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# DISCRETE MATHEMATICS WITH COMBINATORICS

## 离散数学暨 组合数学



James A. Anderson 著

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## **离散数学暨组合数学**

James A. Anderson

*University of South Carolina, Spartanburg*

清华大学出版社

北 京

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# Preface

While there are many discrete mathematics books on the market, none of the available books covers the desired range and depth of topics in discrete mathematics in this book and also works in a theme on how to do proofs. Proofs are introduced in the first chapter and continued throughout the book. Most students taking discrete mathematics are mathematics and computer science majors. While the necessity of learning to do proofs is obvious for mathematics majors, it is also critical for computer science students to think logically. Essentially a logical bug-free computer program is equivalent to a logical proof. Also it is assumed in this book that it is easier to use (or at least not misuse) an application if one understands why it works. With few exceptions, the book is self-contained. Concepts are developed mathematically before they are seen in an applied context.

Calculus is not required for any of the material in this book. College algebra is adequate for the basic chapters. However, although this book is self-contained, some of the remaining chapters require more mathematical maturity than the basic chapters.

This book is intended for either a one- or two-term course in discrete mathematics. The first eight chapters of this book provide a solid foundation in discrete mathematics and would be appropriate for a first-level course at the freshman or sophomore level. These chapters are essentially independent so that the instructor can pick the material he wishes to cover. The remainder of the book contains appropriate material for a second course in discrete mathematics. These chapters expand concepts introduced earlier and introduce numerous advanced topics. Topics are explored from different points of view to show how they may be used in different settings. The range of topics includes

*Logic*—Including truth tables, propositional logic, predicate calculus, circuits, induction, and proofs.

*Set Theory*—Including cardinality of sets, relations, partially ordered sets, congruence relations, graphs, directed graphs and functions.

*Algorithms*—Including complexity of algorithms, search and sort algorithms, the Euclidean algorithm, Huffman's algorithm, Prim's algorithms, Warshall's algorithm, the Ford-Fulkerson algorithm, the Floyd-Warshall algorithm, and Dijkstra's algorithms.

*Graph Theory*—Including directed graphs, Euler cycles and paths, Hamiltonian cycles and paths, planar graphs, and weighted graphs.

*Tree*—Including binary search trees, weighted trees, tree transversal, Huffman's codes, and spanning trees.

*Combinatorics*—Including permutations, combinations, inclusion-exclusion, partitions, generating functions, Catalan numbers, Sterling numbers, Rook polynomials, derangements, and enumeration of colors.

*Algebra*—Including semigroups, groups, lattices, semilattices, Boolean algebras, rings, fields, integral domains, polynomials, and matrices.

There is extensive number theory and algebra in this book. I feel that this is a strength of this text, but realize that others may not want to cover these subjects. The chapters in these areas are completely independent of the remainder of the book and can be covered or not as the instructor desires. This book also contains probability, finite differences, and other topics not usually found in a discrete mathematics text.

## ■ ORGANIZATION

The first three chapters cover logic and set theory. It is assumed in this book that an understanding of proofs is necessary for the logical construction of advanced computer programs.

The basic concepts of a proof are given and illustrated with numerous examples. In Chapter 2, the student is given the opportunity to prove some elementary concepts of set theory. In Chapter 3, the concept of an axiom system for number theory is introduced. The student is given the opportunity to prove theorems in a familiar environment. Proofs using induction are also introduced in this chapter. Throughout the remainder of the book, many proofs are presented and many of the problems are devoted to proofs. Problems, including proofs, begin at the elementary level and continually become more advanced throughout the book.

Relations and graphs are introduced in Chapter 2. Relations lead naturally into functions, which are introduced in Chapter 4. However, the development of functions in Chapter 4 is independent of the material in Chapter 2. Similarly the development of graphs in Chapter 6 does not depend on their development as relations in Chapter 2.

Matrices, permutations, and sequences are introduced in Chapter 4 as special types of functions. Further properties of functions and matrices follow in Chapter 6. Algorithms for matrices are introduced and further properties of matrices are developed, which will be used in later chapters on algebra, counting, and theory of codes.

Permutations are used for counting in Chapter 8 and also for applications in algebra and combinatorics in later chapters. Again the material in Chapter 8, while related to Chapter 4, can be studied independently.

Chapter 5 is independent of the previous chapters except for the matrices in the previous chapter. Algorithms are developed including sorting algorithms. The complexity of algorithms is also developed in this chapter. Prefix and suffix notation are introduced here. They are again discussed in Chapter 15 with regard to traversing binary trees. Binary and hexadecimal numbers are also introduced in this chapter.

Many elementary concepts of graphs, directed graphs, and trees are covered in Chapter 6. These concepts are covered in more depth in Chapters 14-16. Chapter 6 is independent of the previous chapters.

In Chapters 7 and 9 the basics of number theory are developed. These chapters are necessary for applications of number theory in Chapter 23 but are otherwise

completely independent of the other chapters and may be omitted if desired.

Chapter 8 is the beginning of extensive coverage of combinatorics. This is continued in many of the chapters including Chapters 12, 13, and 17. Chapter 8 also covers probability, which is not common in most other discrete mathematics books.

Chapters 9 and 20 cover the basic concepts of algebra including semigroups, groups rings, semilattices, lattices, rings, integral domains, and fields. These chapters use Sections 3.6 and 4.3 for examples of groups and rings. Chapter 9 is necessary for the applications in Chapters 17–21.

In many ways Chapters 11, 12, and 13 form a package. Recursion is continued in Chapter 11. In addition to the standard linear recurrence relations normally covered in a discrete mathematics text, the theory of finite difference is also covered. Chapter 6 should be covered before this chapter unless the student already has some knowledge of recursion. Chapter 12 continues the counting introduced in Chapter 8. It covers topics introduced in Chapter 8 such as occupancy problems and inclusion-exclusion. It also introduces derangements and rook polynomials. It is closely related to Chapter 11. Many of the same topics are covered from different points of view. One example of this is Stirling numbers. However neither chapter is dependent on the other.

Chapters 11 and 12 are tied together in Chapter 13, where generating functions are used to continue the material in both chapters. In particular, generating functions provide a powerful tool for the solution of occupancy problems.

Chapters 14–16 continue the study of trees and graphs begun in Chapter 6. They obviously depend on the material in Chapter 6, but are virtually independent of most of the preceding chapters. One exception is the use of matrices in some of the algorithms. Many of the standard topics of Graphs and Trees are covered including planar graphs, Hamiltonian cycles, binary trees, spanning trees, minimal spanning trees, weighted trees, shortest path algorithms, and network flows.

Chapters 17–23 form another cluster consisting of number theory, algebra, combinatorics and their application. The theory of computation is introduced in Chapter 17. This includes codes, regular languages, automata, grammars and their relationship. This chapter uses semigroups from Section 9.2. Chapter 18 introduces special codes such as error detecting codes and error correcting codes. This chapter requires knowledge of group theory, found in Section 9.4 and a knowledge of matrices, found in Chapters 4 and 5. Codes are explored from yet another direction in Chapter 23 where cryptography is introduced. This chapter is dependent on the previous chapters on number theory.

In Chapter 19, algebra and combinatorics are combined for the development of Burnside's Theorem and Polya's Theorem for the enumeration of colors. It primarily depends on a knowledge of permutations found in Section 9.4.

Chapter 21 is a simple application of groups and semigroups and their mapping onto the complex plane. The prerequisites for this chapter are Sections 9.2 and 9.4.

Chapter 22 gives three important applications of number theory. The study of Hashing functions and cryptography is particularly relevant to computer science.

When teaching a beginning course, I normally cover Chapters 1–5 in their entirety, Sections 8.1–8.3 and try to cover the first three sections of Chapter 6. As mentioned previously, the material in the first eight chapters is arranged for maximal flexibility. The following chart shows the required prerequisites for each chapter.

<i>Chapter</i>	<i>Prerequisite Chapters or Sections</i>
Chapter 1	None
Chapter 2	None
Chapter 3	Sections 1.1–1.4 and 2.1
Chapter 4	None
Chapter 5	Sections 4.1–4.3
Chapter 6	None
Chapter 7	Chapter 3
Chapter 8	None
Chapter 9	Section 3.6
Chapter 10	Chapter 7
Chapter 11	Sections 5.1–5.3
Chapter 12	Chapter 8
Chapter 13	Chapters 11 and 12.
Chapter 14	Chapters 5 and 6.
Chapter 15	Chapters 5 and 6.
Chapter 17	Chapter 9
Chapter 18	Chapters 5 and 9
Chapter 19	Chapter 9
Chapter 20	Chapter 9
Chapter 21	Chapter 9
Chapter 22	Chapter 10

## ■ SUPPLEMENTS

A solutions manual is available from the publisher with complete solutions to all problems. A website is available at [www.prenhall.com/janderson](http://www.prenhall.com/janderson). This website includes links to other interesting sites in discrete mathematics, quizzes, and additional problems.

## ■ ACKNOWLEDGMENTS

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Please feel free to e-mail me with comments and suggestions for future improvements.

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您希望本书在哪些方面进行改进？（可附页）

\_\_\_\_\_

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# TRUTH TABLES, LOGIC, AND PROOFS

---

## ■ 1.1 STATEMENTS AND CONNECTIVES

In this section we develop truth tables and use them to begin the first step in logic. We will find as we continue in this chapter that truth tables are also a basic tool for other important concepts in discrete mathematics. Logic, developed by Aristotle (384–322 BCE), has been used throughout the centuries in the development of many areas of learning including theology, philosophy, and mathematics. It is the foundation on which the whole structure of mathematics is built. Basically it is the science of reasoning, which allows us to determine which statements about mathematics are true and which are false based on a set of basic assumptions called axioms. Logic is also used in computer science to construct computer programs and to show that the programs do what they are designed to do. One of the primary goals of this book is to develop logic and show how to use it in computer science and to develop techniques for analyzing and proving theorems in mathematics.

A **proposition** is a statement or declarative sentence that may be assigned a true or false value. It must make sense to consider the statement being true or false. The true or false value assigned to a statement is called its **truth value**.

Sentences that are not propositions include

*Who are you?*

(a question).

*Read this chapter before the next class.*

(a command or exclamation).

*This sentence is not true.*

(self contradictory).

We will use  $p, q, r, \dots$  to represent propositions. For example,  $p$  could represent the statement *It is going to rain tomorrow* and  $q$  could represent the statement *The square of an integer is positive*.



In English, sentences are combined using connectives and clauses to form more complex compound sentences. Common connectives are *and*, *or*, *not*, *if ... then*, *only if*, and *if and only if*. The logical meaning that we will give to these connectives will be completely determined. The truth of a compound proposition is determined completely by the truth or falsity of the component parts. A statement that contains no connectives is called a **simple** statement. A statement that contains connectives is called a **compound** statement.

Let  $p$  and  $q$  refer to the propositions

$p$  : Jane drives a Ford.

$q$  : Bob has gray hair.

The compound statement

*Jane drives a Ford and Bob has gray hair.*

has two parts joined with the connective *and*. This statement may be expressed symbolically as

$p$  and  $q$

or, more simply, as

$p \wedge q$

where the symbol  $\wedge$  represents the word *and* in the translation from English to symbolic expressions. The expression  $p \wedge q$  is called the **conjunction** of  $p$  and  $q$ .

Similarly, the statement

*Jane drives a Ford or Bob has gray hair.*

is expressed symbolically by

$p$  or  $q$

or

$p \vee q$

where  $\vee$  represents the word *or* in the translation from English to symbols. The expression  $p \vee q$  is called the **disjunction** of  $p$  and  $q$ .

The negation or denial of  $p$  is indicated by

$\sim p$

using the tilde to indicate negation. Thus if  $p$  is the statement *Jane drives a Ford* then  $\sim p$  is the statement *Jane does not drive a Ford*.

If  $r$  is the statement *Joe likes computer science*, then *Jane does not drive a Ford and Bob has gray hair, or Joe likes computer science* would be symbolically indicated