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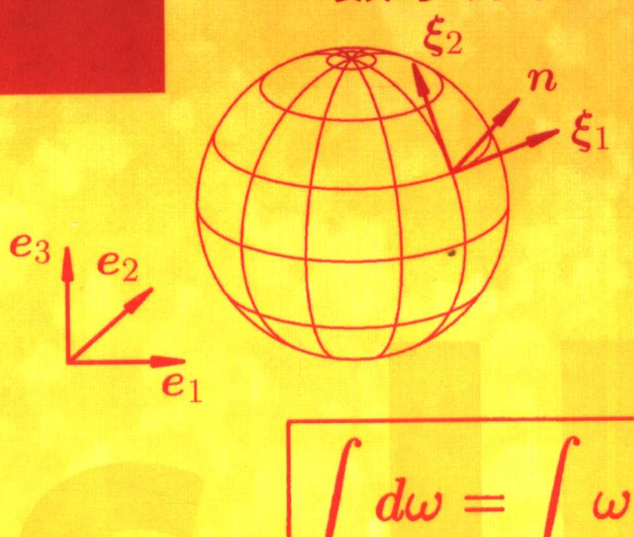
Vladimir A. Zorich

Mathematical Analysis II

数学分析 第2卷



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Vladimir A. Zorich
Moscow State University
Department of Mathematics (Mech-Math)
Vorobievy Gory
Moscow 119992
Russia

Translator:
Roger Cooke
Burlington, Vermont
USA
e-mail: cooke@emba.uvm.edu

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Prefaces

Preface to the English Edition

An entire generation of mathematicians has grown up during the time between the appearance of the first edition of this textbook and the publication of the fourth edition, a translation of which is before you. The book is familiar to many people, who either attended the lectures on which it is based or studied out of it, and who now teach others in universities all over the world. I am glad that it has become accessible to English-speaking readers.

This textbook consists of two parts. It is aimed primarily at university students and teachers specializing in mathematics and natural sciences, and at all those who wish to see both the rigorous mathematical theory and examples of its effective use in the solution of real problems of natural science.

The textbook exposes classical analysis as it is today, as an integral part of Mathematics in its interrelations with other modern mathematical courses such as algebra, differential geometry, differential equations, complex and functional analysis.

The two chapters with which this second book begins, summarize and explain in a general form essentially all most important results of the first volume concerning continuous and differentiable functions, as well as differential calculus. The presence of these two chapters makes the second book formally independent of the first one. This assumes, however, that the reader is sufficiently well prepared to get by without introductory considerations of the first part, which preceded the resulting formalism discussed here. This second book, containing both the differential calculus in its generalized form and integral calculus of functions of several variables, developed up to the general formula of Newton–Leibniz–Stokes, thus acquires a certain unity and becomes more self-contained.

More complete information on the textbook and some recommendations for its use in teaching can be found in the translations of the prefaces to the first and second Russian editions.

Moscow, 2003 *V. Zorich*

Preface to the Fourth Russian Edition

In the fourth edition all misprints that the author is aware of have been corrected.

Moscow, 2002 *V. Zorich*

Preface to the Third Russian Edition

The third edition differs from the second only in local corrections (although in one case it also involves the correction of a proof) and in the addition of some problems that seem to me to be useful.

Moscow, 2001 *V. Zorich*

Preface to the Second Russian Edition

In addition to the correction of all the misprints in the first edition of which the author is aware, the differences between the second edition and the first edition of this book are mainly the following. Certain sections on individual topics – for example, Fourier series and the Fourier transform – have been recast (for the better, I hope). We have included several new examples of applications and new substantive problems relating to various parts of the theory and sometimes significantly extending it. Test questions are given, as well as questions and problems from the midterm examinations. The list of further readings has been expanded.

Further information on the material and some characteristics of this second part of the course are given below in the preface to the first edition.

Moscow, 1998 *V. Zorich*

Preface to the First Russian Edition

The preface to the first part contained a rather detailed characterization of the course as a whole, and hence I confine myself here to some remarks on the content of the second part only.

The basic material of the present volume consists on the one hand of multiple integrals and line and surface integrals, leading to the generalized Stokes' formula and some examples of its application, and on the other hand the machinery of series and integrals depending on a parameter, including

Fourier series, the Fourier transform, and the presentation of asymptotic expansions.

Thus, this Part 2 basically conforms to the curriculum of the second year of study in the mathematics departments of universities.

So as not to impose rigid restrictions on the order of presentation of these two major topics during the two semesters, I have discussed them practically independently of each other.

Chapters 9 and 10, with which this book begins, reproduce in compressed and generalized form, essentially all of the most important results that were obtained in the first part concerning continuous and differentiable functions. These chapters are starred and written as an appendix to Part 1. This appendix contains, however, many concepts that play a role in any exposition of analysis to mathematicians. The presence of these two chapters makes the second book formally independent of the first, provided the reader is sufficiently well prepared to get by without the numerous examples and introductory considerations that, in the first part, preceded the formalism discussed here.

The main new material in the book, which is devoted to the integral calculus of several variables, begins in Chapter 11. One who has completed the first part may begin the second part of the course at this point without any loss of continuity in the ideas.

The language of differential forms is explained and used in the discussion of the theory of line and surface integrals. All the basic geometric concepts and analytic constructions that later form a scale of abstract definitions leading to the generalized Stokes' formula are first introduced by using elementary material.

Chapter 15 is devoted to a similar summary exposition of the integration of differential forms on manifolds. I regard this chapter as a very desirable and systematizing supplement to what was expounded and explained using specific objects in the mandatory Chapters 11–14.

The section on series and integrals depending on a parameter gives, along with the traditional material, some elementary information on asymptotic series and asymptotics of integrals (Chap. 19), since, due to its effectiveness, the latter is an unquestionably useful piece of analytic machinery.

For convenience in orientation, ancillary material or sections that may be omitted on a first reading, are starred.

The numbering of the chapters and figures in this book continues the numbering of the first part.

Biographical information is given here only for those scholars not mentioned in the first part.

As before, for the convenience of the reader, and to shorten the text, the end of a proof is denoted by $\square$. Where convenient, definitions are introduced by the special symbols $:=$ or $=:$ (equality by definition), in which the colon stands on the side of the object being defined.

Continuing the tradition of Part 1, a great deal of attention has been paid to both the lucidity and logical clarity of the mathematical constructions themselves and the demonstration of substantive applications in natural science for the theory developed.

Moscow, 1982 *V. Zorich*

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9 *Continuous Mappings (General Theory)

In this chapter we shall generalize the properties of continuous mappings established earlier for numerical-valued functions and mappings of the type $f : \mathbb{R}^m \to \mathbb{R}^n$ and discuss them from a unified point of view. In the process we shall introduce a number of simple, yet important concepts that are used everywhere in mathematics.

9.1 Metric Spaces

9.1.1 Definition and Examples

Definition 1. A set X is said to be endowed with a *metric* or *a metric space structure* or to be a *metric space* if a function

$$d : X \times X \to \mathbb{R} \tag{9.1}$$

is exhibited satisfying the following conditions:

a) $d(x_1, x_2) = 0 \Leftrightarrow x_1 = x_2$,

b) $d(x_1, x_2) = d(x_2, x_2)$ (symmetry),

c) $d(x_1, x_3) \leq d(x_1, x_2) + d(x_2, x_3)$ (the triangle inequality),

where x_1, x_2, x_3 are arbitrary elements of X.

In that case, the function (9.1) is called a *metric* or *distance* on X.

Thus a *metric space* is a pair (X, d) consisting of a set X and a metric defined on it.

In accordance with geometric terminology the elements of X are called *points*.

We remark that if we set $x_3 = x_1$ in the triangle inequality and take account of conditions a) and b) in the definition of a metric, we find that

$$0 \leq d(x_1, x_2) ,$$

that is, a distance satisfying axioms a), b), and c) is nonnegative.

Let us now consider some examples.

Example 1. The set $\mathbb{R}$ of real numbers becomes a metric space if we set $d(x_1, x_2) = |x_2 - x_1|$ for any two numbers x_1 and x_2, as we have always done.

Example 2. Other metrics can also be introduced on $\mathbb{R}$. A trivial metric, for example, is the discrete metric in which the distance between any two distinct points is 1.

The following metric on $\mathbb{R}$ is much more substantive. Let $x \mapsto f(x)$ be a nonnegative function defined for $x \geq 0$ and vanishing for $x = 0$. If this function is strictly convex upward, then, setting

$$d(x_1, x_2) = f(|x_1 - x_2|) \tag{9.2}$$

for points $x_1, x_2 \in \mathbb{R}$, we obtain a metric on $\mathbb{R}$.

Axioms a) and b) obviously hold here, and the triangle inequality follows from the easily verified fact that f is strictly monotonic and satisfies the following inequalities for $0 < a < b$:

$$f(a + b) - f(b) < f(a) - f(0) = f(a).$$

In particular, one could set $d(x_1, x_2) = \sqrt{|x_1 - x_2|}$ or $d(x_1, x_2) = \frac{|x_1 - x_2|}{1 + |x_1 - x_2|}$. In the latter case the distance between any two points of the line is less than 1.

Example 3. Besides the traditional distance

$$d(x_1, x_2) = \sqrt{\sum_{i=1}^{n} |x_1^i - x_2^i|^2} \tag{9.3}$$

between points $x_1 = (x_1^1, \ldots, x_1^n)$ and $x_2 = (x_2^1, \ldots, x_2^n)$ in $\mathbb{R}^n$, one can also introduce the distance

$$d_p(x_1, x_2) = \left(\sum_{i=1}^{n} |x_1^i - x_2^i|^p \right)^{1/p} , \tag{9.4}$$

where $p \geq 1$. The validity of the triangle inequality for the function (9.4) follows from Minkowski's inequality (see Subsect. 5.4.2).

Example 4. When we encounter a word with incorrect letters while reading a text, we can reconstruct the word without too much trouble by correcting the errors, provided the number of errors is not too large. However, correcting the error and obtaining the word is an operation that is sometimes ambiguous. For that reason, other conditions being equal, one must give preference to the interpretation of the incorrect text that requires the fewest corrections.