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R. Courant
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Supersonic Flow and Shock Waves

超声流和激波

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内容简介

本书初版于1948年，这是1999年的第5次修订印刷版。一部著作历经半个多世纪仍然再版，毫无疑问这是一本经典图书。书中讨论了可压缩流体动力学的基本问题，建立了与气体动力学有关的非线性波传播理论。本书自出版以来一直是流体力学方面的一部重要参考书，其中所涉及的问题至今仍是热门的研究课题。

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Preface

The present book originates from a report issued in 1944 under the auspices of the Office of Scientific Research and Development. Much material has been added and the original text has been almost entirely rewritten. The book treats basic aspects of the dynamics of compressible fluids in mathematical form; it attempts to present a systematic theory of nonlinear wave propagation, particularly in relation to gas dynamics. Written in the form of an advanced textbook, it accounts for classical as well as some recent developments, and, as the authors hope, it reflects some progress in the scientific penetration of the subject matter. On the other hand, no attempt has been made to cover the whole field of nonlinear wave propagation or to provide summaries of results which could be used as recipes for attacking specific engineering problems.

The book has been written by mathematicians seeking to understand in a rational way a fascinating field of physical reality, and willing to accept compromise with empirical approach. The authors hope that it will be helpful to engineers, physicists, and mathematicians alike, and that it will not be rejected by mathematicians as too heavily loaded with physical assumptions or by others as too strictly mathematical.

Dynamics of compressible fluids, like other subjects in which the nonlinear character of the basic equations plays a decisive role, is far from the perfection envisaged by Laplace as the goal of a mathematical theory. Classical mechanics and mathematical physics predict phenomena on the basis of general differential equations and specific boundary and initial conditions. In contrast, the subject of this book largely defies such claims. Important branches of gas dynamics still center around special types of problems, and general features of connected theory are not always clearly discernible. Nevertheless, the authors have attempted to develop and to emphasize as much as possible such general viewpoints, and they hope that this effort will stimulate further advances in this direction.

In a field which during recent years has attracted so many workers and in which such diverse practical and theoretical interests have asserted themselves, the authors found a balanced survey impossible; instead they have followed a path dictated largely by their personal interests and experience. The names of scientists with whom the authors happened to be in close contact appear frequently; names of others may have been omitted. No fair appraisal could be made of the merits of many recent contributions. This is true in particular of the large number of reports issued during the war by various agencies and still not freely accessible. In order to avoid further delay, the authors are publishing this book without a complete survey of the literature.

The book was prepared for publication with the cooperation of members of the staff of the Institute for Mathematics and Mechanics in New York University. The main burden of the editorial work, done for the original report by R. Shaw, has been carried by Cathleen Synge Morawetz, who has also contributed constructive criticism in many details, and whose understanding and competent assistance have been invaluable. L. J. Savage cooperated actively in rewriting the first chapter and other parts of the original report. D. A. Flanders has helped greatly by reading parts of the manuscript and suggesting important improvements. W. Y. Chen, W. M. Hirsch, E. Isaacson, A. Leitner, S. C. Lowell, and M. Sion have assisted in this publication by reading proofs and making useful suggestions. The drawings, many of which represent actual conditions, have been carried out by G. W. Evans and J. R. Knudsen. The preparation of the manuscript was in the competent hands of Edythe Rodermund and Harriet Schoverling.

Much more than a formal acknowledgement is due to the Office of Naval Research, not only for the generous support under Contract N6ori-201, Task Order No. 1, which made possible the preparation of the book, but also for the stimulating active interest of its staff members in the progress of the work.

Thanks also should be expressed to Interscience Publishers for the cooperative attitude of their staff, and for the genuine interest of their officers in the promotion of scientific publications.

The book is dedicated to Warren Weaver. As chief of the Applied Mathematics Panel during the war, he rendered very great services, not only for the problems of the day, but even more so for the lasting

benefit of the mathematical sciences. For us personally his steady interest in the present work has been a source of encouragement. Thus the dedication of the book is as well a token of friendship as a tribute to a man whose energy and vision have contributed so much to the recent development of applied mathematics in this country.

R. COURANT and K. O. FRIEDRICHS

August, 1948

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CHAPTER I

Compressible Fluids

Violent disturbances—such as result from detonation of explosives, from the flow through rocket nozzles, from supersonic flight of projectiles, or from impact on solids—differ greatly from the “linear” phenomena of sound, light, or electromagnetic signals. In contrast to the latter, their propagation is governed by nonlinear differential equations, and as a consequence the familiar laws of superposition, reflection, and refraction cease to be valid; but even more novel features appear, among which the occurrence of *shock fronts* is the most conspicuous. Across shock fronts the medium undergoes sudden and often considerable changes in velocity, pressure, and temperature. Even when the start of the motion is perfectly continuous, shock discontinuities may later arise automatically. Yet, under other conditions, just the opposite may happen; initial discontinuities may be smoothed out immediately. Both these possibilities are essentially connected with the nonlinearity of the underlying equations.

Nature confronts the observer with a wealth of nonlinear wave phenomena, not only in the flow of compressible fluids, but also in many other cases of practical interest. One example, rather different from those mentioned above, is the catastrophic pressure in a crowd of panicky people who rush toward a narrow exit or other obstruction. If they move at a speed exceeding that at which warnings are passed backward, a pressure wave arises much like that behind a shock front receding from a wall. Related phenomena, such as congestion in traffic, seem to be essentially due to similar conditions. In this book, however, we shall concentrate primarily on the theory of compressible fluids.

Understanding and control of nonlinear wave motion is a matter of obvious importance. During a period beginning almost a hundred years ago, Stokes, Earnshaw, Riemann, Rankine, Hugoniot, Lord

Rayleigh, and later Hadamard and others wrote fundamental papers inaugurating this field of research. Then the development was left mainly to a small group of ingenious men in the fields of mechanics and engineering. During the last few years, however, when the barriers between applied and pure science were forced down, a widespread interest arose in nonlinear wave motion, particularly in shock waves and expansion waves.

It is the purpose of the present book to make the mathematical theory of nonlinear waves more accessible, giving particular attention to some recent developments.*

1. Qualitative differences between linear and nonlinear waves

Some characteristics of nonlinear wave motion can be described in general terms. In linear wave motion, as, for example, in the transmission of sound, disturbances are always propagated with a definite speed (relative to the medium) which may vary within the medium. This "sound speed" is a local property of the medium itself and remains the same for every conceivable linear wave motion in the medium. Such a sound speed also plays a role in nonlinear wave motion. Small disturbances or "wavelets," slightly modifying a given primary wave motion, are propagated with a certain speed, again called sound speed, though in this case the sound speed depends not only on the position within the medium but on the state of the medium induced by the primary motion.

The distinctive feature of nonlinear waves, however, concerns disturbances or discontinuities which are not necessarily small. In linear wave motion any initial discontinuity across a surface is preserved as a discontinuity and propagated with sound speed. Nonlinear wave motion behaves in a different manner: Suppose there is an initial discontinuity between two regions of different pressures, densities, and flow velocities. Then there are the following *alternative* possibilities: either the initial discontinuity is resolved immediately and the disturbance, while propagated, becomes continuous, or the initial discontinuity is propagated through one or two *shock fronts*, advancing not at sonic but at supersonic speed relative to the medium

* For the theory of compressible flow reference may be made to [3,4,5]; different approaches are given by Sauer [6] and Liepmann and Puckett [7].

ahead of them. As previously stated, shock fronts are the most conspicuous phenomena occurring in nonlinear wave propagation; even without being caused by initial discontinuities they may appear and be propagated. The underlying mathematical fact is that, unlike linear partial differential equations, nonlinear equations often do not admit solutions which can be continuously extended wherever the differential equations themselves remain regular.

Another striking difference between linear and nonlinear waves concerns the phenomenon of interaction: the principle of superposition holds for linear waves but not for nonlinear waves. As a consequence, for example, excess pressures of interfering sound waves are merely additive; in contrast to this fact, interaction and reflection of nonlinear waves may lead to enormous increases in pressure.

A. General Equations of Flow. Thermodynamic Notions

2. The medium

We shall be primarily concerned with a moving fluid, though many of the results apply to other moving media (e.g. to a solid slab in longitudinal wave motion). In this section we shall set forth the properties of the medium that will be assumed throughout the book and we shall describe certain idealized media of special interest. Moreover, since gas dynamics is thoroughly interwoven with thermodynamical concepts, it is appropriate to insert here a collection of basic notions of thermodynamics in a suitable mathematical form.*

Except where the motion is *discontinuous*, viscosity, heat conduction, and deviation of the medium from thermodynamic equilibrium (at any instant and any point) will be neglected. Some critical comments concerning the neglect of these phenomena will be made in later chapters. In particular it will be shown that viscosity and heat conduction play an important role in forming and maintaining shock discontinuities.

At each instant and each point of the fluid there is a definite state (of thermodynamic equilibrium) defined by:

- p the pressure,
- T the temperature,

*For textbooks on thermodynamics see Epstein [20] and Zemansky [21].

- τ the specific volume (i.e. volume per unit mass),
 ρ the density, with $\rho\tau = 1$,
 S the specific entropy,
 e the specific (internal) energy, and
 i the specific enthalpy,* defined by $i = e + p\tau$.

It is known from thermodynamics that for any given medium only two of the parameters p , T , τ , e , and S are independent. In fact they may all be considered as functions of τ and S .

The internal energy gained by the medium during a change from one state to another is the heat contributed to the medium plus the work done on the medium by compressive action of the pressure forces. For a change from one state to an immediately neighboring one this fundamental fact is expressed by the relation

$$(2.01) \quad de = TdS - p d\tau.$$

In a reversible process, TdS is the heat acquired by conduction; in an irreversible process, TdS is greater than the heat so acquired. If the irreversible process is one that can be described as determined by the action of viscosity, then the excess of TdS over the heat acquired by conduction may conveniently be interpreted as the heat produced by viscous forces.

Suppose that for some medium we know how the specific energy e depends on τ and S . Then the pressure p and temperature T may immediately be found on considering the meaning of relation (2.01). Thus

$$(2.02) \quad p = -e_\tau, \quad T = e_S,$$

the subscripts indicating partial differentiation.**

The functions giving p in terms of ρ , or τ , and S , occurring so frequently in the theory of fluid flow, will consistently be denoted by

$$(2.03) \quad p = f(\rho, S); \quad p = g(\tau, S).$$

Extending slightly the conventional nomenclature, we shall call either of these equations the *caloric equation of state* of the medium.

Neglecting viscosity and heat conduction is tantamount to assuming that as a particle of the medium moves about, the specific

* The notion of enthalpy will be discussed in Section 9.

** Nearly everywhere in the book, we indicate partial derivatives by subscripts.