggestions** that unify the chapter, give specific reminders that may be forgotten, and list exercises for a **Practice Test.**

Projects and Essays Concluding each chapter is a collection of small research projects and topics for student essays that ask the student to research a relevant person or an idea, to compare several different mathematical ideas, or to relate a concept to their lives. This feature is in keeping with recent recommendations from the Mathematical Association of America and the National Council of Teachers of Mathematics. Other more challenging projects can be found in the highly recommended MAA Notes Numbers 27–30, available from the Mathematical Association of America in Washington, D.C.

Accuracy and Proofs All of the answers and other mathematics have been checked carefully by several mathematicians. The statements of definitions and theorem are mathematically accurate. Because the treatment is applied rather than theoretical, intuitive and geometric justifications have often been preferred to formal proofs. When proofs are given, however, they are correct.

Philosophy I wrote this book with several principles in mind. One is that to learn something, it is best to begin doing it as soon as possible. Therefore the preliminary material is brief, so that students begin calculus as soon as possible. An early start allows more time during the course for useful applications and necessary review. Another principle is that the mathematics should be done together with the applications. Consequently every section contains applications (there are no "pure math" sections).

Prerequisites The only prerequisite for this course is some knowledge of algebra and graphing, which are reviewed in Chapter 1. This chapter also covers the preliminary material on functions necessary to begin the study of calculus. Other review material has been placed in relevant locations throughout the book.

Supplements for Students

Student Solutions Manual This booklet, available from your bookstore, contains worked-out solutions to selected exercises.

Math Assistant Software This package, available from Houghton Mifflin, easily plots algebraic and trigonometric functions. It is available for Macintosh, Apple, IBM, and compatible computers.

Supplements for Instructors [Available from Houghton Mifflin]

Instructor's Manual This booklet contains full solutions for all exercises in the book.

Computerized Test Bank (IBM and Macintosh) The test bank contains more than 2000 test questions arranged by chapter and section, allowing instructors to create customized tests efficiently. Many of these test questions are applied problems. Test questions can be selected by section number as well as other criteria. IBM (ESTATEST III) and Macintosh (LXR) versions both have full editing capabilities and high-quality graph reproduction. They produce scrambled and multiple test versions in multiple-choice or free-response format, and provide answer keys. The IBM version also provides new **on-line testing** and **gradebook** functions, and allows importation of files from ASCII, WordStar, and WordPerfect.

Printed Test Bank with Chapter Tests This is a printed version of the Computerized Test Bank for instructors who do not use computers. Also included are two comprehensive Chapter Tests for each chapter (one multiple choice and one free response). Answers to all test questions are included.

Acknowledgments

I am indebted to many people for their useful suggestions, conversations, and correspondence during the writing of this book. I thank Chris Berresford, Anne Burns, Ruth Enoch, Theodore Faticoni, Jeff Goodman, Susan Halter, Brita and Ed Immergut, Ethel Matin, Gary Patric,

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I have had the good fortune to have had supportive and expert editors at Houghton Mifflin: Maureen O'Connor (Senior Sponsoring Editor), Dawn Nuttall (Associate Editor), and Maria Morelli (Senior Project Editor). They made their difficult tasks seem easy, and mine possible. I also express my gratitude to the many others at Houghton Mifflin who made important contributions too numerous to mention.

The following reviewers have contributed greatly to the progression from first draft to finished book:

Paul Allen, *University of Alabama*
Carl G. Arendsen, *Grand Valley State University, MI*
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Gerald Meike, *Wright State University, OH*
Catherine A. Roberts, *University of Rhode Island*
Gary Thomasson, *Palm Beach Community College, FL*

Finally and most importantly, I dedicate this book to my wife, Barbara, to my children, Lee and Chris, for their understanding and patience without which this book would not exist, and to the memory of Janet Oppenheim.

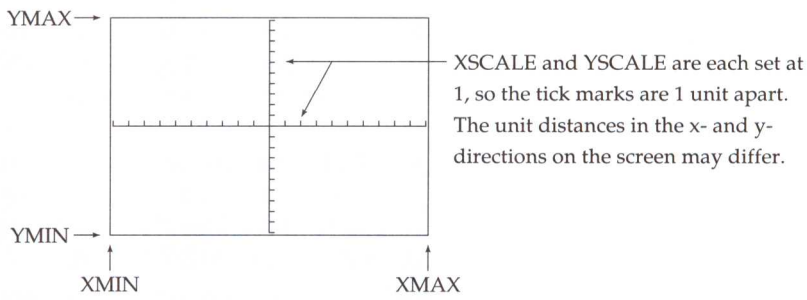
Comments Welcomed

I first wrote this book for my own students. I then rewrote it for publication to include applications from a wider variety of fields. This made the book too long, so I wrote it a third time. With the knowledge that it will never be completely finished, I welcome corrections, criticism, and suggestions from any reader.

GRAPHING CALCULATOR TERMINOLOGY

The graphing calculator applications have been kept as generic as possible for use with any of the popular graphing calculators. It is assumed that either the instructor or the student (or both) is familiar with the operation of the calculator being used. Certain standard calculator terms are capitalized in this book and are described below. Your calculator may use slightly different terminology.

The **viewing window** or **graphing window** is the part of the Cartesian plane shown in the display screen of your graphic calculator. **XMIN** and **XMAX** are the smallest and largest x-values shown, and **YMIN** and **YMAX** are the smallest and largest y-values shown. These values can be set by using the **WINDOW** or **RANGE** command and are changed automatically by using any of the **ZOOM** operations. **XSCALE** and **YSCALE** define the distance between tick marks on the x- and y-axes.



Viewing Window [−10, 10] by [−10, 10]

The viewing window is always [XMIN, XMAX] by [YMIN, YMAX]. We will set XSCALE and YSCALE so that there are a reasonable number of tick marks (generally 2 to 20) on each axis. The x- and y-axes will not be visible if the viewing window does not include the origin.

Pixel, an abbreviation for *pic*ture *el*ement, refers to a tiny rectangle on the screen that can be darkened to represent a dot on a graph. Pixels are arranged in a rectangular array on the screen. In the above window,

the axes and tick marks are formed by darkened pixels. The size of the screen and number of pixels vary with different calculators.

TRACE allows you to move a flashing pixel, or *cursor*, along a curve in the viewing window with the x- and y-coordinates shown at the bottom of the screen.

> *Useful Hint:* To make the x-values in TRACE take simple values like .1, .2, and .3, choose XMIN and XMAX to be multiples of one less than the number of pixels across the screen. For example, on the TI-82, which has 95 pixels across (by actual count), using an x-window like $[-9.4, 9.4]$ or $[-4.7, 4.7]$ or $[940, -940]$ will TRACE with simpler x-values than the standard windows stated in this book.

ZOOM IN allows you to magnify any part of the viewing window to see finer detail around a chosen point. **ZOOM OUT** does the opposite to display a larger portion of the plane. These and other **ZOOM** commands change the viewing window.

VALUE or **EVALUATE** finds the value of an expression at a specified x-value.

SOLVE or **ROOT** finds the x-value that solves $f(x) = 0$, or equivalently, the x-intercepts of a curve. When applied to a difference $f(x) - g(x)$, it finds the x-value where the two functions are equal, or equivalently, where the curves meet (also done by the **INTERSECT** command).

MAX and **MIN** find the maximum and minimum values of an expression between specified x-values.

NDERIV or **DERIV** or $\boldsymbol{dy/dx}$ approximates the *derivative* of a function at a point (explained in Chapter 2). **FNINT** or $\boldsymbol{\int f(x)\, dx}$ approximates the definite integral of a function on an interval (explained in Chapter 5).

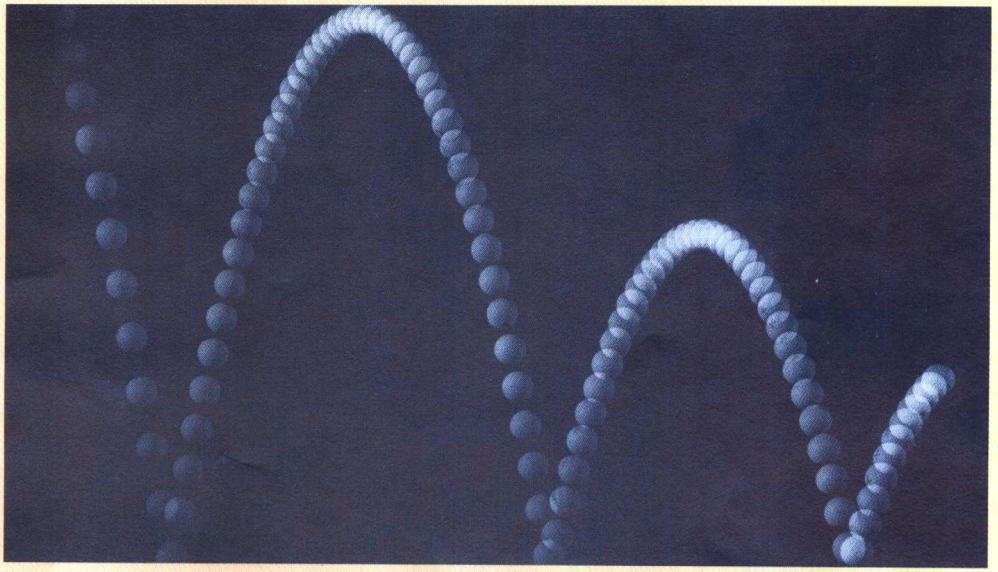
In **CONNECTED MODE** your calculator will try to connect the pixels on a graph to show it as a continuous or unbroken curve. However, this may lead to "false lines," as will be explained later. False lines can be eliminated by using **DOT MODE.**

The **TABLE** command on some calculators lists in table form the values of an expression, just as you have probably done when graphing a curve. The x-values may be chosen by you or by the calculator.

Some of these commands are available only in "graphing mode" and some only in "solving" or "home screen" mode. More information can be found in the manual for your graphing calculator. Other features will be discussed later as needed, including programs to carry out certain mathematical operations.

CONTENTS

1 FUNCTIONS



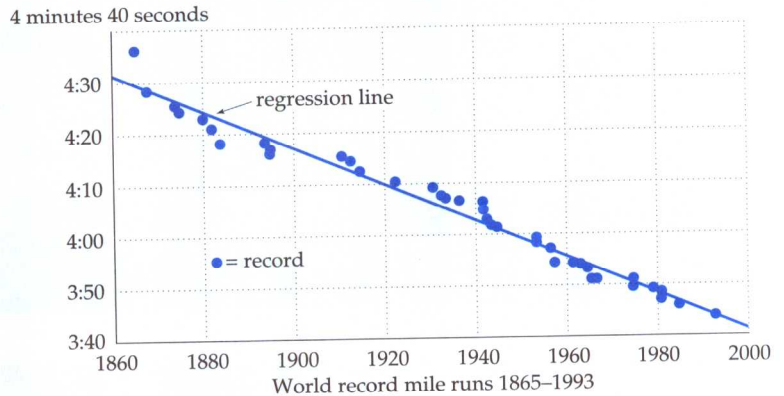
Parabolas described by a bouncing ball

1.1 Real Numbers, Inequalities, and Lines

APPLICATION PREVIEW

World Record Mile Runs

The dots on the graph below show the world record times for the mile run from 1865 to the 1993 world record of 3 minutes 44.39 seconds, set by the Algerian runner Noureddine Morceli. These points fall roughly along a line, called the **regression line.** The regression line is easily found using a graphing calculator, based on a method called **least squares,** which is explained in Chapter 7. Several exercises in this chapter involve using a graphing calculator to find the regression line for a collection of points.



World record mile runs 1865–1993

HISTORY OF THE RECORD FOR THE MILE RUN

Time	Year	Athlete	Time	Year	Athlete
4:36.5	1865	Richard Webster	4:15.6	1895	Thomas Conneff
4:29.0	1868	William Chinnery	4:15.4	1911	John Paul Jones
4:28.8	1868	Walter Gibbs	4:14.4	1913	John Paul Jones
4:26.0	1874	Walter Slade	4:12.6	1915	Norman Taber
4:24.5	1875	Walter Slade	4:10.4	1923	Paavo Nurmi
4:23.2	1880	Walter George	4:09.2	1931	Jules Ladoumegue
4:21.4	1882	Walter George	4:07.6	1933	Jack Lovelock
4:18.4	1884	Walter George	4:06.8	1934	Glenn Cunningham
4:18.2	1894	Fred Bacon	4:06.4	1937	Sydney Wooderson
4:17.0	1895	Fred Bacon	4:06.2	1942	Gunder Hägg

Time	Year	Athlete	Time	Year	Athlete
4:06.2	1942	Arne Andersson	3:51.3	1966	Jim Ryun
4:04.6	1942	Gunder Hägg	3:51.1	1967	Jim Ryun
4:02.6	1943	Arne Andersson	3:51.0	1975	Filbert Bayi
4:01.6	1944	Arne Andersson	3:49.4	1975	John Walker
4:01.4	1945	Gunder Hägg	3:49.0	1979	Sebastian Coe
3:59.4	1954	Roger Bannister	3:48.8	1980	Steve Ovett
3:58.0	1954	John Landy	3:48.53	1981	Sebastian Coe
3:57.2	1957	Derek Ibbotson	3:48.40	1981	Steve Ovett
3:54.5	1958	Herb Elliott	3:47.33	1981	Sebastian Coe
3:54.4	1962	Peter Snell	3:46.31	1985	Steve Cram
3:54.1	1964	Peter Snell	3:44.39	1993	Noureddine Morceli
3:53.6	1965	Michel Jazy			

Source: USA Track & Field

The equation of this regression line is $y = -0.357x + 257.46$, where x represents years after 1900 and y is in seconds. The regression line can be used to predict the world mile record in future years. Notice that the most recent world record would have been predicted quite accurately by this line, since the rightmost dot falls almost exactly on the line. Linear trends, however, must not be extended too far. The downward slope of this line means that it will eventually "predict" mile runs in a fraction of a second, or in *negative* time. Moral: In the real world, linear trends do not continue indefinitely. This and other topics in "linear" mathematics will be developed in Section 1.1.

Introduction

Calculus is, quite simply, the study of rates of change. In this book we will use calculus to analyze rates of inflation, rates of learning, rates of growth of populations, and rates of consumption of natural resources.

When reading this book, it will be helpful (but not necessary) to have a graphing calculator. The **Graphing Calculator Explorations** show how to use a graphing calculator to explore a concept more deeply or to analyze an application in more detail. The parts of the book that require graphing calculators are marked by the symbol  . Exercises that can be done with a graphing *or* scientific *or* business calculator (with keys like ln x and y^x) are marked by  .

Real Numbers and Inequalities

In this book the word "number" means *real* number, a number that can be represented by a point on the number line (also called the *real line*).