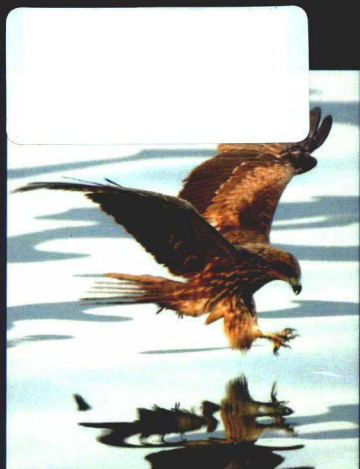


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# An Introduction to **FLAPPING WING AERODYNAMICS**



WEI SHYY • HIKARU AONO  
CHANG-KWON KANG • HAO LIU

CAMBRIDGE

# An Introduction to Flapping Wing Aerodynamics

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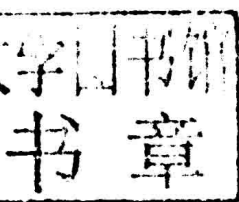
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# Preface

This book is about flapping wing aerodynamics. It presents various aspects of the aerodynamics of natural flyers, such as birds, bats, and insects, and of human-engineered micro air vehicles (MAVs) for both rigid and flexible wing structures. This edition focuses on the many recent developments since the publication of our earlier book titled *Aerodynamics of Low Reynolds Number Flyers*. We have substantially expanded Chapter 1 to offer a general and comprehensive introduction to low Reynolds number flight vehicles for both biological flyers and human-made MAVs. In particular, we summarize the scaling laws to relate the aerodynamics and various flight characteristics to a flyer's size, weight, and speed on the basis of simple geometric and dynamics analyses. In Chapter 2, closely following the previous edition, we discuss the aerodynamics of fixed rigid wings. It considers both two- and three-dimensional airfoils with typically low aspect ratio wings. Both Chapters 3 and 4 have been significantly expanded and updated. Chapter 3 examines the interplay between flapping kinematics and key dimensionless parameters such as the Reynolds number, Strouhal number, and reduced frequency for rigid wings. The various unsteady lift enhancement mechanisms are addressed, including leading-edge vortex, rapid pitch-up and rotational circulation, wake capture, tip vortices, and clap-and-fling. It also discusses both detailed time-dependent and simplified quasi-steady analyses along with experimental observations. Efforts have been made to contrast fixed and flapping wing aerodynamics in the context of geometry and tip, as well as of stall margins. Chapter 3 presents individual and varied objectives in regard to maximizing lift, mitigating drag, and minimizing power associated with flapping wings.

Chapter 4 addresses the role of structural flexibility of low Reynolds number wing aerodynamics. Due to the interplay between structural and fluid dynamics, additional dimensionless parameters appear, resulting in multiple time and length scales. For fixed wings, structural flexibility can further enhance stall margin and flight stability; for flapping wings, passive control can complement and possibly replace active pitching to make the flight more robust and more power efficient. Chapter 4 also discusses the airfoil shape, the time-dependent fluid and structural dynamics, and the spanwise versus chordwise flexibility of a wing. The scaling laws linking lift and power with fluid and structural parameters are of fundamental interest and offer insight into low Reynolds number flight sciences while providing guidelines for

vehicle development. Finally, recent advances and future perspectives are summarized and presented in Chapter 5.

As in the previous edition, we have benefited from collaborations and interactions with many colleagues. In addition to those colleagues named in the previous edition, we would like to acknowledge the generous intellectual and financial support provided by the U.S. Air Force Research Laboratory, in particular the Flight Vehicle Directorate (now Aerospace Systems Directorate) and the Office of Scientific Research.

We feel sure that significant advancements in both scientific and engineering endeavors of flapping wing aerodynamics will continue to be achieved, and we enthusiastically await these new breakthroughs and developments.

Wei Shyy, Hikaru Aono, Chang-kwon Kang, and Hao Liu

## Preface of the First Edition (*Aerodynamics of Low Reynolds Number Flyers*)

Low Reynolds number aerodynamics is important for a number of natural and man-made flyers. Birds, bats, and insects have been of interest to biologists for years, and active study in the aerospace engineering community has been increasing rapidly. Part of the reason is the advent of micro air vehicles (MAVs). With a maximal dimension of 15 cm and nominal flight speeds around 10 m/s, MAVs are capable of performing missions such as environmental monitoring, surveillance, and assessment in hostile environments. In contrast to civilian transport and many military flight vehicles, these small flyers operate in the low Reynolds number regime of  $10^5$  or lower. It is well established that the aerodynamic characteristics, such as the lift-to-drag ratio of a flight vehicle, change considerably between the low and high Reynolds number regimes. In particular, flow separation and laminar-turbulent transition can result in substantial change in effective airfoil shape and reduce aerodynamic performance. Since these flyers are lightweight and operate at low speeds, they are sensitive to wind gusts. Furthermore, their wing structures are flexible and tend to deform during flight. Consequently, the aero/fluid and structural dynamics of these flyers are closely linked to each other, making the entire flight vehicle difficult to analyze.

The primary focus of this book is on the aerodynamics associated with fixed and flapping wings. Chapter 1 offers a general introduction to low Reynolds flight vehicles, including both biological flyers and MAVs, followed by a summary of the scaling laws that relate the aerodynamics and flight characteristics to a flyer's sizing on the basis of simple geometric and dynamics analyses. Chapter 2 examines the aerodynamics of fixed, rigid wings. Both two- and three-dimensional airfoils with typically low aspect ratio wings are considered. Chapter 3 examines structural flexibility within the context of fixed wing aerodynamics. The implications of laminar-turbulent transition, multiple time scales, airfoil shapes, angles-of-attack, stall margin, structural flexibility, and time-dependent fluid and structural dynamics are highlighted.

Unsteady flapping wing aerodynamics is presented in Chapter 4. In particular, the interplay between flapping kinematics and key dimensionless parameters such as the Reynolds number, Strouhal number, and reduced frequency is examined. The various unsteady lift enhancement mechanisms are also addressed, including



leading-edge vortex, rapid pitch-up and rotational circulation, wake capture, and clap-and-fling.

The materials presented in this book are based on our own research, existing literature, and communications with colleagues. At different stages, we have benefited from collaborations and interactions with colleagues: Drs. Peter Ifju, David Jenkins, Rick Lind, Raphael Haftka, Roberto Albertani, and Bruce Carroll of the University of Florida; Drs. Luis Bernal, Carlos Cesnik, and Peretz Friedmann of the University of Michigan; Drs. Michael Ol, Miguel Visbal, and Gregg Abate, and Mr. Johnny Evers of the Air Force Research Laboratory; Dr. Ismet Gursul of the University of Bath; Dr. Charles Ellington of Cambridge University; Dr. Keiji Kawachi of the University of Tokyo; Mr. Hikaru Aono of Chiba University; Dr. Mao Sun of the Beijing University of Aeronautics and Astronautics. In particular, we have followed the flight vehicle development efforts of Dr. Peter Ifju and his group and enjoyed the synergy between us.

MAV and biological flight is now an active and well-integrated research area, attracting participation from a wide range of talents and specialties. The complementary perspectives of researchers with different training and backgrounds enable us to develop new biological insight, mathematical models, physical interpretation, experimental techniques, and design concepts.

Thinking back to the time we started our own endeavor a little more than ten years ago, substantial progress has taken place, and there is every expectation that significantly more will occur in the foreseeable future. We look forward to it!

Wei Shyy, Yongsheng Lian, and Jian Tang Dragos Viieru  
Ann Arbor, Michigan, U.S.A.

Hao Liu  
Chiba, Japan  
December 31, 2006

# List of Abbreviations

<i>Abbreviation</i>	<i>Definition</i>
2D	two-dimensional
3D	three-dimensional
AoA	angle of attack
DNS	direct numerical simulation
LES	large-eddy simulation
LEV	leading-edge vortex
LSB	laminar separation bubble
MAV	micro air vehicle
PIV	particle image velocimetry
RANS	Reynolds-averaged Navier-Stokes
TEV	trailing-edge vortex
TiV	tip vortex
UAV	unmanned air vehicle



# Nomenclature

$AR$	aspect ratio	Eq. (1–7)
$b$	wingspan	Eq. (1–7)
$c$	chord length	Eq. (1–19)
$\mathbf{e}_3$	unit vector in the direction from the leading edge to the trailing edge	Eq. (4–28)
$C_D$	drag coefficient	Eq. (2–22)
$C_{D,F}$	drag coefficient due to skin friction	Eq. (2–22)
$C_{D,P}$	drag coefficient due to pressure	Eq. (2–22)
$C_F$	force coefficient	Eq. (3–35)
$C_L$	lift coefficient	Eq. (1–1)
$C_T$	tension coefficient, thrust coefficient	Eqs. (3–23) and (4–2)
$D_{\text{aero}}$	aerodynamic drag	Eq. (1–29)
$D_{\text{ind}}$	induced drag	Eq. (1–29)
$D_{\text{par}}$	parasite drag (drag on the body)	Eq. (1–29)
$D_{\text{pro}}$	profile drag	Eq. (1–29)
$D_w$	drag on a finite wing	Eq. (1–28)
$e$	span efficiency factor	Eq. (2–22)
$E$	elastic modulus	Eq. (4–1)
$f$	flapping (wing-beat) frequency	Eq. (1–12)
$f_{\text{ext}}$	distributed external force per unit	Eq. (4–1)
$f_n$	natural frequency	Eq. (1–21)
$g$	gravitational acceleration	Eq. (1–3)
$h_a$	flapping amplitude	Eq. (3–4)
$h_s$	thickness of wing, thickness of membrane	Eqs. (4–1) and (4–8)
$h(t)$	time-dependent flapping displacement	Eq. (3–4)
$H$	shape factor	Eq. (2–2)
$I$	moment of inertia	Eq. (1–10)
$J$	advance ratio	Eq. (3–14)
$J_T$	torque	Eq. (1–9)
$k$	reduced frequency, turbulent kinetic energy	Eqs. (1–19) and (2–6)
$l$	characteristic length	Eq. (1–4)
$L$	lift, length of membrane after deformation	Eqs. (1–1) and (4–10)

$L_0$	unstrained membrane length	Eq. (4–3)
$L/D$	lift-to-drag ratio or glide ratio	Eq. (2–20)
$m$	flyer's total mass	Eq. (1–3)
$\tilde{n}$	amplification factor	Eq. (2–12)
$N$	threshold value that triggers turbulent flow in $e^N$ method	Eq. (2–17)
$p$	static pressure	Eq. (2–5)
$P_{\text{aero}}$	total aerodynamic power	Eq. (1–30)
$P_{\text{ind}}$	induced power	Eq. (1–32)
$P_{\text{iner}}$	inertial power	Eq. (1–33)
$P_{\text{par}}$	parasite power	Eq. (1–32)
$P_{\text{pro}}$	profile power	Eq. (1–32)
$P_{\text{tot}}$	total power required for flight	Eq. (1–33)
$q_\infty$	far field dynamic pressure	Eq. (4–13)
$R$	wing length	Eq. (3–24)
$Re$	Reynolds number	
$Re_{f2}$	Reynolds number for 2D flapping motion	Eqs. (3–8a) and (3–8b)
$Re_{f3}$	Reynolds number for 3D flapping motion	Eq. (3–7)
$Re_T$	turbulent Reynolds number	Eq. (2–10)
$Re_\theta$	momentum thickness Reynolds number	Eq. (2–12)
$S$	wing area	Eq. (1–1)
$S^0$	membrane prestress	Eq. (4–8)
$St$	Strouhal number	Eq. (3–9)
$t$	time	Eq. (2–5)
$T$	wing stroke time scale, thrust	Eqs. (1–12) and (1–31)
$u_i$	velocity vector in Cartesian coordinates	Eq. (2–4)
$U$	forward flight velocity (free-stream velocity)	Eq. (1–1)
$U_f$	flapping velocity	Eq. (1–20)
$\dot{U}_{mp}$	velocity for minimum power (forward flight)	Eq. (1–35)
$U_{Mr}$	velocity for maximum range (forward flight)	Eq. (1–35)
$U_r$	relative flow velocity	Eq. (1–20)
$U_{\text{ref}}$	reference velocity	Eq. (1–19)
$w$	vertical velocity in the far wake, transverse deflection	Eqs. (3–26) and (4–1)
$w_i$	downwash (induced) velocity	Eq. (1–14)
$W$	weight	Eq. (1–1)
$W/S$	wing loading	Eq. (1–2)
$x_i$	spatial coordinates vector	Eq. (2–4)
$\alpha$	angle of attack, feathering angle (pitch angle) of a flapping wing	Eqs. (3–3) and (3–5)
$\beta$	stroke plane angle	Eq. (3–25)
$\delta^*$	boundary-layer displacement thickness	Eq. (2–3)
$\phi$	positional angle of a flapping wing	Eq. (3–15)
$\Phi$	stroke angular amplitude	Eq. (3–7)

$\mu$	Coefficient of dynamic viscosity	Table 4.2
$\gamma$	membrane tension, non-dimensional tip, deformation parameter	Eqs. (4–4) and (4–34)
$\Gamma$	circulation	Eq. (2–23)
$\phi$	phase difference between plunging and pitching motion	Eq. (3–5)
$\nu$	kinematic viscosity, Poisson’s ratio	Eqs. (2–5) and (4–23)
$\nu_{Te}$	effective eddy viscosity	Eq. (2–18)
$\nu_T$	turbulent eddy viscosity	Eq. (2–6)
$\Pi_0$	effective inertia	Eq. (4–32)
$\Pi_1$	effective stiffness	Eq. (4–15)
$\Pi_{1, \text{pret}}$	effective pretension	Eq. (4–17)
$\Pi_2$	effective rotational inertia	Eq. (4–25)
$\theta$	gliding angle, boundary-layer momentum thickness, elevation angle of a flapping wing	Eqs. (1–17), (2–5), and Eq. (3–2)
$\rho$	fluid density	Eq. (1–1)
$\rho_s$	structural density	Eq. (4–1)
$\tau_{ij}$	Reynolds-stress tensor	Eq. (2–6)
$\omega$	dissipation rate for k- $\omega$ turbulence model	Eq. (2–7)
$\omega_n$	natural angular frequency of the beam model	Eq. (4–33)
$\dot{\omega}$	angular acceleration	Eq. (1–11)
$\eta$	propulsive efficiency for forward flight	Eq. (4–41)
$\Psi$	the bending angle	Eq. (4–29)
$()^*$	non-dimensional quantity	
$\langle \rangle$	time-averaged quantity	

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Bird, bat, and insect flight has fascinated humans for many centuries. As enthusiastically observed by Dial [1], most species of animals fly. Based on his acute observation of how birds fly, Leonardo da Vinci conceptualized flying machines, which can be seen in documents such as the *Codex on the Flight of Birds*, published circa 1505 [2]; some illustrations of his work are shown in Figure 1.1. Otto Lilienthal was among the most dedicated and successful creators of flying machines at the dawn of human flight. He designed and demonstrated many hang gliders (see Fig. 1.2). Unfortunately, Lilienthal lacked sufficient knowledge of the science of flight and was killed in a fatal fall. For those who wish to explore in greater detail the history and the technology of early flight, John Anderson's *Inventing Flight* [3] offers interesting and well-documented information. Of course, there are ample records of humankind's interest in natural flyers from the artistic angle. Figure 1.3 shows four examples: (Figure 1.3a) decorative art done about 2,500 years ago, in China's Warring Period; (Figure 1.3b) a bronze crane model uncovered from the First Emperor's grave, who died in 210 BC; a pair of bas-reliefs (Figure 1.3c,d) uncovered from the Assyrian palace in today's Iraq, dated back to the 8th century BC; (Figure 1.3e) a stone sculpture of a standing owl from the Shang Dynasty, China, created in the 12th century BC or earlier!

There are nearly a million species of flying insects, and of the non-insects, another 13,000 warm-blooded vertebrate species (including mammals, about 9,000 species of birds, and 1,000 species of bats) take to the skies. In their ability to maneuver a body efficiently through space, birds, bats, and insects represent one of nature's finest locomotion experiments. Although aeronautical technology has advanced rapidly over the past 100 years, nature's flying machines, which have evolved over 150 million years, are still impressive. Considering that humans move at top speeds of 3–4 body lengths per second, a race horse runs approximately 7 body lengths per second, a cheetah accomplishes 18 body lengths per second [4], and a supersonic aircraft such as the SR-71 "Blackbird" traveling near Mach 3 ( $\sim 900$  m/s) covers about 32 body lengths per second, it is remarkable that a Common Pigeon (*Columba livia*) frequently attains speeds of 22.4 m/s, which converts to 75 body lengths per second. A European Starling (*Sturnus vulgaris*) is capable of flying at 120 body lengths per second, and various species of swifts are even faster, flying more than 140 body lengths per second. Whereas the roll rate of highly aerobatic aircraft (e.g., A-4 Skyhawk) is approximately  $720^\circ/\text{s}$ , a Barn Swallow (*Hirundo rustics*) has a roll



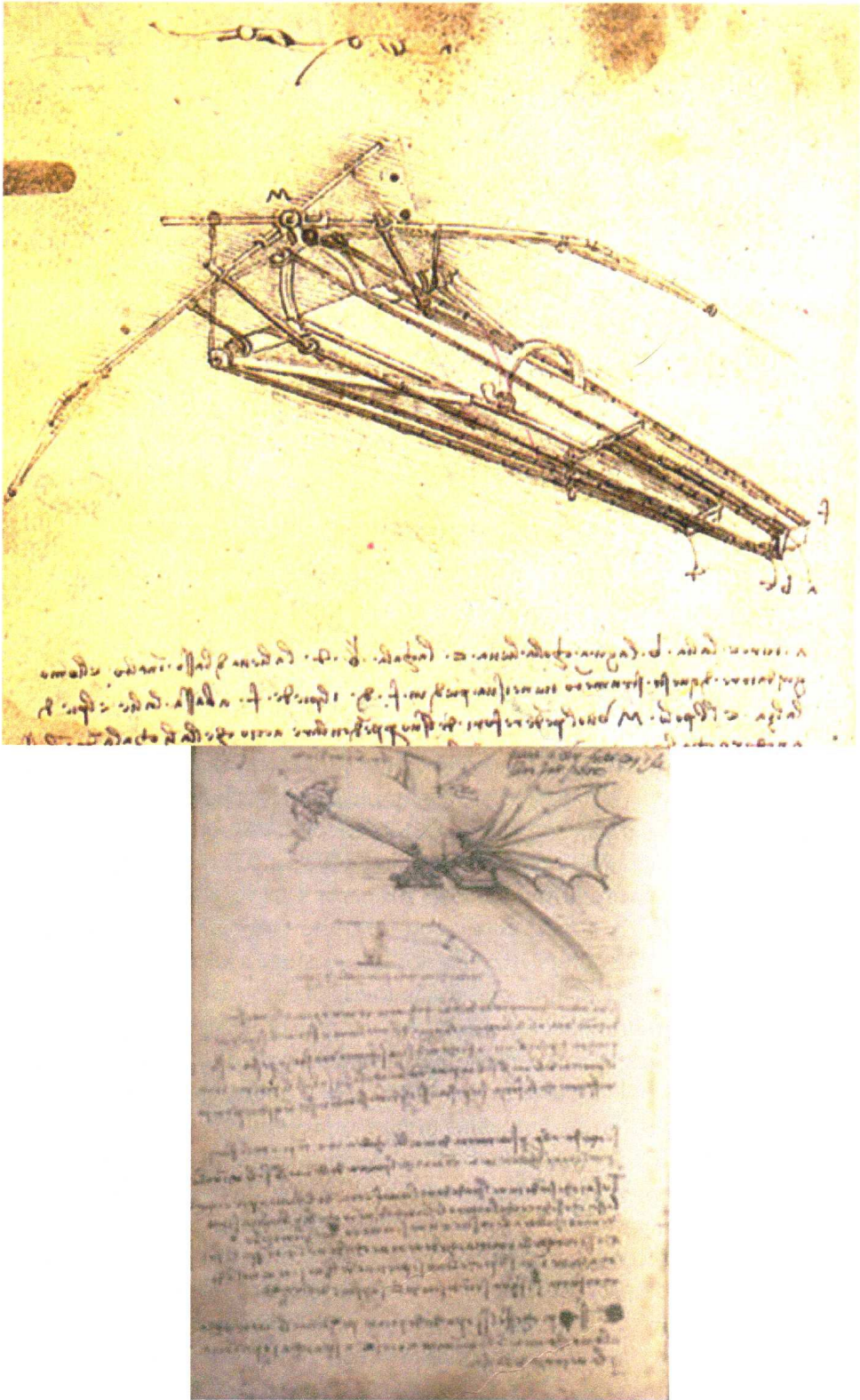


Figure 1.1. A drawing of a design for a flying machine by Leonardo da Vinci (c. 1488). This machine was an ornithopter, with flapping wings similar to a bird, first presented in his *Codex on the Flight of Birds* circa 1505 [2].





Figure 1.2. German engineer Otto Lilienthal flies his hang glider some 2,000 times during 1891–1896 before a fatal fall [3].

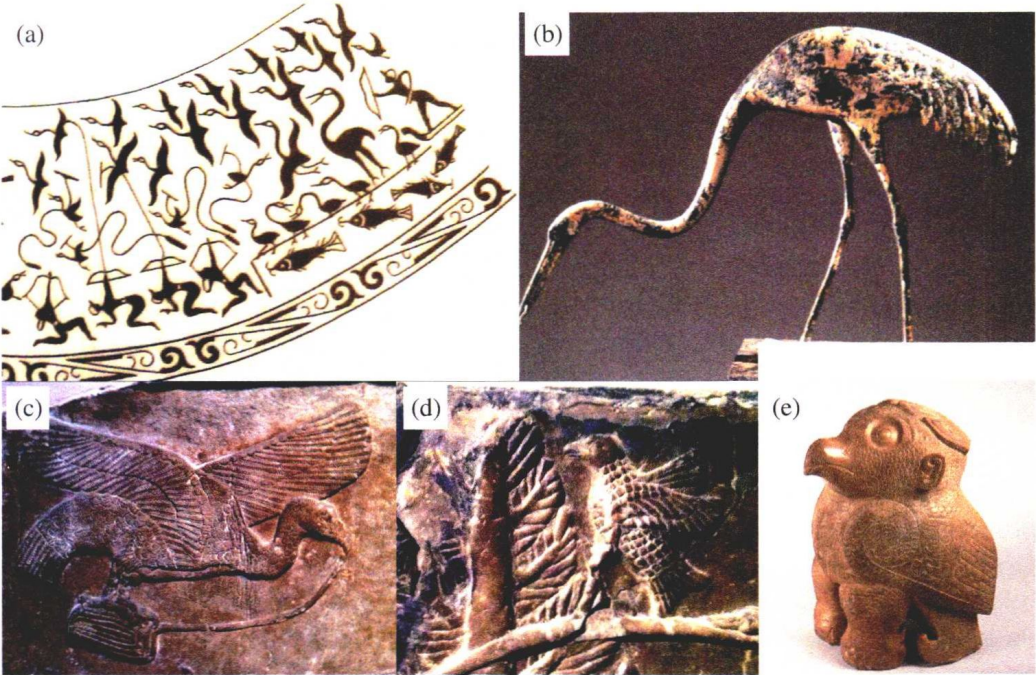


Figure 1.3. Birds recorded in early human history: (a) design of a wine vessel, early Warring period (475-early fourth century BC), China (Shanghai Museum, Shanghai); (b) a bronze crane-eating fish, uncovered inside the First Emperor’s grave site, Xian, China (Museum of Emperor QinShihuang, Xian); (c,d) Assyrian bas-reliefs, circa eighth century BC (British Museum, London); (e) Stone sculpture of a standing owl, Shang Dynasty, China, around the 12th century BC or earlier (Academia Sinica, Taipei).