

Modelling and Forecasting High Frequency Financial Data

Stavros Degiannakis and Christos Floros





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Modelling and Forecasting High Frequency Financial Data

To Aggelos, Andriana and Rebecca

Stavros Degiannakis

To Ioanna, Vasilis-Spyridon, Konstantina-Artemis and Christina-Ioanna Christos Floros

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List of Symbols and Operators

```
indicator function; i.e. d(y_t > 0) = 1 if y_t > 0, and
d(.)
                     d(y_t > 0) = 0 otherwise
                     risk-free rate
rf
AIC
                     Akaike's information criterion
                     annual risk-free interest rate
rannual
                     ask price
P_{ask,t}
                     asymmetric parameter in ARCH model
Yi
                     asymmetric power parameter in ARCH model
8
                     autoregressive coefficients
                     average of predictive loss/evaluation function, i.e.
\bar{\Psi}_{(.)}
                     \tilde{\Psi}_{(MSE)} = \tilde{T}^{-1} \sum_{t=1}^{\tilde{T}} (RV_{t+1|t}^{(\tau)} - RV_{t+1}^{(t)})^2
                     average of the daily realized variances
\mu_2
                     average of the squared closed-to-open log-returns
111
Phid,ti
                     bid price
BV_t(m)
                     Bi-Power variation at m sampling frequency
                     calendar time sampling
CTS
                      coefficients of HAR model
\mu_t^{(m_i)}
                     conditional mean estimation of model m_i
                     conditional mean
\mu_t
                      conditional mean's functional form
\mu(.)
                      conditional standard deviation
\sigma_t
\hat{\sigma}_t^2
                      conditional variance estimate
                      conditional variance's functional form
g(.)
CGR
                      Correlated Gamma Ratio distribution
                      correlation coefficient
P
                      counting process
dq_t
                      covariance of market returns and asset
\sigma_{m,i}
                      covariance of realized variance and squared closed-to-open
712
                      log-returns
F_{X_{(1)}}(x; a, C_{12...n})
                     cumulative distribution function of X_{(1)}
F_I^{*n}(t)
                      cumulative distribution function of durations J
```

$F_{T_n}(t)$	cumulative distribution function of epoch T_n
F(.)	cumulative distribution function
$DM_{(\cdot,\cdot)}$ $\Psi_t^{(\cdot,\cdot)}$	Diebold Mariano Statistic
$\Psi_t^{(s)}$	difference of loss/evaluation functions (evaluation
$k^{(m_i)}$	differential) $C_{m} = C_{m}$
**	dimension of vector of unknown parameters $\beta^{(m_i)}$ durations
$J_i = T_i - T_{i-1} \{T_i\}_{i=1}^M$	epochs
$\tau_{i}^{I_{i}I_{i}=1}$	equidistance points (sub-intervals) in time
	estimated parameters of $RV_{t(HL^*)}^{(\tau)}$
$\hat{\theta}^{(T)}$	
-	estimator of θ based on a sample of size T
d'	exponent of the fractional differencing operator $(1-L)^{d'}$ in ARFIMA models
d	exponent of the fractional differencing operator $(1 - L)^d$ in FIGARCH & FIAPARCH models
$\hat{y}_t^{(m)}$	forecasts of y_t from model m
$L_T(.)$	full sample log-likelihood function based on a sample of
	size T
Γ(.)	Gamma function
X_{m_i}	half sum of squared standardized one-step-ahead prediction
	errors of model m_i
HQ ,	Hannan and Quinn information criterion
cinf .	infimum
I_t	information set
$\varepsilon_{t-i t}$	in-sample fitted error at time $t - i$ based on information available at time t
$y_{t-i t}$	in-sample fitted value of conditional mean at time $t - i$
	based on information available at time t
p(t)	instantaneous (unobserved) asset price
$\log p(t)$	instantaneous logarithmic asset price
$\sigma(t)$	instantaneous variance of the rate of return
$\sigma^{2(IQ)}_{[a,b]}$	integrated quarticity
$\sigma^{2(IV)}_{[a,b]}$	integrated variance over the interval [a, b]
P_{lip,t_i}	interpolated price
Ku	kurtosis
$J_{t,\alpha}^{(LM)}$	L&M statistic
L	lag operator
y_{t_j}	log-return over the sub-interval $[t_j - t_{j-1}]$
Уt	log-returnsover the sub-interval $[t, t-1]$

$\Psi_t^{(.)}$	loss/evaluation function that measures the distance between
	volatility and its forecast
$ \varepsilon_{t_j} = \log P_{t_j} - \log p_{t_j} \mathbf{X}_t^{(m_i)} \sigma_{i,t}^{(MAD)} $	market microstructure noise
$\mathbf{X}_{t}^{(m_{i})}$	matrix of $x_t^{(m_i)}$ explanatory variables
$\sigma^{(MAD)}$	Median Absolute Deviation
MMG	Minimum Multivariate Gamma distribution
	minimum value of X_{m_i}
$X_{(1)}$	
$m_{(1)}$	model with the lowest value of X_{m_i}
d_i $MSE^{(\tau)}$	moving average coefficients
	MSE loss (or evaluation) function
\widehat{T}	number of forecasts for out-of-sample evaluation
n	number of models or variables
$egin{array}{cccc} n & & & & & \\ ilde{T} & & & & & \\ ilde{T} & & & & & \\ ilde{H} & & & & & \\ ilde{T} & & \\$	number of observations for out-of-sample forecasting
$reve{T}$	number of observations for rolling sample
$reve{ heta}$	number of parameters of vector θ
T	number of total observations
$ \begin{cases} P_{t_j} \}_{j=1}^{\tau} \\ RV_{t+1 t}^{(\tau)} \end{cases} $	observed asset price
DV(t)	
$RV_{t+1 t}$	one-day-ahead realized variance at time $t+1$ based on information available at time t
$RV_{(un),t+1 t}^{(\tau)}$	one-day-ahead realized variance at time $t+1$ based on
(un), t+1 t	information available at time <i>t</i> (unbiased estimator)
$y_{t+1 t}$	one-step-ahead conditional mean at time $t+1$ based on
71-111	information available at time t
$y_{t+1 t}^{(m_i)}$	one-step-ahead conditional mean at time $t+1$ based on
$y_{t+1 t}$	
_2	information available at time t of model m_i
$\sigma_{t+1 t}^2$	one-step-ahead conditional variance at time $t+1$ based on
1.2	information available at time <i>t</i>
$h_{t+1 t}^2$	one-step-ahead estimate of integrated quarticity given the
	information available at time t
$\varepsilon_{t+1 t}$	one-step-ahead prediction error at time $t+1$ based on
	information available at time t
$z_{t+1 t}$	one-step-ahead standardized prediction error at time
	t+1based on information available at time t
9	order of ARCH form
p	order of GARCH form
k	order of the autoregressive model
1	order of the moving average model
$d_{i,t}$	outlyingness measure
C(L)	polynomial of autoregressive model - AR
B(L)	polynomial of FIGARCH & FIAPARCH models
$\Phi(L)$	polynomial of FIGARCH & FIAPARCH models
D(L)	polynomial of moving average model - MA
D(L)	Polyholillal of moving average model - MA

1	
P_{pre,t_i}	previous tick price
$lRange_{[a,b]}$	price log-range
$c^{E}(t)$	price of the European call option at time t
$Range_{[4],[a,b]}$	price range, four-data-points
$Range_{[a,b]}$	price range
f(.)	probability density function
$QQ_t(m)$	Quad-power quarticity at m sampling frequency
R	
$RJ_t(m)$	realized jumps at m sampling frequency
$ROWQCov_t$	Realized Outlyingness Weighted Covariation
$ROWQuarticity_t(m)$	Realized Outlyingness Weighted Quarticity at m sampling
	frequency
$ROWVar_t$	Realized Outlyingness Weighted Variance
$RV_{[a,b]}^{[2q]}$	realized power variation of order 2q
\mathbf{RCov}_t	realized quadratic covariation
$RV_t^{(\tau)}$	realized volatility at time t , divided in τ points in time
	realized volatility at time i , divided in i points in time realized volatility for the time interval $[a,b]$
$RV_{[a,b]}$ $RV_{(\tau)}^{(\tau)}$	
T(n)	realized volatility of <i>n</i> -trading-days
$RV_{t(FKO)}^{(\tau)}$ $RV_{t(HL^*)}^{(\tau)}$	realized volatility with Fleming's et al dynamic scaling
$RV_{t(HL^*)}^{(\tau)}$	realized volatility with Hansen and Lunde's interday
	adjustment
$RV_{t(Martens)}^{(\tau)}$	realized volatility with Marten's interday adjustment
$\gamma_d(i)$	sample autocovariance of i th order
$m = \frac{b-a}{\tau-1}$	sampling frequency
SBC	Schwarz information criterion
SH	Shibata information criterion
$f_d(0)$	spectral density at frequency zero
N(.)	standard normal density function
$z_t \sim N(0,1)$	standard normal distribution
W(t)	standard Wiener process
SPA	Superior Predictive Ability statistic
t_j	time index, $t_j \ni [a, b]$
[a,b]	time interval
$TQ_t(m)$	Tri-power quarticity at m sampling frequency
$z_t^{(m_i)}$	unpredictable component of model m_i
ε_t	unpredictable component
η_2	variance of the daily realized variances
σ_m^2	variance of the market
η_1	variance of the squared closed-to-open log-returns
\mathbf{Y}_t	vector of dependent variable y_t
θ	vector of estimated parameters for the conditional mean
	and variance

$\theta^{(t)}$	vector of estimated parameters for the conditional mean and variance	
	at time t	
w	vector of estimated parameters for the density function <i>f</i>	
$\mathbf{x}_{t-1}^{(m_i)}$	vector of explanatory variables of m_i regression model	
β	vector of parameters for estimation in regression model	
$\beta^{(m_i)}$	vector of parameters for estimation of m_i regression model	
v_t	vector of predetermined variables included in I_t	
$\Psi_t^{(ES)}$	loss/evaluation function for Expected Shortfall	
$\Psi_t^{(VaR)}$	loss/evaluation function for VaR	
$ES_t^{(a)}$	Expected Shortfall of a portfolio at confidence level a	
$VaR_t^{(a)}$	Value-at-Risk of a portfolio at confidence level <i>a</i>	
$r_{h,t}$	log-return of the hedged portfolio at time <i>t</i>	
\mathbf{R}_t	conditional correlation matrix	
\mathbf{r}_t	vector of returns	
\mathbf{z}_t	vector of standardized error term (residuals)	
ρ_{ij}	constant correlation of spot and future price returns	
$\sigma_{f,t}^2$	variance of future price returns at time t	
$\sigma_{s,t}\sigma_{f,t}$	covariance of spot and future price returns at time t	
$\sigma_{s,t}^2$	variance of spot price returns at time t	
$\Sigma_t(.)$	conditional variance-covariance matrix	
ε_t	vector of error term (residuals)	
$\mu_{t}(.)$	vector of conditional mean	
Δ	first order log-difference	
Δ	log difference	
R^2	coefficient of determination	
Cov (.,.)	covariance	
MVHR	minimum-variance hedge ratio	
diag(.)	diagonal matrix	