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Math

Workout for the

New SAT

新SAT 数学

Cornelia Cocke
科妮莉亚·科克尔
世界图书出版公司



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Introduction

前言

IN THE BEGINNING . . . 开篇之语

Even though the SAT is designed for juniors and seniors, most of the math on the test bears little resemblance to the type of math found in the high school classroom. Many students find it hard to believe—not to mention a little humiliating—that a test that seems so difficult actually tests little more than basic algebra, arithmetic, and geometry. Even students who are very good at math in school often have trouble on the SAT. Why?

The fact is that while the SAT uses basic mathematical concepts, it's unlike any math test you will ever see in school. The SAT uses basic math problems in very particular ways. This is why preparing for the SAT requires a new set of skills. The SAT does not test how smart you are, how well you will do in school, or what kind of person you are. It only tests *how well you do on the SAT*. And doing well on the SAT is a skill that can be learned.

How can you improve your score on the SAT? First, you need to learn the structure of the test. This will help you develop an overall test-taking strategy. Then you need to learn some powerful test-taking skills, which will help you think your way through SAT-type problems.

Some of our advice may sound a little strange. In fact, if you try some of our techniques in math class, your teacher will probably be unhappy. But remember: This isn't math class. This is the SAT, and it's your job to get as good at SAT math as you can.

STRUCTURE OF THE MATH SECTIONS 数学部分的结构

Of the nine multiple-choice sections on the SAT, three of them will be math. The questions will be presented in two different formats: regular multiple choice and grid-ins. We will discuss how to deal with each of these question formats.

HOW TO USE THIS BOOK 如何使用这本书

This book is designed for students who want concentrated math preparation. It can be used alone or as a supplement to our *Cracking the New SAT*. While we will briefly review the essential Princeton Review test-taking strategies and problem-solving skills. If you want an in-depth guide to these techniques, you'll want to also read *Cracking the New SAT*.

WHERE DOES THE SAT COME FROM? SAT来自哪里?

The SAT is published by the Educational Testing Service (ETS) under the sponsorship of the College Entrance Examination Board (the College Board). ETS and the College Board are both private companies. We'll tell you more about them in Chapter 1.

WHAT IS THE PRINCETON REVIEW?

the Princeton Review是什么?

The Princeton Review is one of the nation's fastest-growing test-preparation companies. We have conducted courses in hundreds of locations around the country, and we prepare more students for the SAT than anyone else. We also prepare students for the PSAT/NMSQT, ACT, GRE, GMAT, LSAT, MCAT, and other standardized tests.

The Princeton Review's techniques are unique and powerful. We developed them after spending countless hours scrutinizing real SATs, analyzing them with computers, and proving our theories with real students. Our methods have been widely imitated, but no one else achieves our score improvements.

This book is based on our extensive experience in the classroom. Our techniques for cracking the SAT will help you improve your SAT scores by teaching you to:

1. think like the test writers at ETS
2. take full advantage of the limited time allowed
3. find the answers to questions you don't understand by guessing intelligently
4. avoid the traps that ETS has laid for you (and use those traps to your advantage)

1

Strategies

策 略

We'll say it again: This isn't the kind of test you get in math class. You need some special techniques for handling SAT problems—techniques that will help you go faster and that take advantage of the format of the questions. Some of the things we suggest may seem awkward at first, so practice them. If you do the math questions on the SAT the way your math teacher taught you, you waste time and throw away points.

ORDER OF DIFFICULTY 难度的区分

To formulate an overall test-taking strategy, the most important thing to learn is the order of difficulty of the math sections. The questions on your SAT are selected with extreme care, in the same way, every time. Knowing how the test is put together is crucial for scoring well. The chart on the next page shows you how the questions are organized on each of the math sections.

As you can see, sections are arranged in order of difficulty, with the easy questions at the beginning, medium questions in the middle, and the hard questions at the end of each section. It is crucial to know the difficulty of a question in order to know the best way to solve it. This is so important that the exercises in this book provide an easy, a medium, and a hard question for each major question type. We've kept the question numbers consistent to help you learn which questions are easy, medium, and hard.

JOE BLOGGS

Joe Bloggs is our name for the average SAT tester. Joe isn't stupid—he's just average. He takes this test as he would take a math test in school, and he gets an average score. If you learn how Joe takes this test, you can learn how to take it better.

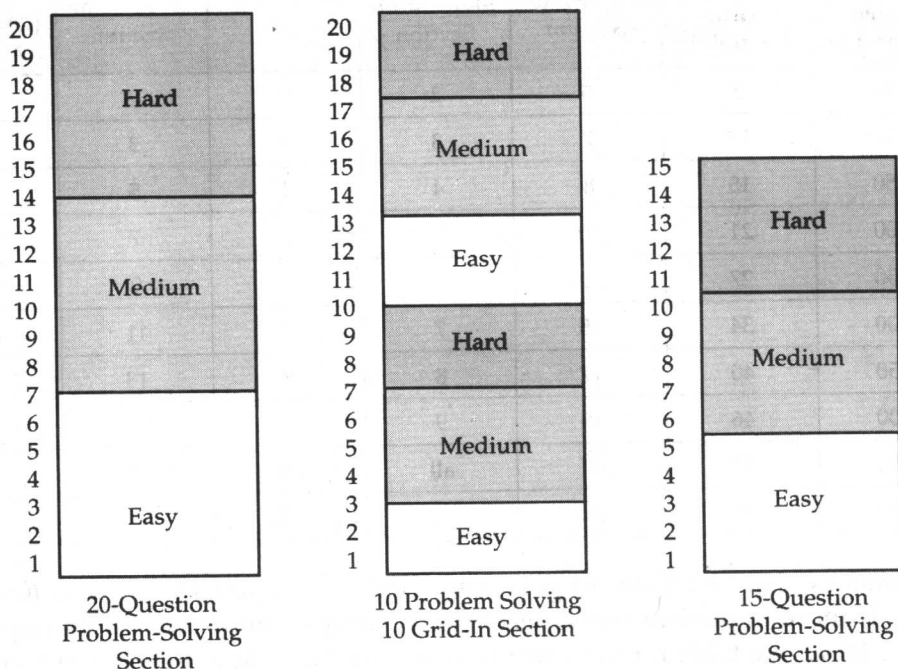
When Joe takes the SAT, he makes two important mistakes. First, he tries to finish the test. This encourages him to rush through the easy problems, which he should get right, but he makes silly mistakes because he's rushing. He rushes all the way into the hard problems, getting almost no questions right. Why? Because of Joe's second mistake: He thinks that he can solve every problem in a straightforward way. While this works on the easy problems, the hard problems are full of trap answers, which cause Joe to spend too much time on each question or pick the wrong choice.

How can you avoid being like Joe? First, learn to do the right number of problems. Second, learn some test-taking techniques that will make harder problems much easier and help you avoid the traps that the test writers have laid for you.

PACING 步骤

Almost everybody works too fast on the SAT, losing a lot of points due to careless errors. The SAT isn't your usual math situation—you don't get *partial credit* for "having the right idea." The only thing that matters is what you bubble in on your answer sheet. Slow down! If you find yourself making careless mistakes, you are throwing points out the window.

Unless you're shooting for a score of 700 or above, do not finish the math sections. Again, this isn't like math class. The test isn't designed for you to finish, and you'll hurt your score by trying to do so. If you miss a total of around five or six questions for all three sections, you're probably hitting the right pace. More mistakes than that, and you're going too quickly. If you aren't missing any questions but aren't finishing, you should guess more aggressively and try to work a bit faster.



SCORING 得分

If you were betting your hard-earned cash, wouldn't you want to know the odds? On the SAT, you're betting for more points, and it's important to understand how the scoring works so you'll play smart.

For each right answer, you earn one raw point. For each wrong answer, you lose one-quarter of a raw point. That's it. If you leave a question blank, nothing happens either way, except that the total number of points you can earn is reduced.

Every right answer earns you one point, whether it's easy or hard.

That's important to understand, because most people spend too much time on hard questions. They aren't going to do anything more for you than easy questions—and you'll hurt your score if you miss easy or medium questions because you're rushing to finish.

Based on a sample conversion table, here are some examples of what you have to do to get a particular score—there are a total of 55 math questions.

Attempt this many questions						
To get: (scaled score)	You need to earn: (raw points)	20-question Section	10-question Section	Grid-Ins	15-question Section	Total # of questions to attempt
350	5	5	2	2	2	11
400	10	7	3	3	3	16
450	15	8	4	4	5	21
500	21	10	5	5	7	27
550	27	12	6	6	9	33
600	34	15	7	7	11	40
650	40	17	8	8	13	46
700	46	19	9	9	all	52
750	51	all	all	all	all	55
800	55	all	all	all	all	55

Amazing, isn't it? Even to get a very high score, *you don't have to finish*. Accuracy is more important than speed! (The examples are based on a sample conversion table; the table for your test may be slightly different—maybe a question or two higher or lower. We put this in just to give you an approximate idea of the score a particular number of right/wrong/blank will give you.)

GUESSING 推测

Say you start to work on a problem and get stuck. Should you just move on? Not if you can cross out any of the wrong answer choices. Are you working on a hard problem? Then cross out any obvious answers and guess from what's left. Are you working on an easy question? Go with your instincts.

If you can eliminate even one answer choice, guess.

Why? Because if you guessed randomly on five questions, without eliminating anything (let your pet monkey pick the answer), you'd have a one out of five chance of picking a right answer. That one right answer earns you (or your pet monkey) one raw point, and the four wrong answers cost you $4 \times \frac{1}{4}$ of a point subtracted. You break even. Eliminating one or more answer choices improves your odds considerably—so take advantage of it!

WHEN TO GUESS 推测的时间

All of that said, we don't mean to suggest that you skip merrily through the sections guessing with abandon. There are smart places to guess and not-so-smart places to guess.

Always be aware of where you are in the section. Use the order-of-difficulty information to guide your guessing: easy questions = easy answers; hard questions = hard answers.

GOOD GUESS 好的推测

- Geometry, drawn to scale. If you can approximate the length or area or angle measurement, go for it. Applies to easy, medium, and hard questions.
- Any grid-in you've got an answer for. No penalty for wrong answers.

BAD GUESS 差的推测

- Obvious answers on hard questions
- Long, complicated word problems at the end of the section. (Spend your time on something shorter and more manageable.)
- Questions you don't have time to read.

CALCULATORS 计算器

Seems like a good deal, doesn't it? Well, maybe. It depends on the problem. Don't grab your calculator too quickly—you have to know how solve the problem first.

Calculators can only calculate; they can't think. You need to figure out how to solve the problem before you can begin calculating.

Calculators are great for helping you avoid silly mistakes in your arithmetic, and you should use them when you can. They can help ensure that you make correct calculations, but they can't tell you which calculations are the right ones to make. So be sure you figure out how to solve the problem before you start punching numbers into your calculator.

Think before you punch.

TIPS TO CALCULATOR HAPPINESS 使用计算器的建议

- Get a calculator that follows the order of operations and has keys for x^2 , y^x , and $\sqrt{\quad}$.
- Use the same calculator every time you practice SAT problems.
- Estimate your answer first.
- Check each number after you punch it in.

CARELESS MISTAKES 粗心错

If you are prone to careless mistakes—and most of us are—you probably make the same kinds of careless mistakes over and over. If you take the time to analyze the questions you get wrong, you will discover which kinds are your personal favorites. Then you can compensate for them when you take the SAT.

In the world, and in math class, it's most important for you to understand concepts and ways to solve problems. On the SAT, it's most important that you bubble in the correct answer. Students typically lose anywhere from 30 to 100 points simply by making careless, preventable mistakes.

Some common mistakes to watch for:

- misreading the question
- computation error
- punching in the wrong thing on the calculator
- on a medium or hard question, stopping after one or two steps, when the question requires three or four steps
- failing to estimate first
- answering a different question from the one asked

If, for example, you find you keep missing questions because you multiply wrong, then do every multiplication twice. Do every step on paper, not in your head. If you make a lot of mistakes on positive/negative, write out each step, and be extra careful on those questions. Correcting careless mistakes is an easy way to pick up more points, so make sure you analyze your mistakes so you know what to look out for.

PLUGGING IN 代值

One of the most powerful math techniques on the SAT is called Plugging In. The idea of Plugging In is to take all of the variables—things like x , y , z —in a problem and replace them with actual numbers. This turns your algebra problems into simple arithmetic and can make even the hardest problem into an easy one.

HOW TO RECOGNIZE A PLUGGING-IN QUESTION 如何识别代值类问题

- There are variables in the answer choices.
- The question says something like *in terms of x*
- Your first thought is to write an equation.
- The question asks for a percentage or fractional part of something, but doesn't give you any actual amounts.

HOW TO SOLVE A PLUGGING-IN QUESTION 如何解答代值类问题

- Don't write an equation.
- Pick an easy number and substitute it for the variable.
- Work the problem through and get an answer. Circle it so you don't lose track of it.
- Plug in your number—the one you chose in the beginning—to the answer choices and see which choice produces your circled answer.

Here's an example:

Jill spent x dollars on pet toys and 12 dollars on socks. If the amount Jill spent was twice the amount she earns each week, how much does Jill earn each week in terms of x ?

(A) $2(x + 12)$

(B) $2x + 24$

(C) $\frac{x}{2} + 12$

(D) $\frac{x + 12}{2}$

(E) $\frac{x - 12}{2}$

Solution: Let's plug in 100 for x . That means Jill spent a total of 112 dollars. If that was twice her weekly salary, then she makes half of 112, or 56 dollars a week. Circle 56. Now we plug our number, 100, into the answers to see which

one gives us 56. (A) $2(100 + 12) = 224$. No good. (B) is 224, which is also too big. (C) $50 + 12 = 62$ (D) $\frac{112}{2} = 56$! Yes! (E) is $\frac{88}{2} = 44$. Nope. The answer is D.

Here's a harder example:

Karl bought x bags of red marbles for y dollars per bag, and z bags of blue marbles for $3y$ dollars per bag. If he bought twice as many bags of blue marbles as red marbles, then in terms of y , what was the average cost, in dollars, per bag of marbles?

- (A) $\frac{3y}{2}$
- (B) $\frac{7y}{3}$
- (C) $3y - y$
- (D) $2y$
- (E) $6y$

Solution: You don't really want to do the algebra, do you? Let's use simple, low numbers and plug in. How about $x = 2$ and $y = 3$? That's 2 bags of red marbles at \$3 each. So he spent \$6 on red marbles. (In word problems, it helps to keep track of what the numbers represent.) If he bought twice as many bags of blue marbles, then $z = 4$. So he bought 4 bags of blue marbles at $3y$ or \$9 a can and spent a total of \$36. Now we figure the average price by adding up the dollars spent and dividing that by the total number of bags. He spent $\$6 + \$36 = \$42$ on $2 + 4 = 6$ bags of marbles. So the average price per bag is $\frac{42}{6} = \$7$. Circle \$7.

Now, since we said $y = 3$, plug 3 in for y in the answer choices and see which one gives you 7. (A) gives us $\frac{9}{2}$, so we can eliminate it. (B) is 7, so that's the answer.

Here's a different kind of example:

At his bake sale, Mr. Heftwhistle sold 30% of his pies to one friend. Mr. Heftwhistle then sold 60% of the remaining pies to another friend. What percent of his original number of pies did Mr. Heftwhistle have left?

- (A) 10%
- (B) 18%
- (C) 28%
- (D) 36%
- (E) 40%

Solution: If you don't plug in, you may make the sad mistake of picking (A) or of working with ugly fractions. Plugging in a number is much easier. Let's say Mr. Heftwhistle had 100 pies. 30% of 100 equals 30, so he's left with 70. 60% of 70 equals 42, so he's left with 28. Here's the great thing about plugging in 100 on percentage problems—28 (left) out of 100 (original number) is simply 28%. That's it. (C) is the answer.

TIPS FOR PLUGGING IN HAPPINESS 代值类问题满分的建议

- Pick easy numbers like 2, 4, 10, 100. The best number to choose depends on the question: Use 100 for percents.
- Avoid picking 0, 1, or any number that shows up in the answer choices.
- If the number you picked leads to ugly computations—fractions, negatives, or anything you need a calculator for—bail out and pick an easier number.
- Practice!

On the next page is a Quick Quiz, so you can practice Plugging In before you continue. The question number corresponds to the difficulty level in the 25-question multiple-choice section. Answers and explanations immediately follow every Quick Quiz.