Mathematics for Industry 6

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Tensegrity Structures

Form, Stability, and Symmetry

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Mathematics for Industry

Volume 6

Aims & Scope

The meaning of “Mathematics for Industry” (sometimes abbreviated as MI or MfI) is different from that of “Mathematics in Industry” (or of “Industrial Mathematics”). The latter is restrictive: it tends to be identified with the actual mathematics that specifically arises in the daily management and operation of manufacturing. The former, however, denotes a new research field in mathematics that may serve as a foundation for creating future technologies. This concept was born from the integration and reorganization of pure and applied mathematics in the present day into a fluid and versatile form capable of stimulating awareness of the importance of mathematics in industry, as well as responding to the needs of industrial technologies. The history of this integration and reorganization indicates that this basic idea will someday find increasing utility. Mathematics can be a key technology in modern society.

The series aims to promote this trend by (1) providing comprehensive content on applications of mathematics, especially to industry technologies via various types of scientific research, (2) introducing basic, useful, necessary and crucial knowledge for several applications through concrete subjects, and (3) introducing new research results and developments for applications of mathematics in the real world. These points may provide the basis for opening a new mathematics-oriented technological world and even new research fields of mathematics.

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Preface

Aims and Scope

Tensegrity structures are now more than 60-years old, since their birth as artworks. However, they are not "old" nor out of fashion! On the contrary, they are becoming more and more present in many different fields, including but not limited to engineering, biomedicine, and mathematics. These applications make use of the unique mechanical as well as mathematical properties of tensegrity structures in contrast to conventional structural forms such as trusses and frames.

Our primary objective in writing this book is to provide a textbook for self-study which is easily accessible not only to engineers and scientists, but also to upper-level undergraduate and graduate students. Both students and professionals will find material of interest to them in the book. With this objective in mind, the presentation of this book is detailed with many examples, and moreover, it is self-contained.

There are already several existing books on tensegrity structures; most of them present approaches to realization and practical applications of those structures. By contrast, this book is devoted to helping the readers achieve a deeper understanding of *fundamental* mechanical and mathematical properties of tensegrity structures. In particular, emphasis is placed on the two key problems in preliminary design of tensegrity structures—*self-equilibrium* and *(super-)stability*, by extensively utilizing the concept of force density and high level of symmetry of the structures.

Subjects and Contents

Tensegrity structures are similar in appearance to conventional bar-joint structures (trusses), however, their members carry forces (prestresses) even when no external load is applied. This means that their nodes and members have to be balanced by the prestresses so as to maintain their equilibrium. Furthermore, most tensegrity structures are intrinsically unstable in the absence of prestresses, and it is the introduction of prestresses that makes them stable. For these reasons, finding the *self-equilibrated configuration* and investigation of *stability* are the two key problems in the preliminary design of tensegrity structures.

Finding the configuration associated with prestresses, in the state of self-equilibrium, is called *form-finding* or shape-finding. It is a common design problem for tension structures, including tensile membrane structures and cable-nets. The problem is difficult because the configuration and prestresses cannot be determined separately as a result of the high interdependency between them. Further difficulties arise from the fact that tensegrity structures maintain their stability without any support.

A structure is stable if and only if it has the locally minimum total potential energy, or strain energy in the absence of external loads. Stability investigation of tensegrity structures is necessary because their stability cannot be guaranteed as can that of cable-nets or membrane structures carrying tension only in their structural elements. This comes from the fact that tensegrity structures are composed of (continuous) tensile members and (discontinuous) compressive members. Moreover, it is possible for tensegrity structures to be *super-stable*, which is a more robust stability criterion, if proper prestresses are associated with the proper connectivity pattern.

In this book, basic concepts and applications of tensegrity structures are introduced in Chap. 1. Chapter 2 formulates the matrices and vectors necessary for the study of self-equilibrium and stability. The analytical conditions for self-equilibrium of several highly symmetric tensegrity structures with simple geometries are given in Chap. 3. Chapter 4 defines the three stability criteria—stability, prestress-stability, and super-stability—and derives the necessary conditions and sufficient conditions for super-stability. The force density method, which guarantees super-stability, is presented in Chap. 5 for numerical form-finding of relatively complex tensegrity structures. Utilizing the analytical formulations for highly symmetric structures given in Appendix D, the self-equilibrium and super-stability conditions are derived for the prismatic tensegrity structures in Chap. 6 and those for the star-shaped structures in Chap. 7; both these classes of structures are of dihedral symmetry. Additionally, Chap. 8 presents the self-equilibrium and super-stability conditions for structures with tetrahedral symmetry.

At the end of the preface, we have to give our deepest thanks to our families, friends, and former and current students for their supports. Part of the work on symmetry has been conducted in close collaboration with Dr. Simon D. Guest of the University of Cambridge and Professor Robert Connelly of Cornell University; they showed us a new way to study tensegrity structures. Mr. Masaki

Okano of Nagoya City University read the first half of the book carefully and found many mistakes, which we then were able to correct. We also appreciate the proposal of writing this book by Dr. Yuko Sumino of Springer Japan; she has always been helpful during the preparation and publication of the book.

Nagoya, December 2014 — Jing Yao Zhang
Higashi-Hiroshima — Makoto Ohsaki

Contents

Chapter 1
Introduction

Abstract In this introductory chapter, we first introduce the basic concepts and some applications of tensegrity structures, and then present their design problems which motivates our study in this book. Finally, a brief review of the existing researches on the design problems is given.

Keywords Applications · Form-finding methods · Stability criteria

1.1 General Introduction

The term *tensegrity* was created by Richard Buckminster Fuller as a contraction of '*tens*ional' and 'int*egrity*' [15]. It refers to the integrity of a stable structure balanced by *continuous* structural members (cables) in tension and *discontinuous* structural members (struts) in compression. Moreover, the cables are flexible and global components, while the struts are stiff and local components.

The first tensegrity structure, called X-column, is considered to be built by Kenneth Snelson in 1948 [35].[1] Snelson came up with the idea of building this structure as an answer to the question posted by Fuller, who was his teacher at Black Mountain College at that time: "Is it possible to build a structure to illustrate the structural principle of nature, which was observed to rely on that continuous tension embraces isolated compression elements?"

There is no strict definition of tensegrity structures up to now that is accepted by all people. Instead of giving a strict definition of our own, we generally accept that a tensegrity structure should have the following characteristics:

[1] More details on the birth of tensegrity structures can be found in the papers [25–27] by Motro.

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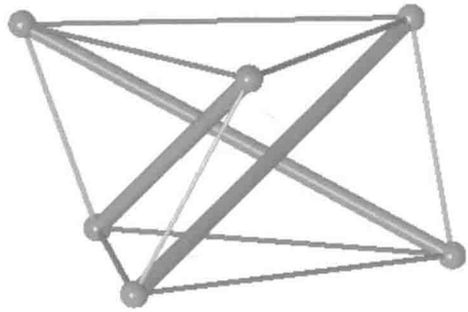

Fig. 1.1 The simplest tensegrity structure in three-dimensional space. The struts in compression are denoted by *thick lines*, and the cables in tension are denoted by *thin lines*

Characteristics of a tensegrity structure:

- The structure is *free-standing*, without any support.
- The structural members are straight.
- There are only two different types of structural members: *struts* carrying compression and *cables* carrying tension.
- The struts do not contact with each other at their ends.[2]

Moreover, we will persist in the entire book that the members in thick lines indicate struts in compression, and the members in thin lines indicate cables in tension, because tensile members are generally flexible and slender.

Figure 1.1 shows the simplest three-dimensional tensegrity structure. The structures having similar appearances are called *prismatic structures*, which will be studied in detail in Chaps. 3 and 6. The struts of the structure do not contact with each other. Moreover, supports or fixed nodes are unnecessary to maintain its (super-)stability with the exclusion of rigid-body motions.

1.2 Applications

Tensegrity structures were originally born in arts; however, they 'exist' universally, from the micro scale to the macro scale. In the micro scale, for example, response of living cells subjected to environmental changes can be interpreted and predicted by tensegrity models; in the intermediate scale, the human body can be modeled as a tensegrity structure; and in the macro scale, structure of the cosmos can also be regarded as a tensegrity structure, where the planets are the nodes and their interactions are the invisible members.

[2] With a very limited exceptions, the struts of some tensegrity structures are allowed to share common nodes, especially in the two-dimensional cases.

Owing to universalness of tensegrity structures, applications of their principles have been consecutively increasing in a great variety of fields, since their birth as art works. This section introduces some of the interesting applications. However, this introduction is no way exhaustive, since any such attempt would become out-of-date very soon due to the rapidly increasing number. The up-to-date information is provided on our homepage.[3]

1.2.1 Applications in Architecture

The members of tensegrity structures are the simplest possible ones, because they are straight and carry only axial forces. Moreover, a tensegrity structure maintains its stability with the minimum possible number of structural members, which is much less than the necessary number for a conventional bar-joint structure (truss) consisting of the same number of nodes. Therefore, tensegrity structures are considered as one of the optimal structural systems, in particular in the engineering view.

Furthermore, tensegrity structures have many other advantages when they are used as long-span structures to cover a large space without columns inside. Some of these advantages by comparison to some other structural systems are listed below.

- Introduction of prestresses into the structures could significantly enhances their structural stiffness, although this is not always the case. Therefore, they can be built with much smaller amounts of materials while having the same capacity of resisting external loads; as a reward, this can also significantly reduce the gravitational loads, which are usually dominant in the design of long-span structures.
- The structural members are of very high mechanical efficiency, because they carry only axial forces such that the stresses (normal stresses only) in a member are uniform.
- The struts in compression that are prone to member buckling might be more slender, because they are local components and much shorter in length than cables. This can also effectively reduce the gravitational loads.
- The cables in tension can make full use of high-strength materials, because large cross sections due to member buckling are not necessary.
- Complex joints connecting different members are not necessary, since the flexible cables are much easier to be attached to the struts, while the struts do not contact with each other.

Using the concept of tensegrity structures, David Geiger designed a permanent long-span structure, called the Georgia Dome [5, 21]. The structure was constructed in 1992 as the main hall for the 1996 Atlanta Summer Olympic Games in U.S. It has a height of 82.5 m, a length of 227 m, a width of 185 m, and a total floor area of 9,490 m^2.

[3] Online sources on tensegrity structures collected by the authors are consecutively updated at http://zhang.AIStructure.net/links/tensegritylinks/.

The great success of Georgia Dome aroused the interests and enthusiasms of many structural engineers and researchers, and a number of tensegrity-domes have been built around the world [46, 49]. However, it should be noted that tensegrity-domes are not 'real' tensegrity structures in strict definition, because they are not free-standing and maintain their stability by being attached to supports at the boundary.

The experimental facility built in Chiba, Japan in 2001 as shown in Fig. 1.2 is one of the earliest attempts to use 'real' tensegrity structure in architectural engineering [20]. Two tensegrity units are used as structural components, and one isolated strut at the top of each unit is used to support the membrane roof. One of the units is 10 m high and the other is 7 m high. The units have the similar shape to the simplest (prismatic) structure as shown in Fig. 1.1, with three additional 'vertical' cables to attain proper rigidity for practical applications.

1.2.2 Applications in Mechanical Engineering

In the filed of mechanical engineering, tensegrity structures are utilized as 'smart' structures [2, 6, 16, 34] and deployable structures [38], the shapes of which are

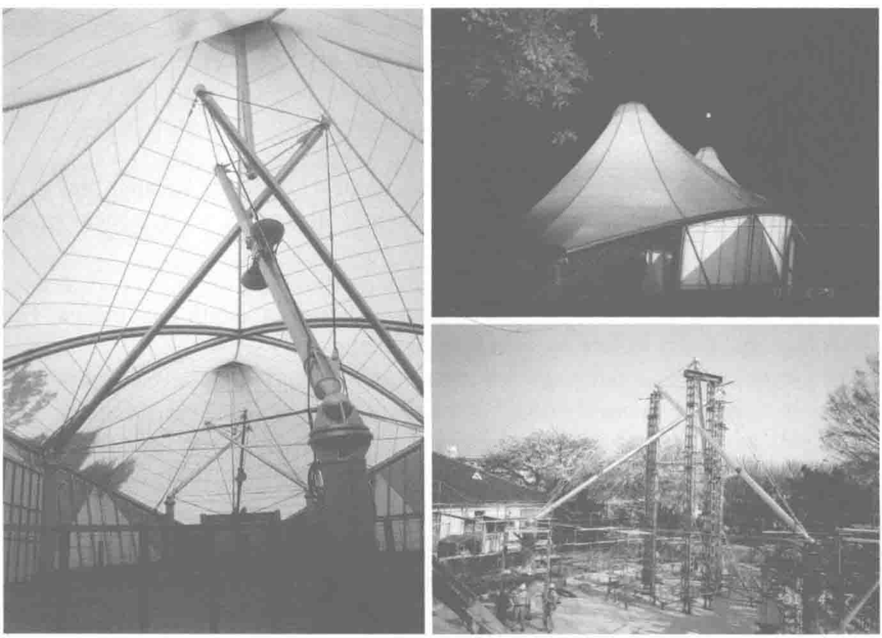

Fig. 1.2 Example of a pair of tensegrity structures used as structural components to support a membrane roof. The structure was constructed in Chiba, Japan in 2001. The *left* photo is the interior view of the building, the *upper-right* photo is its exterior night view, and the *lower-right* photo is one of the tensegrity structures under construction. (Courtesy: Dr. K. Kawaguchi at the University of Tokyo)