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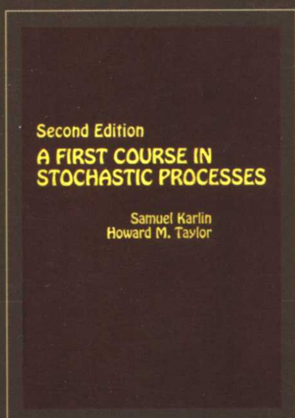
A First Course in Stochastic Processes

随机过程初级教程

（英文版 · 第2版）

[美] Samuel Karlin
Howard M.Taylor 著



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内容提要

本书系统论述随机过程的基本理论和方法，理论与实际应用并重．书中主要内容有：马尔可夫链、连续时间马尔可夫链、更新过程、鞅论、布朗运动、分支过程和平稳随机过程．本书涉及范围十分广泛，含有丰富的背景知识，深入浅出，不需要测度论作为预备知识．

本书可作为高等学校本科生和研究生的教材，也可作为工程技术人员的参考书

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随机过程初级教程（英文版·第2版）

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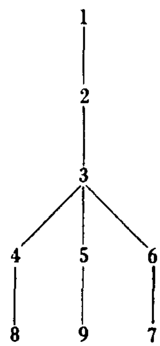
PREFACE

The purposes, level, and style of this new edition conform to the tenets set forth in the original preface. We continue with our tack of developing simultaneously theory and applications, intertwined so that they refurbish and elucidate each other.

We have made three main kinds of changes. First, we have enlarged on the topics treated in the first edition. Second, we have added many exercises and problems at the end of each chapter. Third, and most important, we have supplied, in new chapters, broad introductory discussions of several classes of stochastic processes not dealt with in the first edition, notably martingales, renewal and fluctuation phenomena associated with random sums, stationary stochastic processes, and diffusion theory.

Martingale concepts and methodology have provided a far-reaching apparatus vital to the analysis of all kinds of functionals of stochastic processes. In particular, martingale constructions serve decisively in the investigation of stochastic models of diffusion type. Renewal phenomena are almost equally important in the engineering and managerial sciences especially with reference to examples in reliability, queueing, and inventory systems. We discuss renewal theory systematically in an extended chapter. Another new chapter explores the theory of stationary processes and its applications to certain classes of engineering and econometric problems. Still other new chapters develop the structure and use of diffusion processes for describing certain biological and physical systems and fluctuation properties of sums of independent random variables useful in the analyses of queueing systems and other facets of operations research.

The logical dependence of chapters is shown by the diagram below. Section 1 of Chapter 1 can be reviewed without worrying about details. Only Sections 5 and 7 of Chapter 7 depend on Chapter 6. Only Section 9 of Chapter 9 depends on Chapter 5.



An easy one-semester course adapted to the junior–senior level could consist of Chapter 1, Sections 2 and 3 preceded by a cursory review of Section 1, Chapter 2 in its entirety, Chapter 3 excluding Sections 5 and/or 6, and Chapter 4 excluding Sections 3, 7, and 8. The content of the last part of the course is left to the discretion of the lecturer. An option of material from the early sections of any or all of Chapters 5–9 would be suitable.

The problems at the end of each chapter are divided into two groups. The first, more or less elementary; the second, more difficult and subtle.

The scope of the book is quite extensive, and on this account, it has been divided into two volumes. We view the first volume as embracing the main categories of stochastic processes underlying the theory and most relevant for applications. In *A Second Course* we introduce additional topics and applications and delve more deeply into some of the issues of *A First Course*. We have organized the edition to attract a wide spectrum of readers including theorists and practitioners of stochastic analysis pertaining to the mathematical, engineering, physical, biological, social, and managerial sciences.

The second volume of this work, *A Second Course in Stochastic Processes*, will include the following chapters: (10) Algebraic Methods in Markov Chains; (11) Ratio Theorems of Transition Probabilities and Applications; (12) Sums of Independent Random Variables as a Markov Chain; (13)

Order Statistics, Poisson Processes, and Applications; (14) Continuous Time Markov Chains; (15) Diffusion Processes; (16) Compounding Stochastic Processes; (17) Fluctuation Theory of Partial Sums of Independent Identically Distributed Random Variables; (18) Queueing Processes.

As noted in the first preface, we have drawn freely on the thriving literature of applied and theoretical stochastic processes. A few representative references are included at the end of each chapter; these may be profitably consulted for more advanced material.

We express our gratitude to the Weizmann Institute of Science, Stanford University, and Cornell University for providing a rich intellectual environment, and facilities indispensable for the writing of this text. The first author is grateful for the continuing grant support provided by the Office of Naval Research that permitted an unencumbered concentration on a number of the concepts and drafts of this book. We are also happy to acknowledge our indebtedness to many colleagues who have offered a variety of constructive criticisms. Among others, these include Professors P. Brockwell of La Trobe, J. Kingman of Oxford, D. Iglehart and S. Ghurye of Stanford, and K. Itô and S. Stidham, Jr. of Cornell. We also thank our students M. Nedzela and C. Macken for their assistance in checking the problems and help in reading proofs.

SAMUEL KARLIN
HOWARD M. TAYLOR

PREFACE TO FIRST EDITION

Stochastic processes concern sequences of events governed by probabilistic laws. Many applications of stochastic processes occur in physics, engineering, biology, medicine, psychology, and other disciplines, as well as in other branches of mathematical analysis. The purpose of this book is to provide an introduction to the many specialized treatises on stochastic processes. Specifically, I have endeavored to achieve three objectives: (1) to present a systematic introductory account of several principal areas in stochastic processes, (2) to attract and interest students of pure mathematics in the rich diversity of applications of stochastic processes, and (3) to make the student who is more concerned with application aware of the relevance and importance of the mathematical subleties underlying stochastic processes.

The examples in this book are drawn mainly from biology and engineering but there is an emphasis on stochastic structures that are of mathematical interest or of importance in more than one discipline. A number of concepts and problems that are currently prominent in probability research are discussed and illustrated.

Since it is not possible to discuss all aspects of this field in an elementary text, some important topics have been omitted, notably stationary stochastic processes and martingales. Nor is the book intended in any sense as an authoritative work in the areas it does cover. On the contrary, its primary aim is simply to bridge the gap between an elementary probability course and the many excellent advanced works on stochastic processes.

Readers of this book are assumed to be familiar with the elementary theory of probability as presented in the first half of Feller's classic *Introduction to Probability Theory and Its Applications*. In Section 1, Chapter 1 of my book the necessary background material is presented and the terminology and notation of the book established. Discussions in small print can be skipped on first reading. Exercises are provided at the close of each chapter to help illuminate and expand on the theory.

This book can serve for either a one-semester or a two-semester course, depending on the extent of coverage desired.

In writing this book, I have drawn on the vast literature on stochastic processes. Each chapter ends with citations of books that may profitably be consulted for further information, including in many cases bibliographical listings.

I am grateful to Stanford University and to the U.S. Office of Naval Research for providing facilities, intellectual stimulation, and financial support for the writing of this text. Among my academic colleagues I am grateful to Professor K. L. Chung and Professor J. McGregor of Stanford for their constant encouragement and helpful comments; to Professor J. Lamperti of Dartmouth, Professor J. Kiefer of Cornell, and Professor P. Ney of Wisconsin for offering a variety of constructive criticisms; to Dr. A. Feinstein for his detailed checking of substantial sections of the manuscript, and to my students P. Milch, B. Singer, M. Feldman, and B. Krishnamoorthi for their helpful suggestions and their assistance in organizing the exercises. Finally, I am indebted to Gail Lemmond and Rosemarie Stampfel for their superb technical typing and all-around administrative care.

SAMUEL KARLIN

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Chapter 1

ELEMENTS OF STOCHASTIC PROCESSES

The first part of this chapter summarizes the necessary background material and establishes the terminology and notation of the book. It is suggested that the reader not dwell here assidously, but rather quickly. It can be reviewed further if the need should arise later.

Section 2 introduces the celebrated Brownian motion and Poisson processes, and Section 3 surveys some of the broad types of stochastic processes that are the main concern of the remainder of the book.

The last section, included for completeness, discusses some technical considerations in the general theory. The section should be skipped on a first reading.

1: *Review of Basic Terminology and Properties of Random Variables and Distribution Functions*

The present section contains a brief review of the basic elementary notions and terminology of probability theory. The contents of this section will be used freely throughout the book without further reference. We urge the student to tackle the problems at the close of the chapter; they provide practice and help to illuminate the concepts. For more detailed treatments of these topics, the student may consult any good standard text for a first course in probability theory (see references at close of this chapter).

The following concepts will be assumed familiar to the reader:

(1) A real random variable X.
(2) The distribution function F of X [defined by $F(\lambda) = \Pr\{X \le \lambda\}$] and its elementary properties.
(3) An event pertaining to the random variable X, and the probability thereof.
(4) $E\{X\}$, the expectation of X, and the higher moments $E\{X^n\}$.
(5) The law of total probabilities and Bayes rule for computing probabilities of events.

The abbreviation r.v. will be used for "real random variables." A r.v.

X is called *discrete* if there is a finite or denumerable set of distinct values $\lambda_1, \lambda_2, \ldots$ such that $a_i \equiv \Pr\{X = \lambda_i\} > 0$, $i = 1, 2, 3, \ldots$, and $\sum_i a_i = 1$. If $\Pr\{X = \lambda\} = 0$ for every value of λ, the r.v. X is called *continuous*. If there is a nonnegative function $p(t)$, defined for $-\infty < t < \infty$ such that the distribution function F of the r.v. X is given by

$$F(\lambda) = \int_{-\infty}^{\lambda} p(t)\, dt,$$

then p is said to be the probability density of X. If X has a probability density, then it is necessarily continuous; however, examples are known of continuous r.v.'s which do not possess probability densities.

If X is a discrete r.v., then its mth moment is given by

$$E[X^m] = \sum_i \lambda_i^m \Pr\{X = \lambda_i\}$$

(where the λ_i are as earlier), if the series converges absolutely.

If X is a continuous r.v. with probability density $p(\cdot)$, its mth moment is given by

$$E[X^m] = \int_{-\infty}^{\infty} x^m p(x)\, dx,$$

provided the integral converges absolutely.

The first moment of X, commonly called the *mean*, is denoted by m_X or μ_X. The mth central moment of X is defined as the mth moment of the r.v. $X - m_X$ if m_X exists. The first central moment is evidently zero; the second central moment is called the *variance* (σ_X^2) *of* X. The *median* of a r.v. X is any value v with the property that $\Pr\{X \geq v\} \geq \frac{1}{2}$ and $\Pr\{X \leq v\} \geq \frac{1}{2}$.

If X is a random variable and g is a function, then $Y = g(X)$ is also a random variable. If X is a discrete random variable with possible values $x_1, x_2, \ldots$, then the expectation of $g(X)$ is given by

$$E[g(X)] = \sum_{i=1}^{\infty} g(x_i) \Pr\{X = x_i\} \tag{1.1}$$

provided the sum converges absolutely. If X is continuous and has the probability density function p_X then the expectation of $g(X)$ is computed from

$$E[g(X)] = \int g(x) p_X(x)\, dx. \tag{1.2}$$

The general formula, covering both the discrete and continuous cases is

$$E[g(X)] = \int g(x)\, dF_X(x) \tag{1.3}$$

where F_X is the distribution function of the random variable X. Technically speaking, the integral in (1.3) is called a Lebesgue-Stieltjes integral. We do not require knowledge of such integrals in this text but interpret (1.3) to signify (1.1) when X is a discrete random variable and to represent (1.2) when X possesses a probability density function p_X.

Let $F_Y(y) = \Pr\{Y \leq y\}$ denote the distribution function for $Y = g(X)$. When X is a discrete random variable

$$\begin{aligned} E[Y] &= \sum y_i \Pr\{Y = y_i\} \\ &= \sum g(x_i) \Pr\{X = x_i\} \end{aligned}$$

if $y_i = g(x_i)$ and provided the second sum converges absolutely. In general

$$\begin{aligned} E[Y] &= \int y \, dF_Y(y) \\ &= \int g(x) \, dF_X(x). \end{aligned} \tag{1.4}$$

If X is a discrete random variable then so is $Y = g(X)$. It may be, however, that X is a continuous random variable while Y is discrete (the student should provide an example). Even so, one may compute $E[Y]$ from either form in (1.4) with the same result.

A. JOINT DISTRIBUTION FUNCTIONS

Given a pair (X, Y) of r.v.'s, their joint distribution function is the function F_{XY} of two real variables given by

$$F(\lambda_1, \lambda_2) = F_{XY}(\lambda_1, \lambda_2) = \Pr\{X \leq \lambda_1, \ Y \leq \lambda_2\}.$$

(The subscripts X, Y will usually be omitted unless there is possible ambiguity.)

The function $F(\lambda, +\infty) \equiv \lim_{\lambda_2 \to \infty} F(\lambda, \lambda_2)$ is a probability distribution function, called the ***marginal distribution function*** of X. Similarly, the function $F(+\infty, \lambda)$ is called the marginal distribution of Y. If it happens that $F(\lambda_1, +\infty) \cdot F(+\infty, \lambda_2) = F(\lambda_1, \lambda_2)$ for every choice of λ_1, λ_2, then the r.v.'s X and Y are said to be ***independent***. A joint distribution function F_{XY} is said to possess a (joint) probability density if there exists a function $p_{XY}(s, t)$ of two real variables such that

$$F_{XY}(\lambda_1, \lambda_2) = \int_{-\infty}^{\lambda_2} \int_{-\infty}^{\lambda_1} p_{XY}(s, t) \, ds \, dt$$

for all λ_1, λ_2. If X and Y are independent, then $p_{XY}(s, t)$ is necessarily of the form $p_X(s)p_Y(t)$, where p_X and p_Y are the probability densities of the marginal distribution of X and Y, respectively.

The joint distribution function of any finite collection $X_1, \ldots, X_n$ of random variables is defined as the function

$$\begin{aligned} F(\lambda_1, \ldots, \lambda_n) &= F_{X_1,\ldots,X_n}(\lambda_1, \ldots, \lambda_n) \\ &= \Pr\{X_1 \le \lambda_1, \ldots, X_n \le \lambda_n\}. \end{aligned}$$

The distribution function

$$F_{X_{i_1},\ldots,X_{i_k}}(\lambda_{i_1}, \ldots, \lambda_{i_k}) = \lim_{\lambda_i \to \infty,\, i \ne i_1,\ldots,i_k} F(\lambda_1, \ldots, \lambda_n)$$

is called the marginal distribution of the random variables $X_{i_1}, \ldots, X_{i_k}$.

If $F(\lambda_1, \ldots, \lambda_n) = F_{X_1}(\lambda_1) \cdot \ldots \cdot F_{X_n}(\lambda_n)$ for all values of $\lambda_1, \lambda_2, \ldots, \lambda_n$, the random variables $X_1, \ldots, X_n$ are said to be independent.

A joint distribution function $F(\lambda_1, \ldots, \lambda_n)$ is said to have a probability density if there exists a nonnegative function $p(t_1, \ldots, t_n)$ of n variables such that

$$F(\lambda_1, \ldots, \lambda_n) = \int_{-\infty}^{\lambda_n} \cdots \int_{-\infty}^{\lambda_1} p(t_1, \ldots, t_n)\, dt_1 \cdots dt_n$$

for all real $\lambda_1, \ldots, \lambda_n$.

If X and Y are jointly distributed random variables having means m_X and m_Y, respectively, their covariance (σ_{XY}) is the product moment

$$\sigma_{XY} = E[(X - m_X)(Y - m_Y)].$$

If X_1 and X_2 are independent random variables having the distribution functions F_1 and F_2, respectively, then the distribution function F of the sum $X = X_1 + X_2$ is the *convolution* of F_1 and F_2:

$$\begin{aligned} F(x) &= \int F_1(x-y)\, dF_2(y) \\ &= \int F_2(x-y)\, dF_1(y). \end{aligned}$$

Specializing to the situation where X_1 and X_2 have the probability densities p_1 and p_2, the density function p of the sum $X = X_1 + X_2$ is the convolution of the densities p_1 and p_2:

$$\begin{aligned} p(x) &= \int p_1(x-y)p_2(y)\, dy \\ &= \int p_2(x-y)p_1(y)\, dy. \end{aligned}$$