COLLEGE ALGEBRA

FIFTH EDITION

Gordon Fuller
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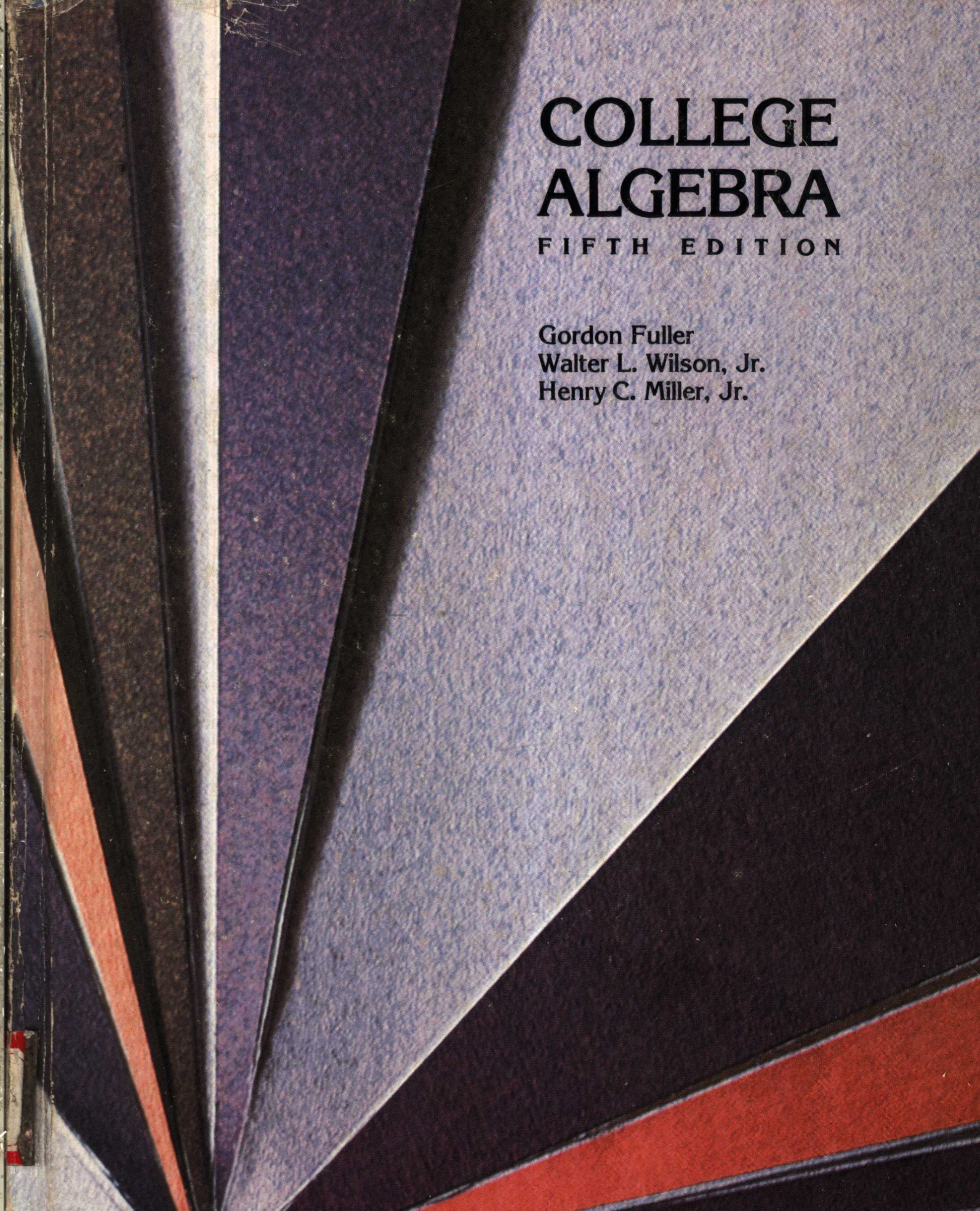

5TH EDITION

COLLEGE ALGEBRA

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PREFACE

The Fifth Edition of *College Algebra* retains the conservative approach of the previous editions. Chapters 2, 3, and 4 contain the essential material of intermediate algebra and may be studied intensively or merely surveyed as the needs of the class require. Chapters 5 through 17 constitute a solid foundation for further study of mathematics, especially calculus. This edition has been designed to offer a variety of flexible teaching and study programs. A few of the new features are listed below.

1. Basic rules of algebra and important formulas have been placed inside the front cover.
2. Review exercises have been extended to include "complete-the-formula" exercises. Answers for all odd-numbered exercises are provided. Detailed solutions are given at appropriate points.
3. The axioms have been modified to simplify the development of the field properties, and less emphasis is placed on set theory.
4. The treatment of relations and functions has been redone, and rational functions and asymptotes are discussed.
5. The chapter on linear equations has been expanded.
6. The presentation of complex numbers has been reworked for greater clarity, and an appendix is provided for those who wish to do a rigorous development. There is no use of trigonometry.

Meticulous care has been taken to find and correct errors in the text, the exercises, and answers to exercises.

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CONTENTS

CHAPTER 1

THE REAL NUMBERS

1·1 INTRODUCTION

This first chapter presents a rather formal development of certain rules of algebra. The next three chapters contain less formal developments. For easy reference, some of these rules are listed on the inside front cover of this text. You must be sufficiently familiar with these rules to use them, in various contexts, as tools for manipulating symbols that denote numbers. To help you learn these rules, a list of "complete the formulas" questions is provided at the end of the review exercises for each of the first four chapters.

The remainder of this section contains some remarks about sets. The concept of *set* is basic in the study of mathematics. We will make use of the following language of sets at certain points in the course.

A set is determined by its *elements*, that is, the objects that belong to it. To define a set we may list its elements or we may use a prose sentence, or notation for this prose sentence, which prescribes the elements of the set. For example, if A is the set of months of the year having thirty-one days (prose description), we may write

$A = \{$January, March, May, July, August, October, December$\}$

$A = \{x|x$ is a month of the year having thirty-one days$\}$

This last form is called *set-builder notation*. It is read: "A equals the set of all x such that x is a month of the year having thirty-one days." January is an element of A, and we write January $\in A$.

Some elementary properties of sets are described in the following definitions.

DEFINITION 1·1

Two sets A and B are said to be *equal*, written $A = B$, if each element of set A is an element of set B and each element of set B is an element of set A.

DEFINITION 1·2

If each element of a set A is an element of set B, then A is called a *subset* of B, written $A \subseteq B$. If A is a subset of B and if B has one or more elements not belonging to A, then A is a *proper subset* of B, written $A \subset B$.

DEFINITION 1·3

A set which contains no elements is called the *null* or *empty* set, often denoted by $\varnothing$.

DEFINITION 1·4

The *union* of two sets A and B, denoted by $A \cup B$, consists of all elements which belong to A or to B or to both A and B:

$$A \cup B = \{x | x \in A \quad \text{or} \quad x \in B\}$$

DEFINITION 1·5

The *intersection* of two sets A and B, written $A \cap B$, is the set of all elements that belong both to A and to B:

$$A \cap B = \{x | x \in A \quad \text{and} \quad x \in B\}$$

DEFINITION 1·6

An *ordered pair* is either*

1 A set with two distinct elements one of which is designated as first and the other as second, or
2 A set with one element which is designated as being both first and second

In either case, if x is first and y is second, the ordered pair is denoted by (x, y). In the second case we have $y = x$.

DEFINITION 1·7

If X and Y are sets, the set of all ordered pairs (x, y) such that $x \in X$ and $y \in Y$ is called the *cartesian product*, or product set, of X and Y:

$$X \times Y = \{(x, y) | x \in X \quad \text{and} \quad y \in Y\}$$

*For a more sophisticated definition of ordered pair, see P. R. Halmos, *Naive Set Theory*, D. Van Nostrand, 1961.

1 · 2 A DEDUCTIVE SYSTEM

Recall from high school plane geometry that a theorem consists of a hypothesis and a conclusion, and that the proof of the theorem is accomplished by a process of logical reasoning which shows that the conclusion is a consequence of the hypothesis. Among the theorems which are established by such a process, there must be a first theorem. The proof of the first theorem clearly cannot rest on other theorems; the proof must involve an unproved statement or statements. The unproved statements are called *axioms* or *postulates*; these are assumed to be true.

There are also numerous definitions of terms in plane geometry. The first of a succession of definitions must describe a term in words whose meanings are assumed to be known. Such words are called *undefined terms*. The words "point" and "line," for example, are classed as undefined terms.

The undefined terms and axioms form the basis (starting point) or foundation for proving theorems. With the proof of each theorem the basis for proving other theorems is enlarged.

Algebra, like geometry and many other areas of mathematics which stem from the undefined terms and axioms, is a ***deductive***, or *axiomatic, system*. In this chapter we will introduce and study the set of axioms and undefined terms upon which algebra is based. We will consider a set of undefined elements called *real numbers* and indicate the set by R. Even though real numbers are not defined, we will gain an intuitive understanding of their nature by studying some of their properties.

The set of integers is an infinite subset of the system of real numbers R. Such numbers as $\sqrt{3}$, $\sqrt[3]{7}$, $\frac{5}{3}$, and 2.74 are also part of the system of real numbers.

The set of real numbers has two basic undefined operations called ***addition*** and ***multiplication***. The symbol + denotes the operation of addition and the symbols · and × denote multiplication. Thus, if a and b stand for two real numbers, we write $a + b$ for addition and $a \cdot b$, $a \times b$, or just ab to indicate multiplication. The result of addition is called the *sum*; the result of multiplication is called the *product*.

Later we shall discuss numbers that are different from real numbers. In the meantime, for brevity, we use the word *number* to mean real number (not necessarily an integer!).

The equals sign in the number system denotes the relation of ***equality***. Thus $a = b$ means that a and b are symbols for the same number. For example, we may write VI = 6 and $\frac{3}{2} = 1.5$. A statement such as $a = b$ is called an ***equation***. There are five axioms which characterize this equality relation.

AXIOM E · 1 *Reflexive property.* For any $a \in R$, $a = a$.

AXIOM E · 2 *Symmetric property.* For any $a, b \in R$, if $a = b$, then $b = a$.

AXIOM E · 3 *Transitive property.* For any $a, b, c \in R$, if $a = b$ and $b = c$, then $a = c$.

AXIOM E · 4 *Addition property.* For any $a, b, c, d \in R$, if $a = b$ and $c = d$, then
$a + c = b + d$
and since $d = c$,
$a + c = b + c$

AXIOM E · 5 *Multiplication property.* For any $a, b, c, d \in R$, if $a = b$ and $c = d$, then
$ac = bd$
and since $d = c$,
$ac = bc$

The last three properties are sometimes described, respectively, by the following statements:

1. Quantities equal to the same quantity are equal to each other.
2. If equals are added to equals, the sums are equal.
3. If equals are multiplied by equals, the products are equal.

1 · 3 FIELD AXIOMS FOR THE REAL NUMBERS

We shall now introduce six axioms, called *axioms of a field*, for the real numbers R. In Chapter 9 additional axioms will be stated to complete our study of real numbers. The grouping symbols ( ) and [ ] in this discussion indicate that the enclosed numbers are to be considered as a single quantity; that is, we agree that $(a) = [a] = a$.

AXIOM 1 *The closure law.* For any $a, b \in R$,

$a + b \in R$ and $a \cdot b \in R$

This axiom simply states that the operations of addition and multiplication on two numbers which belong to R yield numbers which also belong to R. That is, the sum $a + b$ and the product $a \cdot b$ are real numbers. Hence the set R is said to be *closed* under these operations.

AXIOM 2

> *The commutative law.* For any $a, b \in R$,
>
> $a + b = b + a$ and $a \cdot b = b \cdot a$

The commutative law tells us that the sum or product of two numbers does not depend on the order in which the addition or multiplication is performed. Thus $3 + 7 = 7 + 3$ and $3 \cdot 7 = 7 \cdot 3$.

The sum and the product of two real numbers are each unique. That is, the two numbers have only one sum and only one product. To show that the sum is unique, suppose that c is the sum of a and b and that another number d is also the sum. Then we can write

$$a + b = c \qquad \text{and} \qquad d = a + b$$

and it follows from Axiom E · 3 that the supposed sum d must equal c.

AXIOM 3

> *The associative law.* For any $a, b, c \in R$,
>
> $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

The associative law tells us that the sum, and also the product, of any three real numbers is independent of the way in which the numbers are grouped for these operations. In fact, Axioms 1 to 3 permit us to add or multiply any number of real numbers in any chosen order. Hence we can interpret $a + b + c$ to mean $(a + b) + c$ or $a + (b + c)$, and abc to mean $(ab)c$ or $a(bc)$. So we have at once two ways of representing the sum and the product of three real numbers. For example, we may write

$$3 + 4 + 5 = (3 + 4) + 5 = 3 + (4 + 5) = 12$$
$$3 \cdot 4 \cdot 5 = (3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5) = 60$$

AXIOM 4

> *The distributive law.* For any $a, b, c \in R$,
>
> $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

The distributive law, in each form, states that a particular product is equal to a sum and, conversely, the sum is equal to a product. This is true since equality is symmetric (Axiom E · 2). A consequence of Axiom 4 is the fact that the product of a number and the sum of three or more numbers is equal to the sum of the products of the first number by each of the numbers forming the sum. Thus, for the sum of three numbers, we have

$$\begin{aligned} a[b+c+d] &= a[b+(c+d)] && \textit{Axiom 3} \\ &= ab + a(c+d) && \textit{Axiom 4} \\ &= ab + (ac+ad) && \textit{Axiom 4} \\ &= ab + ac + ad && \textit{Axiom 3} \end{aligned}$$

Similarly, $(b+c+d)a = ba + ca + da$.

AXIOM 5

The identity elements. There exists exactly one* real number, called "zero" and denoted by 0, such that for any $a \in R$

$$a + 0 = a \quad \text{and} \quad 0 + a = a$$

There exists exactly one real number, called "one" and denoted by 1, such that for any $a \in R$

$$a \cdot 1 = a \quad \text{and} \quad 1 \cdot a = a$$

The number 0 is called the *identity element for addition*, and the number 1 is called the *identity element for multiplication*.

AXIOM 6

The inverse elements. For each $a \in R$, there exists exactly one* real number, denoted by $-a$, such that

$$a + (-a) = 0 \quad \text{and} \quad (-a) + a = 0$$

For each nonzero number $a \in R$, there exists exactly one real number, denoted by $1/a$, such that

$$a \cdot \frac{1}{a} = 1 \quad \text{and} \quad \frac{1}{a} \cdot a = 1$$

The number $-a$ is called the *additive inverse of a*, the *negative of a*, or *minus a*. The number $1/a$ is called the *multiplicative inverse of a* or *the reciprocal of a*.

EXAMPLE 1 Show that $(a+b) + (-a) = b$.

*Actually, it is necessary to assume only existence. Uniqueness can be proved.