

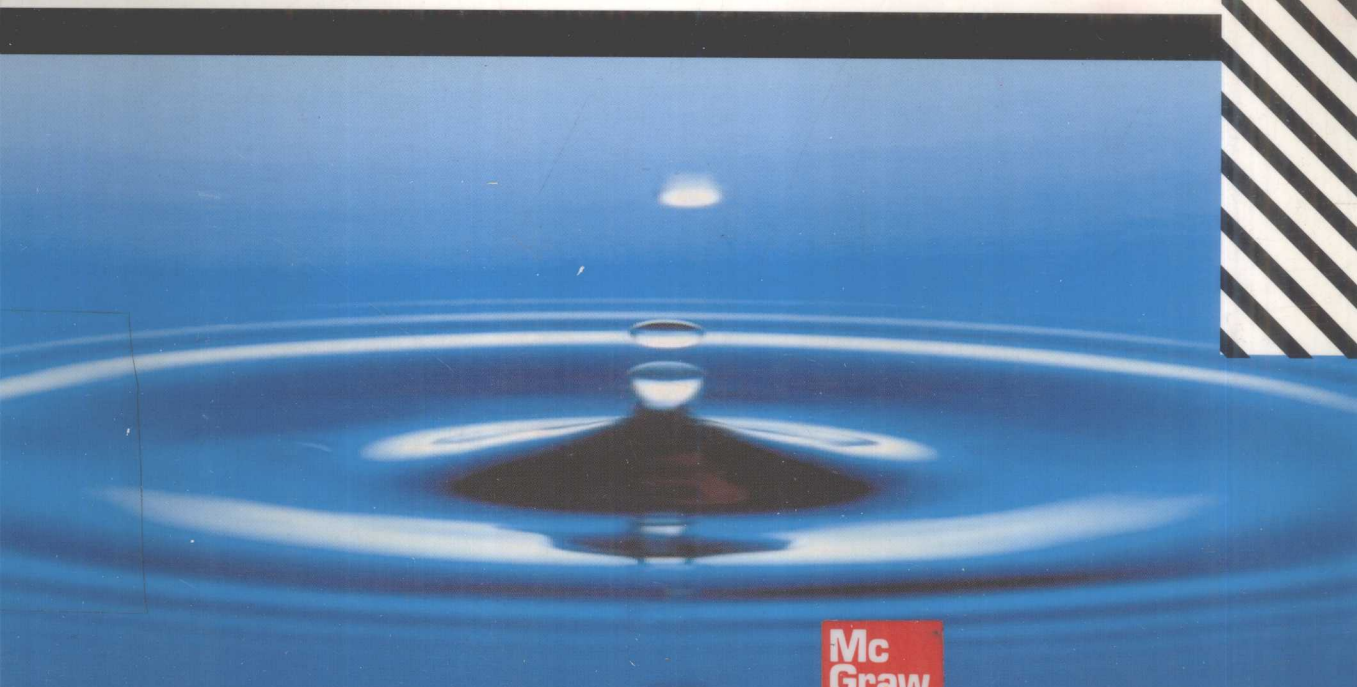
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大学计算机教育国外著名教材、教参系列 (影印版)

Introduction to Logic Design

Alan B. Marcovitz

逻辑设计基础



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清华大学出版社

(京) 新登字 158 号

Introduction to Logic Design
Alan B. Marcovitz

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书 名：Introduction to Logic Design
作 者：Alan B. Marcovitz
出版者：清华大学出版社（北京清华大学学研大厦，邮编 100084）
[http:// www.tup.tsinghua.edu.cn](http://www.tup.tsinghua.edu.cn)
印刷者：清华大学印刷厂
发行者：新华书店总店北京发行所
开 本：787×960 1/16 印张：36.5
版 次：2002 年 8 月第 1 版 2002 年 8 月第 1 次印刷
书 号：ISBN 7-302-05717-6/TP·3374
印 数：0001~4000
定 价：50.00 元

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2002.3

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10. Introduction to Logic Design 2002/Alan B. Marcovitz (逻辑设计基础 584 页)

This book is intended as an introductory logic design book for students in computer science, computer engineering, and electrical engineering. It has no prerequisites, although the maturity attained through an introduction to engineering course or a first programming course would be helpful.

The book stresses fundamentals. It teaches through a large number of examples. The philosophy of the author is that the only way to learn logic design is to do a large number of design problems. Thus, in addition to the numerous examples in the body of the text, each chapter has a set of Solved Problems, that is, problems and their solutions, as well as a large set of Exercises. In addition, there are a set of laboratory experiments that tie the theory to the real world. The Appendix provides the background to do these experiments with a standard hardware laboratory (chips, switches, lights, and wires), a breadboard simulator (for the PC or Macintosh), and two schematic capture tools. The course can be taught without the laboratory, but the student will benefit significantly from the addition of 8–10 selected experiments.

Although computer-aided tools are widely used for the design of large systems, the student must first understand the basics. The basics provide more than enough material for a first course. The schematic capture laboratory exercises and a section on Hardware Design Languages in Chapter 6 provide some material for a transition to a second course based on one of the computer-aided tool sets.

Chapter 1 gives a brief overview of number systems as it applies to the material of this book. (Those students who have studied this in an earlier course can skip to Section 1.2.) It then discusses the steps in the design process for combinational systems and the development of truth tables.

Chapter 2 introduces switching algebra and the implementation of switching functions using common gates—AND, OR, NOT, NAND, NOR, and Exclusive-OR. We are only concerned with the logic behavior of the gates, not the electronic implementation.

Chapter 3 deals with two simplification techniques, the Karnaugh map and Iterated Consensus. It provides methods for solving problems (up to six variables with the map) with both single and multiple outputs.

Chapter 4 is concerned with the design of larger combinational systems. It introduces a number of commercially available larger devices, including adders, decoders, encoders and priority encoders, and multiplexers. That is followed by a discussion of the use of logic arrays—ROMs, PLAs, and PALs for the implementation of medium-scale combinational systems. Finally, two larger systems are designed.

Chapter 5 introduces sequential systems. It starts by examining the behavior of latches and flip flops. The design process for sequential systems is then presented. Before designing sequential systems, several systems are analyzed. The special case of counters is studied next. Finally, the solution of word problems, developing the state table or state diagram from a verbal description of the problem is presented in detail.

Chapter 6 looks at larger sequential systems. It starts by examining the design of shift registers and counters. Then, PLDs are presented. Two techniques that are useful in the design of more complex systems, ASM diagrams and HDLs, are discussed next. Finally, two examples of larger systems are presented.

Chapter 7 deals with state reduction and state assignment issues. First, a tabular approach for state reduction is presented. Then partitions are utilized both for state reduction and for achieving a state assignment that will utilize less combinational logic.

A feature of this text is the Solved Problems. Each chapter has a large number of problems, illustrating the techniques developed in the body of the text, followed by a detailed solution of each problem. Students are urged to solve each problem (without looking at the answer) and then compare their solution with the one shown.

Each chapter concludes with a large set of exercises. Solution to these will be made available through the Web.

Another unique feature of the book is the laboratory exercises, included in the Appendix. Four platforms are presented—a hardware based Logic Lab (using chips, wires, etc.); a hardware lab simulator that allows the student to “connect” wires on the computer screen; and two circuit capture programs, LogicWorks IV and Altera Max+plus II. Enough information is provided about each to allow the student to perform a variety of experiments. A set of 25 laboratory exercises are presented. Several of these have options, to allow the instructor to change the details from one term to the next.

We teach this material as a 4-credit course that includes an average of 3 1/2 hours per week of lecture, plus, typically, eight laboratory exercises. (The lab is unscheduled; it is manned by Graduate Assistants 40 hours per week; they grade the labs.) In that course we cover

Chapter 1: all of it

Chapter 2: all but 2.11

Chapter 3: all of 3.1

Chapter 4: all but 4.8. However, there is a graded design problem based on that material (10 percent of the grade; students usually working in groups of 2 or 3).

Chapter 5: all, though sometimes we skip 5.6

Chapter 6: 6.1, 6.2, and 6.3. We sometimes have a second project based on 6.6.

Chapter 7 and Section 3.2: We often have some time to look at one of these. We have never been able to cover both.

With less time, the coverage of Section 2.10 could be minimized. Section 3.1.5 is not needed for continuity; Section 3.1.6 is used somewhat in the discussion of PLAs in Section 4.7.2. Chapter 4 is not needed for anything else in the text, although many of the topics are useful to students elsewhere. Sections 5.5 and 5.6 could be eliminated without loss of continuity. As is the case for Chapter 4, the instructor can pick and choose among the topics of Chapter 6. With a limited amount of time, Section 7.1 could be covered. With more time, it could be skipped and state reduction taught using partitions (7.2 and 7.3).

ACKNOWLEDGMENTS

I want to thank my wife, Allyn, for her encouragement and for enduring endless hours when I was closeted in my office working on the manuscript. Several of my colleagues at Florida Atlantic University have read parts of the manuscript and have taught from earlier drafts. I wish to acknowledge especially Mohammad Ilyas, Imad Mahgoub, Oge Marques, and Abhi Pandya for their help. In addition, I wish to express my appreciation to my two chairs, Mohammad Ilyas and Roy Levow, who made assignments that allowed me to work on the book. Even more importantly, I want to thank my students who provided me with the impetus to write a more suitable text, who suffered through earlier drafts of the book, and who made many suggestions and corrections. I want to thank Visram Rathnam for his contributions to the section on Altera tools. The reviewers

Nader I. Rafla, *Boise State University*;
Nicholas C. K. Phillips, *Southern Illinois University*;
Walter B. Ligon, III, *Clemson University*;
Jonathan Hill, *Worcester Polytechnic Institute*;
Anura Jayasumana, *Colorado State University*;
James K. Archibald, *Brigham Young University*;
Charles B. Silio, Jr., *University of Maryland*;
Gary J. Minden, *University of Kansas*;
and Michael McCool, *University of Waterloo*,

provided many useful comments and suggestions. The book is much better because of their efforts. Finally, the staff at McGraw-Hill, particularly Melinda Dougharty, Catherine Fields Shultz, Kay Brimeyer, Betsy Jones, Michelle Meerdink, Wayne Harms, Phil Meek, Sandy Ludovissy, Audrey Reiter, and John Wannemacher, have been indispensable in producing the final product, as have Brittney Corrigan-McElroy and her staff at Interactive Composition Corporation.

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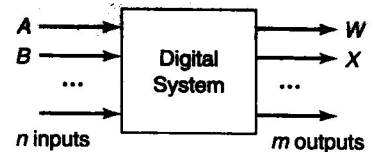
Introduction

This book concerns the design of digital systems, a process often referred to as logic design. A digital system is one in which all of the signals are represented by discrete values. Computers and calculators are obvious examples, but most electronic systems contain a large amount of digital logic. Internally, digital systems usually operate with two-valued signals, which we will label 0 and 1. Such a system, as shown in Figure 1.1, may have an arbitrary number of inputs (A, B, \dots) and an arbitrary number of outputs (W, X, \dots).

In addition to the data inputs shown, some circuits require a timing signal, called a clock (which is just another input signal that alternates between 0 and 1 at a regular rate). We will discuss the details of clock signals in Chapter 5.

A simple example of digital systems is shown in Example 1.1.

Figure 1.1 A digital system.



A system with three inputs, A, B , and C , and one output, Z , such that $Z = 1$ if and only if¹ two of the inputs are 1.

EXAMPLE 1.1

The inputs and outputs of a digital system represent real quantities. Sometimes, as in Example 1.1, these are naturally binary, that is, they take on one of two values. Other times, they may be multivalued. For example, an input may be a decimal digit or the output might be the letter grade for this course. Each must be represented by a set of binary digits (often called bits). This process is referred to as coding the inputs and outputs into binary. (We will discuss the details of this later.)

¹The term *if and only if* is often abbreviated *iff*. It means that the output is 1 if the condition is met and is not 1 (which means it must be 0) if the condition is not met.

The physical manifestation of these binary quantities may be one of two voltages, for example, 0 volts or ground for logic 0 and 5 volts for logic 1, as in the laboratory implementations we will be discussing in the Appendix. It may also be a magnetic field in one direction or another (as on diskettes), a switch in the up or down position (for an input), or a light on or off (as an output). Except in the discussion of specific laboratory experiments and in the translation of verbal descriptions into more formal ones, the physical representation will be irrelevant in this text; we will be concerned with 0's and 1's.

Table 1.1 A truth table for Example 1.1.

A	B	C	Z
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

We can describe the behavior of a digital system, such as that of Example 1.1, in tabular form. Since there are only eight possible input combinations, we can list all of them and what the output is for each. Such a table (referred to as a truth table) is shown in Table 1.1. We will leave the development of truth tables (including one similar to this) to later in the chapter.

Two other examples are given in Examples 1.2 and 1.3.

EXAMPLE 1.2

A system with eight inputs, representing two 4-bit binary numbers, and one 5-bit output, representing the sum. (Each input number can range from 0 to 15; the output can range from 0 to 30.)

EXAMPLE 1.3

A system with one input, *A*, plus a clock, and one output, *Z*, which is 1 iff the input was one at the last three consecutive clock times.

The first two examples are *combinational*, that is, the output depends only on the present value of the input. In the Example 1.1, if we know the value of *A*, *B*, and *C* right now, we can determine what *Z* is now.² Example 1.3 is *sequential*, that is, it requires *memory*, since we need to know something about inputs at an earlier time (previous clock times).

We will concentrate on combinational systems in the first half of the book and leave the discussion about sequential systems until later. As we will see, sequential systems are composed of two parts, memory and combinational logic. Thus, we need to be able to design combinational systems before we can begin designing sequential ones.

A word of caution about natural language in general, and English in particular, is in order. English is not a very precise language. The examples given above leave some room for interpretation. In Example 1.1, is the output to be 1 if all three of the inputs are 1, or only if exactly two

²In a real system, there is a small amount of delay between the input and output, that is, if the input changes at some point in time, the output changes a little after that. The time frame is typically in the nanosecond (10^{-9} sec) range. We will ignore those delays almost all of the time, but we will return to that issue in Chapter 4.

inputs are 1? One could interpret the statement either way. When we wrote the truth table, we had to decide; we interpreted “two” as “two or more” and thus made the output 1 when all three inputs were 1. (In problems in this text, we will try to be as precise as possible, but even then, different people may read the problem statement in different ways.)

The bottom line is that we need a more precise description of logic systems. We will develop that for combinational systems in the first two chapters and for sequential systems in Chapter 5. ■

1.1 A BRIEF REVIEW OF NUMBER SYSTEMS

This section gives an introduction to some topics in number systems, primarily those needed to understand the material in the remainder of the book. We will only deal with integers. If this is familiar material from another course, skip to Section 1.2.

Integers are normally written using a positional number system, where each digit represents the coefficient in a power series

$$N = a_{n-1}r^{n-1} + a_{n-2}r^{n-2} + \cdots + a_2r^2 + a_1r + a_0$$

where n is the number of digits, r is the radix or base, and the a_i are the coefficients, where each is an integer in the range

$$0 \leq a_i < r$$

For decimal, $r = 10$, and the a 's are in the range 0 to 9. For binary, $r = 2$, and the a 's are all either 0 or 1. Other commonly used notations in computer documentation are octal, $r = 8$, and hexadecimal, $r = 16$. In binary, the digits are usually referred to as *bits*, a contraction for *binary digits*.

The decimal number 7642 (sometimes written 7642_{10} to emphasize that it is radix 10, that is, decimal) thus stands for

$$7642_{10} = 7 \times 10^3 + 6 \times 10^2 + 4 \times 10 + 2$$

and the binary number

$$\begin{aligned} 101111_2 &= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2 + 1 \\ &= 32 + 8 + 4 + 2 + 1 = 47_{10} \end{aligned}$$

From this last example,³ it is clear how to convert from binary to decimal; just evaluate the power series. To do that easily, it is useful to know the powers of 2, rather than compute them each time they are needed.

³Section 1.6, Solved Problems, contains additional examples of each of the types of problems discussed in this chapter. There is a section of Solved Problems in each of the other chapters.

(It would save a great deal of time and effort if at least the first ten powers of 2 were memorized; the first 20 are shown in the Table 1.2.)

Table 1.2 Powers of 2.

n	2^n	n	2^n
1	2	11	2,048
2	4	12	4,096
3	8	13	8,192
4	16	14	16,384
5	32	15	32,768
6	64	16	65,536
7	128	17	131,072
8	256	18	262,144
9	512	19	524,288
10	1,024	20	1,048,576

We will often be using the first 16 positive binary integers, and sometimes the first 32, as shown in the Table 1.3. (As in decimal, leading 0's are often left out, but we have shown the 4-bit number including leading 0's for the first 16.) When the size of the storage place for a positive binary number is specified, then leading 0's are added so as to obtain the correct number of bits.

Table 1.3 First 32 binary integers.

Decimal	Binary	4-bit	Decimal	Binary
0	0	0000	16	10000
1	1	0001	17	10001
2	10	0010	18	10010
3	11	0011	19	10011
4	100	0100	20	10100
5	101	0101	21	10101
6	110	0110	22	10110
7	111	0111	23	10111
8	1000	1000	24	11000
9	1001	1001	25	11001
10	1010	1010	26	11010
11	1011	1011	27	11011
12	1100	1100	28	11100
13	1101	1101	29	11101
14	1110	1110	30	11110
15	1111	1111	31	11111

Note that the number one less than 2^n consists of n 1's (for example, $2^4 - 1 = 1111 = 15$ and $2^5 - 1 = 11111 = 31$).

An n -bit number can represent the positive integers from 0 to $2^n - 1$. Thus, for example, 4-bit numbers have the range of 0 to 15, 8-bit numbers 0 to 255 and 16-bit numbers 0 to 65,535.

To convert from decimal to binary, we could evaluate the power series of the decimal number, by converting each digit to binary, that is

$$746 = 111 \times (1010)^{10} + 0100 \times 1010 + 0110$$

but that requires binary multiplication, which is rather time-consuming.

There are two straightforward algorithms using decimal arithmetic. First, we can subtract from the number the largest power of 2 less than that number and put a 1 in the corresponding position of the binary equivalent. We then repeat that with the remainder. A 0 is put in the position for those powers of 2 which are larger than the remainder.

For 746, $2^9 = 512$ is the largest power of 2 less than or equal to 746, and thus there is a 1 in the 2^9 (512) position. We then compute $746 - 512 = 234$. The next smaller power of 2 is $2^8 = 256$, but that is larger than 234 and thus, there is a 0 in the 2^8 position. Next, we compute $234 - 128 = 106$, putting a 1 in the 2^7 position. (Now, the binary number begins 101.) Continuing, we subtract 64 from 106, resulting in 42 and a 1 in the 2^6 position (and now the number begins with 1011). Since 42 is larger than 32, we have a 1 in the 2^5 position, and compute $42 - 32 = 10$. At this point, we can continue subtracting (8 next) or recognize that there is no $2^4 = 16$, and that the binary equivalent of the remainder, 10, is 1010, giving

$$\begin{aligned} 746_{10} &= 1 \times 2^9 + 0 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 \\ &\quad + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 0 \\ &= 1011101010_2 \end{aligned}$$

EXAMPLE 1.4

The other approach is to divide the decimal number by 2 repeatedly. The remainder each time gives a digit of the binary answer, starting at the least significant bit (a_0). The remainder is then discarded and the process is repeated.

Converting 746 from decimal to binary, we compute $746/2 = 373$ with a remainder of 0. Then, $373/2 = 186$ with a remainder of 1, giving the last two bits of the answer as 10. Continuing, $186/2 = 93$ with a remainder of 0, $93/2 = 46$, remainder 1; $46/2 = 23$, remainder 0, giving 01010 so far; $23/2 = 11$, remainder 1, $11/2 = 5$, remainder 1; $5/2 = 2$, remainder 1; $2/2 = 1$, remainder 0; and $1/2 = 0$, remainder 1. Thus the answer is 1011101010 as before. In this method, we could also stop when we recognize the number that is left and convert it to binary. Thus, when we had 23, we could recognize that as 10111 (from Table 1.3) and place that in front of the bits we had produced, giving 1011101010.

EXAMPLE 1.5

EXAMPLE 1.6

Convert 105 to binary

$$\begin{array}{rll}
 105/2 = 52, \text{ rem } 1 & \text{produces} & 1 \\
 52/2 = 26, \text{ rem } 0 & & 01 \\
 26/2 = 13, \text{ rem } 0 & & 001 \\
 \text{but } 13 = 1101 & & 1101\ 001
 \end{array}$$

The method works because all of the terms in the power series except the last divide evenly by 2. Thus,

$$\begin{aligned}
 746/2 &= 373 \text{ and remainder of } 0 \\
 &= 1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 \\
 &\quad + 1 \times 2^2 + 0 \times 2 + 1 + \text{rem } 0
 \end{aligned}$$

The last bit became the remainder. If we repeat the process, we get

$$\begin{aligned}
 373/2 &= 186 \text{ and remainder of } 1 \\
 &= 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 \\
 &\quad + 0 \times 2^2 + 1 \times 2 + 0 + \text{rem } 1
 \end{aligned}$$

That remainder is the second digit from the right. On the next division, the remainder will be 0, the third digit. This process continues until the last bit is found.

[SP 1, 2; EX 1, 2]⁴

1.1.1 Octal and Hexadecimal⁵

Octal ($r = 8$) and *hexadecimal*, often referred to as *hex* ($r = 16$) are two other bases that are commonly used in computer documentation. Each is just a shorthand notation for binary. In octal, binary digits are grouped in threes (starting at the least significant). For example, a 9-bit number,

$$\begin{aligned}
 N &= (b_8 2^8 + b_7 2^7 + b_6 2^6) + (b_5 2^5 + b_4 2^4 + b_3 2^3) \\
 &\quad + (b_2 2^2 + b_1 2^1 + b_0) \\
 &= 2^6(b_8 2^2 + b_7 2^1 + b_6) + 2^3(b_5 2^2 + b_4 2^1 + b_3) \\
 &\quad + (b_2 2^2 + b_1 2^1 + b_0) \\
 &= 8^2 o_2 + 8 o_1 + o_0
 \end{aligned}$$

where the o_i represent the octal digits and must fall in the range 0 to 7. Each term in parentheses is just interpreted in decimal. If the binary number does not have a multiple of 3 bits, leading 0's are added.

⁴At the end of most sections, a list of solved problems and exercises that are appropriate to that section is given.

⁵This section may be omitted; the material is not needed elsewhere in the text.

(from Examples 1.4 and 1.5)

$$\begin{aligned} 1011101010_2 &= 001\ 011\ 101\ 010_2 \\ &= 1\ 3\ 5\ 2_8 \end{aligned}$$

EXAMPLE 1.7

To convert from octal to binary, we just replace each octal digit by its 3-bit binary equivalent, the inverse of the last step in Example 1.7.

To convert from octal to decimal, we can evaluate the power series (where the powers of 8 can be obtained from Table 1.2 since $8^i = 2^{3i}$).

$$\begin{aligned} 1352_8 &= 1 \times 8^3 + 3 \times 8^2 + 5 \times 8 + 2 \\ &= 512 + 3 \times 64 + 40 + 2 \\ &= 746_{10} \end{aligned}$$

EXAMPLE 1.8

To convert from decimal to octal, we can first convert to binary, or we can (more easily) adapt the second algorithm used to convert from decimal to binary, replacing divide by 2 by divide by 8.

$746/8 = 93$	rem 2	produces	2
$93/8 = 11$	rem 5		52
$11/8 = 1$	rem 3		352
$1/8 = 0$	rem 1		1352 ₈

EXAMPLE 1.9

Since it involves less work to convert decimal to octal than to binary, we often first convert to octal and then go to binary. Thus,

$$746_{10} = 1352_8 = 001\ 011\ 101\ 110_2$$

Hexadecimal ($r = 16$) groups bits by 4's. Each digit can then be in the range 0 to 15, where the digits above 9 are represented by the first 6 letters of the alphabet (upper case):

- 10 A
- 11 B
- 12 C
- 13 D
- 14 E
- 15 F