APPLIED FINITE MATHEMATICS

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PREFACE

Our purpose in *Applied Finite Mathematics* is to introduce the mathematical tools most needed by students of business, economics, biology, and social science. We have tried to demonstrate the use of these tools to increase students' motivation. The material in this book is more than sufficient to provide a one-semester course in applied finite mathematics.

The book is intended for students who have had between one and two years of high school algebra. We have begun the book with two chapters of primarily review material. Depending upon the preparation of the students in the class, the instructor may include both of these chapters or refer to them as needed.

Important features of the book are as follows:

Modeling Approach A modeling approach is used throughout the book. This teaches students to think about how to attack a problem, rather than simply "plunging in." As a result, the modeling approach provides a better way to understand the mathematics in the book.

Writing Style The book is written so that students can read it. Through the use of a conversational, casual style, we neither talk down to students nor do we bore them.

Examples All examples in the book are completely worked out. Thus we ensure that students will not miss the purpose of an example because they can't supply a missing algebraic (or arithmetic) manipulation. Moreover, each chapter contains many examples.

Practice Problems Most examples are followed by a practice problem (and answer) to reinforce the concept demonstrated by the example.

Problem From Life Each chapter begins with a "Problem From Life" to help answer the questions "What good is this stuff?" and "Why do I need mathematics?" Each of these problems is worked as an example in the chapter.

Chapter Reviews Each chapter ends with a review and a set of review problems. The chapter review provides a summary, in alphabetical order, of the important terms and procedures the student should know. This provides an excellent starting point for study and a ready reference for review.

Exercise Sets Each chapter contains several exercise sets with numerous problems. Problems range in difficulty from basic drill to more-advanced exercises that are designed to test basic comprehension. Answers to the odd-numbered problems are provided in the back of the book.

Calculator The use of a calculator is integrated into the book, with examples that demonstrate explicitly the steps required to use a calculator.

Supplements An instructor's manual, which includes detailed answers to the even-numbered problems, is available. In addition, a student solutions manual, which provides detailed answers to the odd-numbered problems in the book, is also available.

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Any errors that remain are solely our responsibility.

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Richmond, Virginia

R.J.C.
P.M.U.

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APPLIED FINITE MATHEMATICS

1

NUMBERS, CALCULATORS, AND SETS

Gary and Georgia Vanning ate dinner with another couple, the Coopers. The Vannings paid $14.50 for the drinks in the bar before dinner. The bill for dinner (including sales tax of 5%) was $107.50. The Coopers and the Vannings decide to split the entire cost fifty–fifty (each couple will pay one half the total). The tip should be 15% of the cost of the dinner *without* tax. How much should each couple pay? How much more do the Vannings need to pay? (To be solved in Example 1, Section 1.1.)

PROBLEM FROM LIFE

INTRODUCTION

The world today, especially in the area of business, is far more complex than it was 30 years ago. The problems you encounter today are complicated because the world around you is not only complex but also changing rapidly. Approaches that might have been acceptable 30 years ago are likely to be inadequate today. Thus the more tools that you can use to make decisions, the better your decisions will be. Some of the tools that are currently used in business include accounting, economics, finance, statistics, operations research, and computers. To understand and utilize these tools properly, you **must** have the appropriate mathematical skills. We intend to help you learn these skills.

In this chapter, we'll begin a review of the basics that you'll need to be able to understand the material in this book. The chapter includes a review of the real number system and of sets and also a section that will help you use your calculator more effectively. In addition, we'll begin the chapter with a brief discussion of problem solving and modeling and how they relate to the mathematics you'll learn with this book.

1.1

PROBLEM SOLVING AND MODELING

As you go through this book, we not only want you to learn a set of skills, but we also want you to develop the ability to use these skills to solve problems. Here's a simple description of a quantitative approach to problem solving.

A Quantitative Approach to Solving Problems

1. Define the problem. This requires you to wade through the detail and determine what needs to be done.
2. Build a mathematical model of the problem. A **model** is an abstraction of the real world. Thus a mathematical model is a set of symbols and relationships that gives a reasonable portrayal of the real problem.
3. Analyze the model and determine the solution.
4. Implement the solution in the real world. This requires you to translate your paper solution into something that is meaningful in actual practice.

Most people believe that step 3, analyzing the model (performing the calculations), is the hardest part of problem solving. Actually, it's one of the easier parts. Steps 1 and 2, defining the problem and then converting it into symbols, are more difficult. As you work your way through the book, try to learn not only how to solve a problem, but how to recognize it and how to model it. We'll help you achieve this by providing practice in modeling and problem recognition.

You should have noticed the Problem from Life that began the chapter. We'll begin every chapter with one, which we'll analyze after you've learned the necessary skills. Now, let's take a look at the dinner problem presented to the Vannings and the Coopers. Although we won't solve it now, we do want to take you through the first two steps of our problem-solving approach.

EXAMPLE 1

Problem from Life Garry and Georgia Vanning ate dinner with another couple, the Coopers. The Vannings paid $14.50 for the drinks in the bar before dinner. The bill for dinner (including sales tax of 5%) was $107.50. The Coopers and the Vannings decide to split the entire cost fifty-fifty (each couple will pay one half the total). The tip should be 15% of the cost of the dinner *without* tax. How much should each couple pay? How much more should the Vannings pay?

SOLUTION

How can we solve this problem? It certainly doesn't appear to require any advanced mathematics. In fact, it is quite easy to solve once you devote some thought to it. The first step of our four-step approach is to define the problem. In this case it means that you should begin by *reading the problem completely and carefully*. Determine what value(s) you must know in order to solve the problem. Make note of the facts you do know.

Let's try that. The next-to-last sentence tells us that we want to determine how much each couple should pay. This means that we need to compute the total cost of the dinner (including drinks), because each couple should pay one half the total. Once we know the total cost of the dinner, it will be easy to compute how much more the Vannings owe. We know the following facts;

1. The cost of the drinks before dinner was \$14.50 (paid by the Vannings).
2. The cost of the dinner including tax was \$107.50.
3. The tax on the meal is 5%.
4. The tip should be 15% of the cost of the meal (not including tax).

The second step in the four-step approach is to develop a mathematical model. A good way to begin is to develop a simple *verbal* description of how to combine the different values to arrive at the solution. Once you have done this, you can decide which facts are relevant and which values are missing. It will then be much easier to develop the mathematical model. Don't be discouraged if your first model turns out to be incorrect. Sometimes you have to try several models. If a model does turn out to be incorrect (or inadequate), think about what's wrong with it before you revise it. In this case, we might develop the model shown in Fig. 1.1. Study it for a moment. Is it correct?

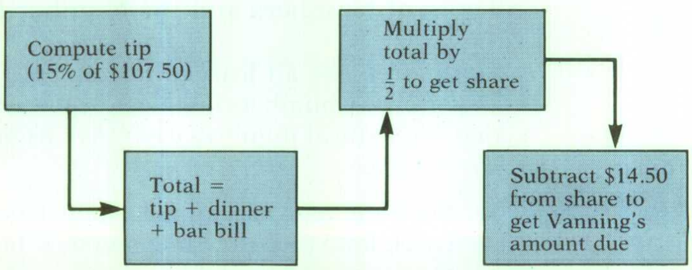

Figure 1.1 First model, Vanning-Cooper dinner (Example 1).

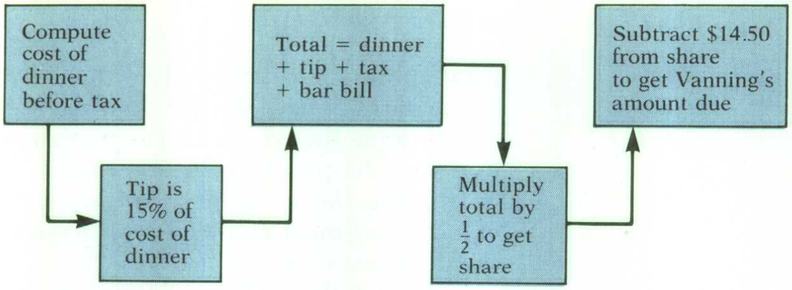

Figure 1.2 Second model, Vanning-Cooper dinner (Example 1).

Congratulate yourself if you said no. In Fig. 1.1 the tip is computed with the tax included. The correct method is to determine the tax, subtract it from the given cost, and then compute the tip. (See Fig. 1.2.)

For this example, there is no need to introduce a formal model with symbols and mathematical notation. You could proceed to calculate the answer directly from the model shown in Fig. 1.2 (step 3). (We'll do so in Section 1.3 of this chapter when we discuss calculators.) Moreover, because the situation described is so simple, there would be no difficulty in deciding how to arrive at a practical solution based on your mathematical model. □

1.2 THE REAL NUMBER SYSTEM

Numbers are a very important part of the language of mathematics. In order to be able to use numbers properly in both modeling and solving problems, you must be familiar with the "grammar" of numbers: what is allowed and what isn't allowed. You've seen various types of numbers such as 2, -3, $\frac{3}{8}$, 1.85, $\sqrt{2}$, and π (pi). Let's see which ones we need and how they relate to one another.

Types of Numbers and the Number Line: Rounding Off

As children, we all learned to count with the numbers 1, 2, 3, ... This group of numbers is called the **natural numbers.** We also learned that the natural numbers can't give us answers to certain questions. For instance,

1. Suppose your checking account has $100 in it and you write a check for $150. How much does it have in it now?
2. How can you divide two apples evenly among three friends?

To answer the first question we must expand the natural numbers to include the number 0 and the negative numbers such as -1, -2, -3. . . . This expanded group is called the **integers.** Integers can be positive (the natural numbers), 0, or negative. To answer the second question, we must introduce numbers such as $\frac{2}{3}$. The **rational numbers** are ratios (quotients) of the form m/n, where both m and n are integers and $n \neq 0$ (n is not equal to 0). Thus numbers such as $\frac{1}{2}$, $\frac{5}{3}$, $-\frac{6}{7}$, and $-10/-3$ are all rational numbers. *We cannot divide by zero.* Quotients of the form $m/0$ are often called **undefined.**

WARNING

BE CAREFUL OF QUOTIENTS THAT INCLUDE 0

$$\frac{0}{n} = 0 \qquad \text{whenever } n \neq 0,$$

$$\frac{m}{0} \qquad \text{is undefined (}\textit{not}\text{ 1).}$$

We will find it very useful to represent numbers on a line called the **number line.** As Fig. 1.3 shows, we can associate each of the types of numbers we discussed with a point on the number line. The natural numbers lie to the right of 0. The inclusion of 0 and the negative integers extends the line to the left. The rational numbers are represented by points in between the integers. Do you think each of the points on the number line corresponds to one of the numbers we've discussed? Are there any "gaps" in the number line?

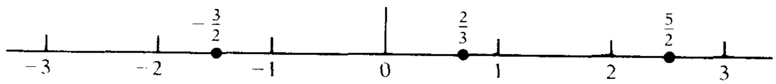

Figure 1.3 **The number line.**

In many cases we convert noninteger rational numbers to their **decimal equivalent.** This conversion is done by simply dividing the **denominator** (bottom) of the ratio into the **numerator** (top). Because a decimal number is just another way of expressing a rational number, it is graphed on the number line in the same place as its rational equivalent. (See Fig. 1.4.)

Some decimal numbers can be written with only one or two places to the right of the decimal point: For example, $\frac{1}{4}$ is 0.25, and

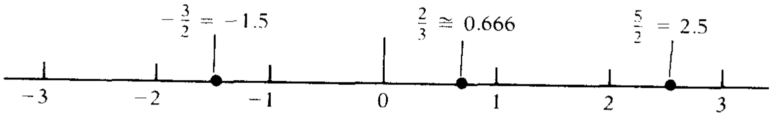

Figure 1.4 **Decimal numbers are equivalent to rational numbers.**

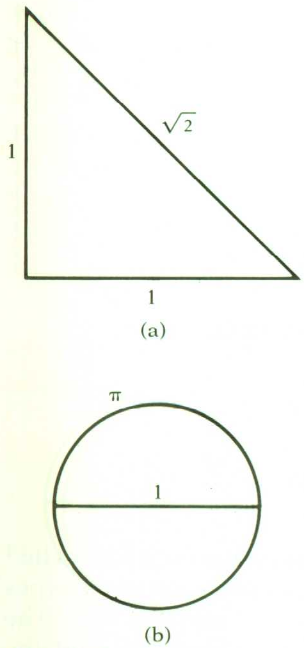

Figure 1.5 Irrational numbers can occur in simple geometrical figures.

$\frac{375}{1000}$ is 0.375. Other rational numbers require an infinite number of places to the right of the decimal place to be represented properly: For example, $\frac{1}{3} = 0.33\underline{3}$, where the underscore indicates that the digit 3 repeats endlessly. Sometimes a group of digits will repeat endlessly: For example, $\frac{1}{7} = 0.142857\underline{142857}$, where the group of digits 142857 repeats endlessly. Is it possible to have a decimal number that requires an endless number of places to the right of the decimal point and yet does *not* repeat? The answer to this question is yes. It is indeed possible for a decimal number with an endless number of places not to be repeating. Such numbers are called **irrational** because they cannot be expressed as the ratio of two integers. Moreover, the irrational numbers are needed to complete the number line because without them there would be gaps between the rational numbers.

You are probably familiar with several irrational numbers. Both $\sqrt{2}$ and π are irrational numbers that arise quite naturally from simple geometry. For example, suppose we construct a right triangle with both legs of length 1. By the Pythagorean theorem, the length of the hypotenuse (the side opposite the right angle) is the square root of the sum of the squares of the lengths of the other two sides. Thus we have a hypotenuse with length $\sqrt{1+1} = \sqrt{2}$. (See Fig. 1.5a.) As another example, consider a circle with a diameter of 1. The circumference of a circle is always π times the diameter, so our circle has a circumference of π. (See Fig. 1.5b.)

All the different groups of numbers we've discussed are referred to collectively as the **real numbers.** A representation of the real number system is shown in Fig. 1.6. The group of all decimal numbers is exactly the real number system if we include whole numbers (integers), decimal numbers that terminate or repeat endlessly (ra-

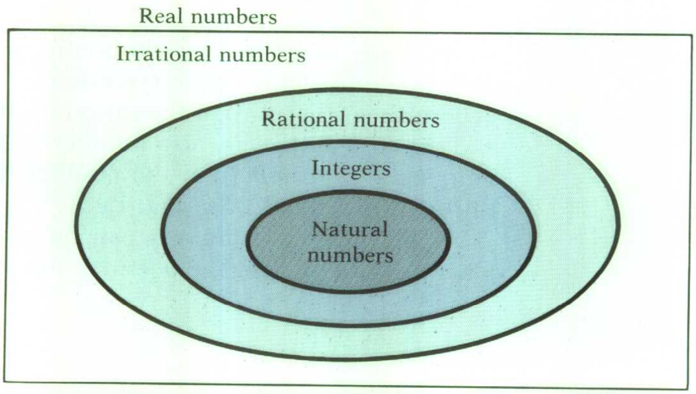

Figure 1.6 The real number system and its members.