国外数学名著系列（续一）
（影印版）55

R. V. Gamkrelidze (Ed.)

Geometry I

Basic Ideas and Concepts of Differential Geometry

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微分几何基本思想与概念

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格(Springer)出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。 这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005 年 12 月 3 日

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Basic Ideas and Concepts of Differential Geometry

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Translated from the Russian
by E. Primrose

Contents

Preface

Apparently, at the basis of geometrical thinking there lie mechanisms, so far unknown, that enable us to extract and use structurally formed elements of information flow. This point of view is developed in more detail in Chapter 1. Starting from this, we make an attempt to realize the variety of modern geometrical theories, imitating the physicists who, starting from the "big bang" hypothesis, explain the existing state of the universe.

In putting together this survey of what seem to us the fundamental concepts, ideas and mthods of modern differential geometry, we have not intended that it should be read systematically from beginning to end. Therefore within each chapter and the book as a whole the presentation gradually speeds up, so that the reader can start or stop wherever it is natural for him. Any new theme begins with "general conversations"; the process of turning these into precise formulae is traced as far as possible. We have drawn attention to this aspect, since the art of a geometer is determined to a large extent by the ability to organize this process.

Our understanding of geometry as a whole has changed significantly in the process of writing this survey, and we shall be satisfied if the benefit that we ourselves have gained turns out to be not only the property of the authors.

In conclusion we wish to thank sincerely our friends and colleagues in the Laboratory of Problems of High Dimension of the Institute of Program Systems of the USSR Academy of Sciences for the very substantial help they have given us in preparing the manuscript for the press, and the Chief Editor of this series, Corresponding Member of the USSR Academy of Sciences R.V. Gamkrelidze.

Chapter 1
Introduction: A Metamathematical View of Differential Geometry

"Geometry is the ruler of all mental investigation".
M.V. Lomonosov

§ 1. Algebra and Geometry – the Duality of the Intellect

It is known from physiology that in the process of thinking the hemispheres of the human brain fulfil different functions. The left one is the site of the "rational" mind. In other words, this part of the brain carries out formal deductions, reasons logically, and so on. On the other hand, imagination, intuition, emotions and other components of the "irrational" mind are the product of the right hemisphere. This division of labour can have the following explanation.

The process of solving some problem or other by a human being or an artificial mechanism involves the need to draw correct conclusions from correct premises. The logical computations that carry out these functions in various specific circumstances can easily be formulated algorithmically and thereby carried out on modern computers. There are good reasons for supposing that the human brain acts in a similar way and the left hemisphere is its "logical block". However, the ability to argue logically is only half the problem, and apparently the simpler half. In fact, to solve any complicated problem it is necessary to construct a rather long chain, consisting of logically correct elements of the type "premise-conclusion". However, from given premises it is possible to draw very many correct conclusions. Therefore it is practically impossible to find the solution of a complicated problem by randomly building up logically correct chains of the form mentioned, in view of the large number of variants that arise. Thus the problem we are posing is: in which direction should we reason? We can solve it only if there are various mechanisms of selection and motivation, that is, mechanisms that induce the thinking apparatus to consider only expedient versions. Man solves this problem by using intuition and imagination. Thus, we can think that the process of evolution of nature has led to the two most important aspects of any thought process – the formally logical and the motivational – being provided by the two functional blocks of the brain – its left and right hemispheres, respectively.

Mathematics is the science that deals with pure thought. Therefore the two main aspects of mental activity mentioned above must be revealed in its structure. In fact, this is familiar to everyone from the school division of mathematics into algebra and geometry. Apparently we can regard it as established experimentally that an algebraist or analyst is a mathematician with a pronounced dominant left hemisphere, while for a geometer it is the right hemisphere. Thus, if we consider the body of mathematicians linked by various lines of communication, having a common memory, in which there are mechanisms of stimulation and repression, etc., as a thinking system, then geometry is the product of its right hemisphere.

It is clear a priori that successful functioning of the intellect (natural or artificial) can be ensured by balanced interaction of its right and left hemispheres, acting on different levels. Hypertrophy of the function of the left hemisphere leads to a phenomenon that can appropriately be called thought bureaucracy (formalism, scholasticism, and so on). On the other hand, hypertrophy of the right hemisphere leads to unsound fantasies and wandering in the clouds. For example, F. Klein (Klein [1926]) writes: "We state here as a principle that we will always combine the analytical and geometrical treatment of problems, and we shall not, like many mathematicians, take a one-sided point of view. An analytical treatment does not give a visual idea of the results obtained, while a geometrical examination can only give an approximate basis for proof...".

We liken the development of thought in the solution of a problem to the process of propagation of an electromagnetic wave, as a consequence of the inductive connection of an electric field (E) and a magnetic field (H) (Fig. 1). We venture to compare the left hemisphere (algebra) with E, and the right hemisphere (geometry) with H, since a magnetic field does not have sources. Short-wave oscillations of algebra-geometry type inevitably arise when one or several investigators are working on a specific problem. Long waves of this kind, which often interfere with one another in an odd way, are the waves of mathematical history.

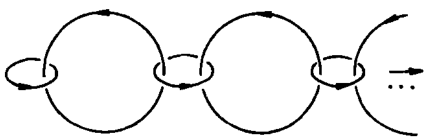

Fig. 1

If 20 years ago the formal algebraic spirit, personified by Nicolas Bourbaki, dominated, nowadays the geometrical spirit is on the crest of the wave, with the typical growth of interest shown by mathematicians in problems of physics and nature in general. The chain

Analysis →	Differential topology →	Homological algebra
↓ initial "algebra"	"geometry" directing it	secondary "algebra" regulating this "geometry"

is an example of a large-scale wave in modern mathematics.

§2. Two Examples: Algebraic Geometry, Propositional Logic and Set Theory

2.1. In ancient manuscripts of geometrical content, instead of a proof of a theorem there is often displayed a convenient diagram together with the instruction "look". Thereby it is implicitly assumed that the contemplation of the diagram is capable of arranging the thoughts of the observer into a conclusive chain of deductions. Modern algebraic geometry is an approach to the solution of problems of commutative algebra by way of an intelligent and systematic construction of the necessary visual images. The source of this approach is an idea that goes back to Descartes: it is possible to obtain a visual representation of the set of solutions of a system of polynomial equations, interpreting it by introducing coordinates as an "algebraic subvariety" of affine space. In modern algebraic geometry the main way of visualizing is to interpret an arbitrary commutative algebra as the algebra of functions on some set. Its description in general outline reduces to the following (the definitions of the simplest concepts of commuatative algebra used below can be found, for example, in Volume 11 of the present series).

Let A be a commuatative algebra with unity over a field k. Consider the set Spec A of all prime ideals of this algebra. An element $a \in A$ can be thought of as a "function" on this set, whose value at a point $\mathfrak{p} \in \operatorname{Spec} A$ is equal to the image of a under the natural homomorphism $A \to A/\mathfrak{p}$. As a result we obtain the following picture: to each point $\mathfrak{p}$ of Spec A we "attach" an algebra $A/\mathfrak{p}$ without divisors of zero (since $\mathfrak{p}$ is a prime ideal) and the "function" mentioned above, associated with the element $a \in A$, is the map $\mathfrak{p} \mapsto [a] \in A/\mathfrak{p}$. The algebra $A/\mathfrak{p}$, generally speaking, depends on $\mathfrak{p}$. If this dependence did not exist, that is, all the algebras $A/\mathfrak{p}$ were the same, then the construction we have described would lead us to the usual $A/\mathfrak{p}$-valued functions on Spec A.

Example. Let $k = \mathbb{C}$ (the field of complex numbers) and let $A = \mathbb{C}[x]$ be the algebra of polynomials with complex coefficients in the variable x. Any non-zero prime ideal $\mathfrak{p} \subset A$ consists of polynomials divisible by $x - c$ (the number $c \in \mathbb{C}$ is fixed). Dividing a given polynomial $p(x) \in A$ by $x - c$ and taking the remainder, we obtain a unique representation in the form $p(x) = p_0 + (x - c)h(x)$, where $p_0 \in \mathbb{C}$. Then $A/\mathfrak{p} = \mathbb{C}$ if $\mathfrak{p} \neq \{0\}$, and we can represent the operation of factoriza-