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A GUIDE-BOOK TO MATHEMATICS FOR TECHNOLOGISTS AND ENGINEERS

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A GUIDE-BOOK TO MATHEMATICS FOR TECHNOLOGISTS AND ENGINEERS

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FROM THE AUTHORS' PREFACE TO THE FIRST RUSSIAN EDITION

It was a very difficult task to write a guide-book of a small size designed to contain the fundamental knowledge of mathematics which is most necessary to engineers and students of higher technical schools. In our tendency to the compactness and brevity of the exposition, we attempted, however, to produce a guide-book which would be easy to understand, convenient to use and as accurate as possible (as much as it is required in engineering).

It should be pointed out that this book is neither a handbook nor a compendium, but a guide-book. Therefore it is not written as systematically as a handbook should be written. Hence the reader should not be surprised to find, for example, l'Hôpital's rule in the section devoted to computation of limits which is a part of the chapter "Introduction to the analysis" placed before the concept of the derivative, or information about the Gamma function in the chapter "Algebra"—just after the concept of the factorial. There are many such "imperfections" in the book. Thus a reader who wants to acquire certain information is advised to use not only the table of contents but also the alphabetical index inserted at the end of the book. If a problem mentioned in the text is explained in detail in another place of the book, then the corresponding page is indicated in a footnote.

PREFACE TO THE ENGLISH EDITION

The English translation is completed by two chapters on calculus of variations and integral equations. Both are translated from German. The first has been published already, the second one will be published in the next German edition.

MATHEMATICAL NOTATIONS (1)

I. Relations between quantities

$=$	equal
$\equiv$	identically equal
$\neq$	not equal
$\approx$	approximately equal
$<$	less
$>$	greater
$\leqslant$	less or equal
$\geqslant$	greater or equal

II. Algebra

$\|a\|$	absolute value of the number a
$+$	(plus)—addition
$-$	(minus)—subtraction
$\cdot$ or $\times$	multiplication, for example, $a \cdot b$ or $a \times b$; the multiplication sign is often omitted, for example, ab
: or —, or /	division ($a : b$ or $\frac{a}{b}$, or a/b)
a^m	"a to the power m"
$\sqrt{\ }$	square root, for example, $\sqrt{a}$
$\sqrt[n]{\ }$	root of the nth degree, for example, $\sqrt[n]{a}$
$\log_b$	logarithm to the base b, for example, $5 = \log_2 32$ (p. 156)
log	common logarithm, for example, $2 = \log 100$ (p. 156)
ln	natural logarithm, for example, $1 = \ln e$ (p.156)
(), [], {}	parentheses of brackets (denote succession of operations)
!	factorial, for example, $a!$; $6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$ (p. 190)

---

(1) Numbers in brackets (p. ...) denote the pages on which the corresponding notions are explained.

III. Geometry

$\perp$	perpendicular
$\parallel$	parallel
$\#$	equal and parallel
$\sim$	similar, for example, $\Delta ABC \sim \Delta DEF$
Δ	triangle
$\sphericalangle$	angle (sometimes $\angle$), for example, $\sphericalangle ABC$, $\angle ABC$
$\smile$	arc, for example, $\breve{AB}$
$^\circ$	degree
$'$	minute
$''$	second

degree, minute, second: in the degree measure, for example, $32^\circ 14' 11''.5$

IV. Trigonometry, hyperbolic functions

sin	the sine	(p. 213)
cos	the cosine	(p. 213)
tan	the tangent	(p. 213)
cot	the cotangent	(p. 213)
sec	the secant	(p. 213)
cosec	the cosecant	(p. 213)
Arc sin	the inverse sine	(p. 223)
Arc cos	the inverse cosine	(p. 223)
Arc tan	the inverse tangent	(p. 223)
Arc cot	the inverse cotangent	(p. 223)
arc sin	the principal branch of the inverse sine	(p. 223)
arc cos	the principal branch of the inverse cosine	(p. 223)
arc tan	the principal branch of the inverse tangent	(p. 223)
arc cot	the principal branch of the inverse cotangent	(p. 223)
sinh	the hyperbolic sine	(p. 230)
cosh	the hyperbolic cosine	(p. 230)
tanh	the hyperbolic tangent	(p. 230)
coth	the hyperbolic cotangent	(p. 230)
sech	the hyperbolic secant	(p. 230)
cosech	the hyperbolic cosecant	(p. 230)
ar sinh	the inverse hyperbolic sine	(p. 232)
ar cosh	the inverse hyperbolic cosine	(p. 232)
ar tanh	the inverse hyperbolic tangent	(p. 232)
ar coth	the inverse hyperbolic cotangent	(p. 232)

V. Notations for constants

const	a constant quantity
$\pi = 3.14159\ldots$	the ratio of the length of a circumference to its diameter (p. 199)
$e = 2.71828\ldots$	base of the natural logarithms (p. 331)
$C = 0.57722\ldots$	Euler's constant (p. 331)

VI. Mathematical analysis

lim	limit
$\to$	tends to ...
∞	infinity
	for example, $\lim\limits_{N\to\infty}\left(1+\frac{1}{N}\right)^N = e$ (pp. 318, 328)
Σ	sum
$\sum\limits_{i=1}^{n}$	sum in which i varies from 1 to n
$f(), \varphi()$	notations for functions, for example, $y = f(x)$, $u = \varphi(x, y, z)$
Δ	increment, for example, Δx
d	differential, for example, dx (p. 363)
d_x, d_y etc.	partial differential, for example, $d_x u$ (p. 363)
$'$, $''$, $'''$, $^{\mathrm{IV}}$ or $^{\bullet}$, $^{\bullet\bullet}$, $^{\bullet\bullet\bullet}$, $^{\bullet\bullet\bullet\bullet}$	notation for successive derivatives of functions of one variable; for example, if $y = f(x)$: $f'(x)$, $f''(x)$, $f'''(x)$, $f^{\mathrm{IV}}(x)$, $y', y'', y''', y^{\mathrm{IV}}$, $y^{\bullet}, y^{\bullet\bullet}, y^{\bullet\bullet\bullet}$ (pp. 360, 364, 365)
$\frac{d}{dx}, \frac{d^2}{dx^2}, \ldots$	first derivative, second derivative etc. — for example, $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ etc. (pp. 360, 364)
D	symbol of derivative (differentiation operator), for example, $Dy = y'$, $D^2y = y''$ etc. (pp. 360, 364)
f'_x, f''_{xx}, f''_{xy} or $\frac{\partial}{\partial x}, \frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial x\,\partial y}$	partial derivatives, for example, $f'_x(u), \frac{\partial f}{\partial x}, \frac{\partial^2 f}{\partial x^2}$, etc. (pp. 362, 365)
$\int$	integral (p. 394)
$\int\limits_a^b$	definite integral from the lower limit a to the upper limit b (p. 455)
$\int\limits_{(K)}$	line integral taken over the arc K or over a projection of K (pp. 486, 490)
$\int\limits_S, \int\limits_V$	integral over the surface S or over the volume V (pp. 495–497)
$\iint$	double integral (pp. 495–497)
$\iiint$	triple integral (pp. 495–497)

VII. Complex numbers

i (sometimes j)	imaginary unit ($i^2 = -1$) (p. 585)
re a	the real part of the number a (p. 585)
im a	the imaginary part of the number a (p. 585)
$\|a\|$	absolute value (modulus) of a (p. 586)
arg a	argument of a (p. 586)
$\bar{a}$	the conjugate of a, for example, $a = 2 + 3i$, $\bar{a} = 2 - 3i$ (p. 587)
Ln	(natural) logarithm of a complex number (p. 592)

VIII. Vector calculus

$\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$, or $\bar{a}, \bar{b}, \bar{c}$	symbols of vectors (p. 613)
$\boldsymbol{a}^0$	unit vector of the direction of the vector $\boldsymbol{a}$ (p. 614)
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	unit vectors of the rectangular coordinate system (p. 615)
$\|\boldsymbol{a}\|$ or a	length (absolute value) of the vector $\boldsymbol{a}$ (p. 613)
$\boldsymbol{a} = \boldsymbol{b}$, $\boldsymbol{a} + \boldsymbol{b}$, $\boldsymbol{a} - \boldsymbol{b}$	equality, composition and subtraction of vectors (pp. 613, 614)
$\alpha\boldsymbol{a}$	multiplication of a vector by a scalar (p. 614)
$\boldsymbol{ab}$	scalar product of vectors (p. 616)
$\boldsymbol{a} \times \boldsymbol{b}$ or $[\boldsymbol{ab}]$	vector product of vectors (p. 616)
$\boldsymbol{abc} = \boldsymbol{a}(\boldsymbol{b} \times \boldsymbol{c})$	box product of three vectors (p. 618)
a_x, a_y, a_z	coordinates of the vector $\boldsymbol{a}$ in the Cartesian coordinate system (p. 615)
∇	Hamilton's differential operator (nabla) (p. 643)
Δ	Laplace's operator (p. 645)
grad	gradient of a scalar field (grad $\varphi = \nabla\varphi$) (p. 632)
div	divergence of a vector field (div $\boldsymbol{V} = \nabla\boldsymbol{V}$) (p. 640)
rot (or curl)	rotation (or curl) of a vector field (curl $\boldsymbol{V} = \nabla \times \boldsymbol{V}$) (p. 641)
$\dfrac{\partial U}{\partial \boldsymbol{c}}$	derivative of a scalar field in the direction $\boldsymbol{c}$ (p. 632)

PART ONE

TABLES AND GRAPHS

I. TABLES

Interpolation. Most of the tables inserted below give the values of functions to four significant figures for three significant figures of the argument. In the cases, when the argument is given with a greater accuracy, and the desired value of the function cannot be obtained directly from the tables, interpolation should be used. The simplest form is *linear interpolation* in which we assume that the increment of the function is proportional to the increment of the argument. If the desired value of the argument x lies between the values x_0 and $x_1 = x_0 + h$ in the tables and the corresponding values of the function are

$$y_0 = f(x_0) \quad \text{and} \quad y_1 = f(x_1) = y_0 + \Delta,$$

then we assume that

$$f(x) = f(x_0) + \frac{x - x_0}{h}\Delta.$$

The *interpolation correction* $\frac{x - x_0}{h}\Delta$ can be easily computed by using the tables of proportional parts on pp. 84, 85 and also by using the supplement to this guide which gives the products of the difference Δ (from 11 to 90) times 0.1, 0.2, ..., 0.9.

Examples. (1) Find 1.6754^2. We find in the table (p. 23) $1.67^2 = 2.789$, $1.68^2 = 2.822$, $\Delta = 33$ (1). From the tables of proportional parts we have $0.5 \cdot 33 = 16.5$, $0.04 \cdot 33 = 1.3$, $\frac{x - x_0}{h}\Delta = 16.5 + 1.3 \approx 18$, hence $1.6754^2 = 2.807$.

(2) Find tan 79°24′. From the tables (pp. 60 and 85), tan 79°20′ = 5.309, tan 79°30′ = 5.396, $\Delta = 87$; $0.4 \cdot 87 \approx 35$, hence tan 79°24′ = 5.344.

The error in linear interpolation does not exceed one unit of the last significant figure, provided that the two consecutive

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(1) The difference Δ is usually expressed in the units of the last order of the value of the function, without the first zeros or the decimal point.