GENERAL MATHEMATICS

BOOK ONE

Arithmetic	*Algebra*
Geometry	*Trigonometry*

GINN AND COMPANY

GENERAL MATHEMATICS

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PREFACE

The purpose of this book, as implied in the introduction, is as follows: to *obtain a vital, modern scholarly course in introductory mathematics that may serve to give such careful training in quantitative thinking and expression as well-informed citizens of a democracy should possess.* It is, of course, not asserted that this ideal has been attained. Our achievements are not the measure of our desires to improve the situation. There is still a very large "safety factor of dead wood" in this text. The material purposes to present such simple and significant principles of algebra, geometry, trigonometry, practical drawing, and statistics, along with a few elementary notions of other mathematical subjects, the whole involving numerous and rigorous applications of arithmetic, as the average man (more accurately the modal man) is likely to remember and to use. There is here an attempt to teach pupils things worth knowing and to discipline them rigorously in things worth doing.

The argument for a thorough reorganization need not be stated here in great detail. But it will be helpful to enumerate some of the major errors of secondary-mathematics instruction in current practice and to indicate briefly how this work attempts to improve the situation. The following serve to illustrate its purpose and program:

1. The conventional first-year algebra course is characterized by *excessive formalism*, and there is much drill work largely on nonessentials. The excessive formalism is

greatly reduced in this text and the emphasis placed on those topics concerning which there is general agreement, namely, *function*, *equation*, *graph*, and *formula*. The time thus gained permits more ample illustrations and applications of principles and the introduction of more significant material.

2. Instead of crowding the many difficulties of the traditional geometry course into one year, geometry instruction is spread over the years that precede the formal course, and the relations are taught inductively by experiment and by measurement. Many foreign schools and an increasing number of American schools proceed on this common-sense basis. This gives the pupil the vocabulary, the symbolism, and the fundamental ideas of geometry. If the pupil leaves school or drops mathematics, he nevertheless has an effective organization of geometric relations. On the other hand, if he later pursues a formal geometry course, he can work far more effectively because he can concentrate on the logical organization of space relations and the formal expression of these relations. The longer "time exposure" minimizes the difficulties met in beginning the traditional geometry courses and avoids the serious mistake of forcing deductive logic and philosophic criticism in these early years.

3. The traditional courses delay the consideration of much interesting and valuable material that the field of secondary mathematics has to offer, and which may well be used to give the pupil very early an idea of what mathematics means and something of the wonderful scope of its application. The material of the seventh, eighth, and ninth years is often indefensibly meaningless when compared with that of many foreign curricula. Trigonometry,

containing many easy real problems, furnishes a good example of this delay. Other examples are found in the use of logarithms, the slide rule, standardized graphical methods, the notion of function, the common constructions of practical drawing, the motivation of precise measurement, a study of the importance of measurement in modern life, and the introductory ideas of the calculus. It appears that the mathematics student should be given an opportunity to use these important tools very early in his study. They lend to the subject a power and interest that drills on formal material cannot possibly give.

Particular emphasis is given to graphical representation of statistics. The growing complexity of our social life makes it necessary that the intelligent general reader possess elementary notions of statistical methods. The hundreds of articles in the current magazines so extensively read demand an elementary knowledge of these things in order that the pupil may not remain ignorant of the common, everyday things of life. Brief chapters on logarithms and the slide rule have been introduced in order that a greater number of students may use these practical labor-saving devices and in order that these devices may function in the student's subsequent work, whether in everyday life or in the classroom. Actual classroom experience with these chapters has proved them to be relatively simple and good material for eighth-grade and ninth-grade students.

4. Mathematics needs to be reorganized on the side of method. The information we now possess of individual differences and effective devices in supervised study should make the study of mathematics more nearly a laboratory course, in which more effective work can be done.

5. The teaching of algebra, geometry, and trigonometry in separate fields is an artificial arrangement that does not permit the easy solution of problems concerning projects that correlate with problems met in the physical and biological sciences or the manual and fine arts. To reject the formalism of algebra, to delay the demands of a logical unit in geometry, and to present the simple principles of the various branches of mathematics in the introductory course opens the door to a greater variety of problems that seem to be real applications. The pupil sees the usefulness of the various modes of treatment of the facts of quantity. Power is gained because the pupil is equipped with more tools, in that the method of attack is not limited to one field.

6. One of the most curious characteristics of American secondary-mathematics instruction is the obscurity in the teaching of the function notion. It is generally agreed that functional thinking (the dependence of one magnitude upon another) constitutes one of the most fundamental notions of mathematics. Because of the interrelations of the equation, the formula, the function, the graph, and the geometric relations inductively acquired, the material is easily correlated around the function idea as the *organizing and unifying principle.* The function concept (implicitly or explicitly) dominant throughout helps to lend concreteness and coherence to the subject. However, it would be false to assume that this material is presented to establish the principle of correlation. On the contrary, it happens that correlation around the function notion, though incidental, is a valuable instrument for accomplishing the larger aim, which is to obtain a composite introductory course in mathematics that all future citizens of our democracy should be required to take as a matter of general scholarship.

7. The traditional reticence of texts has made mathematics unnecessarily difficult for pupils in the early years. The style of this book, though less rigidly mathematical, is more nearly adapted to the pupils' mental age. The result is a misleading length of the book. The book can easily be taught in a school year of approximately one hundred and sixty recitations. In the typical high school it will be taught in the first year. (The Minnesota high schools taught it in this grade, five recitations per week.) In schools which control the seventh and eighth years the following are also possibilities which have been tested by the authors and coöperating teachers: (1) in the eighth year with daily recitations; (2) half of the book in the eighth year and the remainder in the ninth year, with three recitations per week (it was so used in the Lincoln School); (3) the course may be started in the seventh year provided the class has achieved good results in previous arithmetic work.

Specific references are given where material which is not of the common stock has been taken consciously, the purpose, however, being chiefly to stimulate pupils and teachers to become familiar with these books for reasons other than the obligations involved. Something of human interest is added by relating some of the well-known stories of great mathematicians. We are indebted to Professor David Eugene Smith on questions relating to historical material. In our thinking we are particularly indebted to Professors Nunn, Smith, Breslich, and Myers. We shall be obliged to all teachers who may think it worth while to point out such errors as still exist.

THE AUTHORS

INTRODUCTION

The movement to provide an introductory course in general mathematics is a part of an extensive movement toward making the materials of study in secondary education more concrete and serviceable. The trend of education expresses a determination that the seventh, eighth, and ninth school years should be enriched by the introduction of such significant experiences of science, civics, art, and other knowledge of human life as all enlightened citizens of a democracy should possess. The work of these grades cannot be liberalized by "shoving down" the conventional material a year or so. The reorganization must be more fundamental in order to revitalize and socialize the mathematics of these grades.

Competent authorities in mathematics have from time to time asserted, first, that American secondary-mathematics teaching has been characterized by a futile attempt to induce all pupils to become technical college mathematicians. Secondly, that instead of giving pupils an idea of the real meaning of mathematics and the wide range of its applications, they are forced to waste a great deal of time on abstract work in difficult problems in radicals, fractions, factoring, quadratics, and the like, which do not lead to anything important in mathematics. And, thirdly, that this meaningless juggling of symbolism fails to meet the needs of the great number of pupils who go rather early into their careers; it also wastes time and effort on the part of pupils with especial ability in the subject, who ought to get an early insight into the scope and power of the real science of mathematics.

Quantitative thinking and expression play so large a part in human experience that proper training in these matters will always be important. The growing complexity of social and industrial life is responsible for corresponding changes in the use of quantitative relationships. Old applications in many instances are disappearing, but new ones growing out of present-day relations are being introduced to take their places. These changes require a new kind of introductory text in mathematics. Action is forced by the demand that there shall be justification of the time and effort given to each subject and each item in the subject. New subjects which appear necessary in the proper training for citizenship are crowding the curriculum. Mathematics too must justify its place "in the sun" by a thorough reorganization that will meet modern needs. This is what is meant by revitalizing mathematics.

The practical administrator will be impressed by the fact that this program raises no administrative difficulties. The pupil may be expected to develop greater power in algebra, because the elimination of material which wastes time and effort has made possible the emphasizing of the topics concerning which there is general agreement. The supplementary material which is drawn from the other subjects constitutes a preparation for further study in these fields; for example, the text gives the pupil the vocabulary, the symbolism, and many of the ideas of plane geometry.

This type of introductory course should appeal to the progressive educator because of a number of other features. The "problem method" of teaching is followed throughout. Rationalized drills are provided in abundance. The course has been used in mimeographed form by experienced teachers. Scores of prospective teachers have found the treatment simple and easy to present. Inexperienced teachers have gone out into difficult situations and have taught the material with satisfaction. Pupils following this course have made better progress than pupils following the traditional course, and both pupils and teachers manifest a degree of interest seldom seen in the ordinary class in mathematics. The tests prepared by the authors will save time for teachers and enable them, if they desire, to diagnose their own situations and to compare their results with those obtained in other institutions using the same material. If the question is raised as to what students completing such a course will do when they get to college, it may be replied that enough of them have already entered college to convince the unbiased that they experience no handicap. The more important point, however, is that such a course enables one to understand and to deal with the quantitative world in which he lives.

This course in reorganized introductory mathematics, although but a part of a large movement in secondary education which looks toward more concrete teaching and more serviceable materials of study, has a further highly significant aspect. It is a potent and encouraging evidence that high-school teachers have become students of their own teaching, and as a result are preparing their own textbooks in the midst of real teaching situations, as the outcome of intelligent constructive experimentation.

Probably very few books have been subjected previous to publication to such thorough tests of teaching situations. The authors have been shaping this course for many years. During the last three years the manuscript as originally accepted by the publishers has been taught in mimeograph form to more than a thousand pupils distributed in a selection of typical schools, among these being the following: Minneapolis Central High School (large city high school), Bremer Junior High School, Seward Junior High School, University of Minnesota High School, Owatonna High School, Mabel High School (small town), and the Lincoln School of Teachers' College. Numerous consultations with the teachers in these schools resulted in many valuable suggestions which contributed directly toward making the text easily teachable.

This text contains a wealth of material in the way of *desirable options.* It is not probable that any one school will desire to cover all of these options; for example, some schools may decide to omit the chapters on the Slide Rule and Logarithms, and others may desire to omit Articles 286 to 298 in the chapter on Statistics. A modern text must offer the opportunity for choice.

Each of the authors has taught secondary mathematics for more than ten years in large public and private schools; they have supervised many teachers in training; they have taught teacher-training courses; and each during most of this time has had unusual opportunities for free experimentation. The text may be regarded by fellow teachers of mathematics as a report which shows in organization and subject matter the things that have seemed most useful.

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OTIS W. CALDWELL

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GENERAL MATHEMATICS

FIRST YEAR

CHAPTER I

THE EQUATION

1. A problem introducing the use of the equation. In order to find the weight of a bag of candy, it was placed on one pan of perfectly balanced scales (Fig. 1). The candy, together with a 4-ounce weight, balanced 10 oz. of weights on the other pan. How much did the bag of candy weigh?

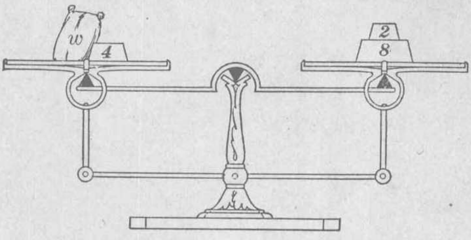

FIG. 1. THE BALANCED SCALES ILLUSTRATE THE MEANING OF AN EQUATION

It is a familiar principle of balanced scales that if the same number of ounces be taken from each pan, the balance is not disturbed. Hence, if we suppose that a 4-ounce weight could be removed from each pan, the candy would be balanced by 6 oz.

This solves the problem, but let us analyze it a little further. The important fact in the situation above is that an unknown number of ounces of candy plus 4 oz. in one pan balances 10 oz. in the other pan. If we agree to let the letter w represent the number of ounces of weight in the bag of candy and use the sign of equality (=) to denote the perfect balance of the scales, the preceding mathematical fact may be conveniently translated into the following expression: $w+4=10$, where $w+4$ denotes the weight in the left pan and 10 the weight in the right pan. The abbreviated ("shorthand") statement, $w+4=10$, expresses equality and is called an *equation.* The number to the left side of the equality sign is called the *left member* of the equation, the number to the right is the *right member.*

Just as the scales will balance if the same number of ounces are taken from each pan, so *we may subtract the same number from both sides of an equation and get another equation.* In the preceding problem the written work may be arranged thus:

Let $w = \left\{\begin{array}{l}\text{number of ounces of weight}\\ \text{in the bag of candy.}\end{array}\right.$

Then $w+4=10$

Subtracting 4 from each member of the equation, $\left.\begin{array}{r} 4 = 4 \\ \hline w \quad = 6\end{array}\right.$

Thus, the bag of candy weighs 6 oz.

The preceding problem illustrates the principle that *if the same number be subtracted from both members of an equation, the remainders are equal*; that is, another equation is obtained. [***Subtraction Law***]

EXERCISES

Find the value of the unknown numbers in the following equations, doing all you can orally:

1. $x+2=6.$
2. $x+6=10.$
3. $x+7=13.$
4. $x+11=18.$
5. $x+13=23.$
6. $x+9=26.$
7. $x+10=27.$
8. $x+14=21.$
9. $x+33=44.$