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时间序列分析的小波方法

（英文版）

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Wavelet Methods for Time Series Analysis

Donald B. Percival & Andrew

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Preface

The last decade has seen an explosion of interest in wavelets, a subject area that has coalesced from roots in mathematics, physics, electrical engineering and other disciplines. As a result, wavelet methodology has had a significant impact in areas as diverse as differential equations, image processing and statistics. This book is an introduction to wavelets and their application in the analysis of discrete time series typical of those acquired in the physical sciences. While we present a thorough introduction to the basic theory behind the discrete wavelet transform (DWT), our goal is to bridge the gap between theory and practice by

- emphasizing what the DWT actually means in practical terms;
- showing how the DWT can be used to create informative descriptive statistics for time series analysts;
- discussing how stochastic models can be used to assess the statistical properties of quantities computed from the DWT; and
- presenting substantive examples of wavelet analysis of time series representative of those encountered in the physical sciences.

To date, most books on wavelets describe them in terms of continuous functions and often introduce the reader to a plethora of different types of wavelets. We concentrate on developing wavelet methods in discrete time via standard filtering and matrix transformation ideas. We purposely avoid overloading the reader by focusing almost exclusively on the class of wavelet filters described in Daubechies (1992), which are particularly convenient and useful for statistical applications; however, the understanding gained from a study of the Daubechies class of wavelets will put the reader in a excellent position to work with other classes of interest. For pedagogical purposes, this book in fact starts (Chapter 1) and ends (Chapter 11) with discussions of the continuous case. This organization allows us at the beginning to motivate ideas from a historical perspective and then at the end to link ideas arising in the discrete analysis to some of the widely known results for continuous time wavelet analysis.

Topics developed early on in the book (Chapters 4 and 5) include the DWT and the 'maximal overlap' discrete wavelet transform (MODWT), which can be regarded as a generalization of the DWT with certain quite appealing properties. As a whole, these two chapters provide a self-contained introduction to the basic properties of wavelets, with an emphasis both on algorithms for computing the DWT and MODWT and also on the use of these transforms to provide informative descriptive statistics for time series. In particular, both transforms lead to both a scale-based decomposition of the sample variance of a time series and also a scale-based additive decomposition known as a multiresolution analysis. A generalization of the DWT and MODWT that are known in the literature as 'wavelet packet' transforms, and the decomposition of time series via matching pursuit, are among the subjects of Chapter 6. In the second part of the book, we combine these transforms with stochastic models to develop wavelet-based statistical inference for time series analysis. Specific topics covered in this part of the book include

- the wavelet variance, which provides a scale-based analysis of variance complementary to traditional frequency-based spectral analysis (Chapter 8);
- the analysis and synthesis of 'long memory processes,' i.e., processes with slowly decaying correlations (Chapter 9); and
- signal estimation via 'thresholding' and 'denoising' (Chapter 10).

This book is written 'from the ground level and up.' We have attempted to make the book as self-contained as possible (to this end, Chapters 2, 3 and 7 contain reviews of, respectively, relevant Fourier and filtering theory; key ideas in the orthonormal transforms of time series; and important concepts involving random variables and stochastic processes). The text should thus be suitable for advanced undergraduates, but is primarily intended for graduate students and researchers in statistics, electrical engineering, physics, geophysics, astronomy, oceanography and other physical sciences. Readers with a strong mathematical background can skip Chapters 2 and 3 after a quick perusal. Those with prior knowledge of the DWT can make use of the Key Facts and Definitions toward the end of various sections in Chapters 4 and 5 to assess how much of these sections they need to study. Drafts of this book have been used as a textbook for a graduate course taught at the University of Washington for the past five years, but we have also designed it to be a self-study work-book by including a large number of exercises embedded within the body of the chapters (particularly Chapters 2 to 5), with solutions provided in the Appendix. Working the embedded exercises will provide readers with a means of progressively understanding the material. For use as a course textbook, we have also provided additional exercises at the end of each chapter (instructors wishing to obtain a solution guide for the exercises should follow the guidance given on the Web site detailed below).

The wavelet analyses of time series that are described in Chapters 4 and 5 can readily be carried out once the basic algorithms for computing the DWT and MODWT (and their inverses) are implemented. While these can be immediately and readily coded up using the pseudo-code in the Comments and Extensions to Sections 4.6 and 5.5, links to existing software in `S-Plus` and Lisp can be found by consulting the Web site for this book, which currently is at

`http://www.staff.washington.edu/dbp/wmtsa.html`

(alternatively the reader can go to the site for Cambridge University Press – currently at `http://www.cup.org` – and search for the page describing this book, which should have a link to the Web site). The reader should also consult this Web site to obtain a current errata sheet, updates to references at the end of the book that are yet to appear, and references to additional material. Additionally readers can use the Web site to download the coefficients for various scaling filters (as discussed in Sections 4.8 and 4.9), the values for all the time series used as examples in this book, and certain computed values that can be used to check computer code. To facilitate preparation of overheads for courses and seminars, the Web site also allows access to pdf files with all the figures and tables in the book (please note that these figures and tables are the copyright of Cambridge University Press and must not be further distributed or used without written permission).

The book was written using Donald Knuth's superb typesetting system TEX as implemented by Blue Sky Research in their product TEXtures for Apple Macintosh™ computers. The figures in this book were created using either the plotting system GPL written by W. Hess (whom we thank for many years of support) or S-Plus, the commercial version of the S language developed by J. Chambers and co-workers and marketed by MathSoft, Inc. The computations necessary for the various examples and figures were carried out using either S-Plus or P$_I$TSS$_A$ (a Lisp-based object-oriented program for interactive time series and signal analysis that was developed in part by one of us (Percival)).

We thank R. Spindel and the late J. Harlett of the Applied Physics Laboratory, University of Washington, for providing discretionary funding that led to the start of this book. We thank the National Science Foundation, the National Institutes of Health, the Environmental Protection Agency (through the National Research Center for Statistics and the Environment at the University of Washington), the Office of Naval Research and the Air Force Office of Scientific Research for ongoing support during the writing of this book. Our stay at the Isaac Newton Institute for Mathematical Sciences (Cambridge University) during the program on Nonlinear and Nonstationary Signal Processing in 1998 contributed greatly to the completion of this book; we thank the Engineering and Physical Science Research Council (EPSRC) for the support of one of us (Percival) through a Senior Visiting Fellowship while at Cambridge.

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Contents

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Introduction to Wavelets

1.0 Introduction

Wavelets are mathematical tools for analyzing time series or images (although not exclusively so: for examples of usage in other applications, see Stollnitz *et al.*, 1996, and Sweldens, 1996). Our discussion of wavelets in this book focuses on their use with time series, which we take to be any sequence of observations associated with an ordered independent variable t (the variable t can assume either a discrete set of values such as the integers or a continuum of values such as the entire real axis – examples of both types include time, depth or distance along a line, so a time series need not actually involve time). Wavelets are a relatively new way of analyzing time series in that the formal subject dates back to the 1980s, but in many aspects wavelets are a synthesis of older ideas with new elegant mathematical results and efficient computational algorithms. Wavelet analysis is in some cases complementary to existing analysis techniques (e.g., correlation and spectral analysis) and in other cases capable of solving problems for which little progress had been made prior to the introduction of wavelets.

Broadly speaking (and with apologies for the play on words!), there have been two main waves of wavelets. The first wave resulted in what is known as the continuous wavelet transform (CWT), which is designed to work with time series defined over the entire real axis; the second, in the discrete wavelet transform (DWT), which deals with series defined essentially over a range of integers (usually $t = 0, 1, \ldots, N-1$, where N denotes the number of values in the time series). In this chapter we introduce and motivate wavelets via the CWT. The emphasis is on conveying the ideas behind wavelet analysis as opposed to presenting a comprehensive mathematical development, which by now is available in many other places. Our approach will concentrate on what exactly wavelet analysis can tell us about a time series. We do not presume extensive familiarity with other common analysis techniques (in particular, Fourier analysis). After this introduction in Sections 1.1 and 1.2, we compare and contrast the DWT with the CWT and discuss why we feel the DWT is a natural tool for discrete time series analysis. The remainder of the book will then be devoted to presenting the DWT (and certain closely related transforms) from the ground level up (Chapters 2 to 6), followed by a discussion in Chapters 7 to 10 of the statistical analysis of time series via the DWT. We return to the CWT only in Chapter 11, where we deepen our understanding of the DWT by noting its connection to the CWT in the elegant theory of multiresolution analysis for functions defined over the entire real axis.

1.1 The Essence of a Wavelet

What is a wavelet? As the name suggests, a wavelet is a 'small wave.' A small wave grows and decays essentially in a limited time period. The contrasting notion is obviously a 'big wave.' An example of a big wave is the sine function, which keeps on oscillating up and down on a plot of $\sin(u)$ versus $u \in (-\infty, \infty)$. To begin to quantify the notion of a wavelet, let us consider a real-valued function $\psi(\cdot)$ defined over the real axis $(-\infty, \infty)$ and satisfying two basic properties.

[1] The integral of $\psi(\cdot)$ is zero:

$$\int_{-\infty}^{\infty} \psi(u)\, du = 0. \tag{2a}$$

[2] The square of $\psi(\cdot)$ integrates to unity:

$$\int_{-\infty}^{\infty} \psi^2(u)\, du = 1 \tag{2b}$$

(for the sine function, the above integral would be infinite, so $\sin^2(\cdot)$ cannot be renormalized to integrate to unity).

If Equation (2b) holds, then for any ϵ satisfying $0 < \epsilon < 1$, there must be an interval $[-T, T]$ of finite length such that

$$\int_{-T}^{T} \psi^2(u)\, du > 1 - \epsilon.$$

If we think of ϵ as being very close to zero, then $\psi(\cdot)$ can only deviate insignificantly from zero outside of $[-T, T]$: its nonzero activity is essentially limited to the finite interval $[-T, T]$. Since the length of the interval $[-T, T]$ is vanishingly small compared to the infinite length of the entire real axis $(-\infty, \infty)$, the nonzero activity of $\psi(\cdot)$ can be considered as limited to a relatively small interval of time. While Equation (2b) says $\psi(\cdot)$ has to make some excursions away from zero, Equation (2a) tells us that any excursions it makes above zero must be canceled out by excursions below zero, so $\psi(\cdot)$ must resemble a wave. Hence Equations (2a) and (2b) lead to a 'small wave' or wavelet.

Three such wavelets are plotted in Figure 3. Based on their definitions below, the reader can verify that these functions indeed satisfy Equations (2a) and (2b) (that they integrate to zero is evident from the plots). The first is called the Haar wavelet function:

$$\psi^{(\mathrm{H})}(u) \equiv \begin{cases} -1/\sqrt{2}, & -1 < u \le 0; \\ 1/\sqrt{2}, & 0 < u \le 1; \\ 0, & \text{otherwise} \end{cases} \tag{2c}$$

(a slightly different formulation of this wavelet is discussed in detail in Section 11.6). The above is arguably the oldest wavelet, being named after A. Haar, who developed

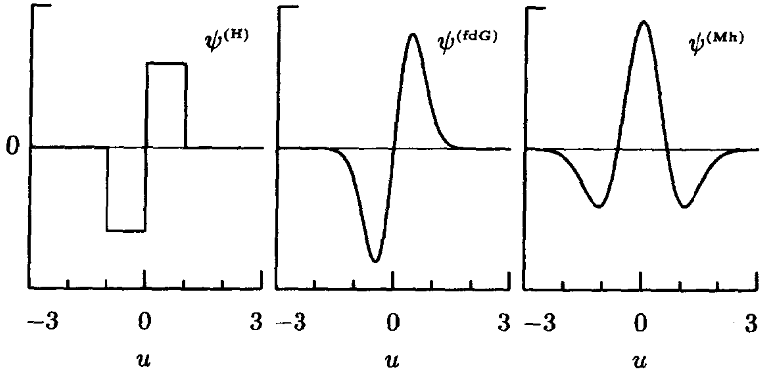

Figure 3. Three wavelets. From left to right, we have one version of the Haar wavelet; a wavelet that is related to the first derivative of the Gaussian probability density function (PDF); and the Mexican hat wavelet, which is related to the second derivative of the Gaussian PDF.

it as an analysis tool in an article in 1910. To form the other two wavelets, we start with the Gaussian (normal) probability density function (PDF) for a random variable with mean zero and variance σ^2:

$$\phi(u) \equiv \frac{e^{-u^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}, \quad -\infty < u < \infty.$$

The first derivative of $\phi(\cdot)$ is

$$\frac{d\phi(u)}{du} = -\frac{ue^{-u^2/2\sigma^2}}{\sigma^3\sqrt{2\pi}}.$$

If we renormalize the negative of the above to satisfy Equation (2b), we obtain the wavelet

$$\psi^{(\mathrm{fdG})}(u) \equiv \frac{\sqrt{2}\,ue^{-u^2/2\sigma^2}}{\sigma^{3/2}\pi^{1/4}}, \tag{3a}$$

which is shown in the middle of Figure 3 with $\sigma \doteq 0.44311$. With proper renormalization again, the negative of the second derivative of $\phi(\cdot)$ also yields a wavelet, usually referred to as the Mexican hat:

$$\psi^{(\mathrm{Mh})}(u) \equiv \frac{2\left(1-\frac{u^2}{\sigma^2}\right)e^{-u^2/2\sigma^2}}{\pi^{1/4}\sqrt{3\sigma}}. \tag{3b}$$

The origin of its name should be apparent from a glance at the right-hand plot of Figure 3, in which $\sigma \doteq 0.63628$.

In summary, a wavelet by definition is any function that integrates to zero and is square integrable (see, however, item [1] in the Comments and Extensions below).

Comments and Extensions to Section 1.1

[1] We have intentionally given just a 'bare bones' definition of a wavelet so that we can focus on presenting the key concepts behind the subject. To get wavelets of practical utility, it is necessary to impose conditions beyond Equations (2a) and (2b). Much of the mathematical research in wavelets has been to determine what conditions are required to achieve certain types of analysis with wavelets (e.g., edge detection, singularity analysis, etc.). We mention one important and common additional condition here, namely, the so-called *admissibility condition.* A wavelet $\psi(\cdot)$ is said to be admissible if its Fourier transform, namely,

$$\Psi(f) \equiv \int_{-\infty}^{\infty} \psi(u) e^{-i2\pi f u}\, du,$$

is such that

$$C_\psi \equiv \int_0^{\infty} \frac{|\Psi(f)|^2}{f}\, df \text{ satisfies } 0 < C_\psi < \infty \tag{4a}$$

(Chapter 2 has a review of Fourier theory, including a summary of the key results for functions such as $\psi(\cdot)$ in the last part of Section 2.7). This condition allows the reconstruction of a function $x(\cdot)$ from its continuous wavelet transform (see Equation (11a)). For additional discussion on the admissibility condition, see, e.g., Daubechies (1992, pp. 24–6).

[2] To simplify our exposition in the main part of this chapter, we have assumed $\psi(\cdot)$ to be real-valued, but complex-valued wavelets are often used, particularly in geophysical applications (see, e.g., the articles in Foufoula–Georgiou and Kumar, 1994). One of the first articles on wavelet analysis (Goupillaud *et al.*, 1984) was motivated by Morlet's involvement in geophysical signal analysis for oil and gas exploration. He wanted to analyze signals containing short, high-frequency transients with a small number of cycles, as well as long, low-frequency transients. The examples given used the complex wavelet

$$\psi(u) = Ce^{-i\omega_0 u}\left(e^{-u^2/2} - \sqrt{2}e^{-\omega_0^2/4}e^{-u^2}\right), \tag{4b}$$

where C and ω_0 are constants. It is well known that

$$\int_{-\infty}^{\infty} e^{-i\omega_0 u} e^{-u^2/2} du = \sqrt{2\pi} e^{-\omega_0^2/2}$$

(see e.g., Bracewell, 1978, p. 386, or Percival and Walden, 1993, p. 67), from which it follows that $\int_{-\infty}^{\infty} \psi(u) du = 0$, so that Equation (2a) is satisfied. For Morlet, the fact that Equation (2a) holds was not just a mathematical result, but also a physical necessity: the seismic reflection time series under analysis also 'integrate to zero' since compressions and rarefactions must cancel out. The constant C is chosen so that the complex-valued version of Equation (2b), namely, $\int |\psi(u)|^2\, du = 1$, holds for a particular choice of ω_0. For example, when $\omega_0 = 5$, we have $C \doteq 0.7528$. As ω_0 is increased further, the negative term in (4b) becomes negligible; when $\omega_0 = 10$ with $C = \pi^{-1/4}$, we have $\int_{-\infty}^{\infty} |\psi(u)|^2 du \doteq 1$ to nine decimal places accuracy. Hence for large ω_0,

$$\psi(u) \approx \psi_{\omega_0}^{(\mathrm{M})}(u) \equiv \pi^{-1/4} e^{-i\omega_0 u} e^{-u^2/2},$$

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