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The FINITE ELEMENT METHOD

有限元法

Volume 1

THE BASIS

FIFTH EDITION

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The Finite Element Method

Fifth edition

Volume 1: The Basis

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Dedication

This book is dedicated to our wives Helen and Mary Lou and our families for their support and patience during the preparation of this book, and also to all of our students and colleagues who over the years have contributed to our knowledge of the finite element method. In particular we would like to mention Professor Eugenio Oñate and his group at CIMNE for their help, encouragement and support during the preparation process.

Preface

It is just over thirty years since *The Finite Element Method in Structural and Continuum Mechanics* was first published. This book, which was the first dealing with the finite element method, provided the base from which many further developments occurred. The expanding research and field of application of finite elements led to the second edition in 1971, the third in 1977 and the fourth in 1989 and 1991. The size of each of these volumes expanded geometrically (from 272 pages in 1967 to the fourth edition of 1455 pages in two volumes). This was necessary to do justice to a rapidly expanding field of professional application and research. Even so, much filtering of the contents was necessary to keep these editions within reasonable bounds.

It seems that a new edition is necessary every decade as the subject is expanding and many important developments are continuously occurring. The present fifth edition is indeed motivated by several important developments which have occurred in the 90s. These include such subjects as adaptive error control, meshless and point based methods, new approaches to fluid dynamics, etc. However, we feel it is important not to increase further the overall size of the book and we therefore have eliminated some redundant material.

Further, the reader will notice the present subdivision into three volumes, in which the first volume provides the general basis applicable to linear problems in many fields whilst the second and third volumes are devoted to more advanced topics in solid and fluid mechanics, respectively. This arrangement will allow a general student to study Volume 1 whilst a specialist can approach their topics with the help of Volumes 2 and 3. Volumes 2 and 3 are much smaller in size and addressed to more specialized readers.

It is hoped that Volume 1 will help to introduce postgraduate students, researchers and practitioners to the modern concepts of finite element methods. In Volume 1 we stress the relationship between the finite element method and the more classic finite difference and boundary solution methods. We show that all methods of numerical approximation can be cast in the same format and that their individual advantages can thus be retained.

Although Volume 1 is not written as a course text book, it is nevertheless directed at students of postgraduate level and we hope these will find it to be of wide use. Mathematical concepts are stressed throughout and precision is maintained, although little use is made of modern mathematical symbols to ensure wider understanding amongst engineers and physical scientists.

In Volumes 1, 2 and 3 the chapters on computational methods are much reduced by transferring the computer source programs to a web site.[1] This has the very substantial advantage of not only eliminating errors in copying the programs but also in ensuring that the reader has the benefit of the most recent set of programs available to him or her at all times as it is our intention from time to time to update and expand the available programs.

The authors are particularly indebted to the International Center of Numerical Methods in Engineering (CIMNE) in Barcelona who have allowed their pre- and post-processing code (GiD) to be accessed from the publisher's web site. This allows such difficult tasks as mesh generation and graphic output to be dealt with efficiently. The authors are also grateful to Dr J.Z. Zhu for his careful scrutiny and help in drafting Chapters 14 and 15. These deal with error estimation and adaptivity, a subject to which Dr Zhu has extensively contributed. Finally, we thank Peter and Jackie Bettess for writing the general subject index.

OCZ and RLT

[1] Complete source code for all programs in the three volumes may be obtained at no cost from the publisher's web page: http://www.bh.com/companions/fem

Contents

1

Some preliminaries: the standard discrete system

1.1 Introduction

The limitations of the human mind are such that it cannot grasp the behaviour of its complex surroundings and creations in one operation. Thus the process of subdividing all systems into their individual components or 'elements', whose behaviour is readily understood, and then rebuilding the original system from such components to study its behaviour is a natural way in which the engineer, the scientist, or even the economist proceeds.

In many situations an adequate model is obtained using a finite number of well-defined components. We shall term such problems *discrete*. In others the subdivision is continued indefinitely and the problem can only be defined using the mathematical fiction of an infinitesimal. This leads to differential equations or equivalent statements which imply an infinite number of elements. We shall term such systems *continuous*.

With the advent of digital computers, *discrete* problems can generally be solved readily even if the number of elements is very large. As the capacity of all computers is finite, *continuous* problems can only be solved exactly by mathematical manipulation. Here, the available mathematical techniques usually limit the possibilities to oversimplified situations.

To overcome the intractability of realistic types of continuum problems, various methods of *discretization* have from time to time been proposed both by engineers and mathematicians. All involve an *approximation* which, hopefully, approaches in the limit the true continuum solution as the number of discrete variables increases.

The discretization of continuous problems has been approached differently by mathematicians and engineers. Mathematicians have developed general techniques applicable directly to differential equations governing the problem, such as finite difference approximations,[1,2] various weighted residual procedures,[3,4] or approximate techniques for determining the stationarity of properly defined 'functionals'. The engineer, on the other hand, often approaches the problem more intuitively by creating an analogy between real discrete elements and finite portions of a continuum domain. For instance, in the field of solid mechanics McHenry,[5] Hrenikoff,[6] Newmark[7], and indeed Southwell[9] in the 1940s, showed that reasonably good solutions to an elastic continuum problem can be obtained by replacing small portions of the continuum by an arrangement of simple elastic bars. Later, in the same context, Argyris[8] and Turner *et al.*[9] showed that a more direct, but no less intuitive, substitution of properties can be made much more effectively by considering that small portions or 'elements' in a continuum behave in a simplified manner.

It is from the engineering 'direct analogy' view that the term 'finite element' was born. Clough[10] appears to be the first to use this term, which implies in it a direct use of a *standard methodology applicable to discrete systems*. Both conceptually and from the computational viewpoint, this is of the utmost importance. The first allows an improved understanding to be obtained; the second offers a unified approach to the variety of problems and the development of standard computational procedures.

Since the early 1960s much progress has been made, and today the purely mathematical and 'analogy' approaches are fully reconciled. It is the object of this text to present a view of the finite element method as *a general discretization procedure of continuum problems posed by mathematically defined statements.*

In the analysis of problems of a discrete nature, a standard methodology has been developed over the years. The civil engineer, dealing with structures, first calculates force–displacement relationships for each element of the structure and then proceeds to assemble the whole by following a well-defined procedure of establishing local equilibrium at each 'node' or connecting point of the structure. The resulting equations can be solved for the unknown displacements. Similarly, the electrical or hydraulic engineer, dealing with a network of electrical components (resistors, capacitances, etc.) or hydraulic conduits, first establishes a relationship between currents (flows) and potentials for individual elements and then proceeds to assemble the system by ensuring continuity of flows.

All such analyses follow a standard pattern which is universally adaptable to discrete systems. It is thus possible to define a *standard discrete system*, and this chapter will be primarily concerned with establishing the processes applicable to such systems. Much of what is presented here will be known to engineers, but some reiteration at this stage is advisable. As the treatment of elastic solid structures has been the most developed area of activity this will be introduced first, followed by examples from other fields, before attempting a complete generalization.

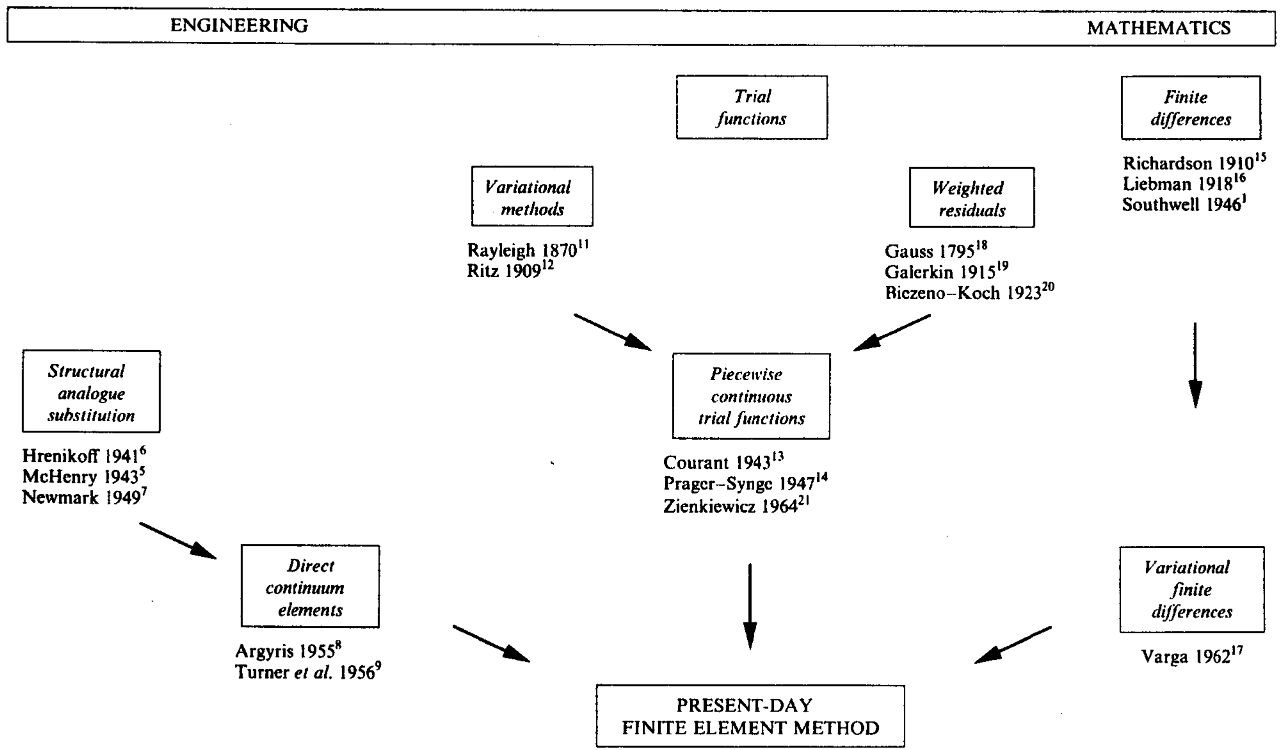
The existence of a unified treatment of 'standard discrete problems' leads us to the first definition of the finite element process as a method of approximation to continuum problems such that

(a) the continuum is divided into a finite number of parts (elements), the behaviour of which is specified by a finite number of parameters, and
(b) the solution of the complete system as an assembly of its elements follows precisely the same rules as those applicable to *standard discrete problems*.

It will be found that most classical mathematical approximation procedures as well as the various direct approximations used in engineering fall into this category. It is thus difficult to determine the origins of the finite element method and the precise moment of its invention.

Table 1.1 shows the process of evolution which led to the present-day concepts of finite element analysis. Chapter 3 will give, in more detail, the mathematical basis which emerged from these classical ideas.[11–20]

Table 1.1


ENGINEERING
MATHEMATICS
Trial functions
Finite differences
Richardson 1910[15]
Liebman 1918[16]
Southwell 1946[1]
Variational methods
Rayleigh 1870[11]
Ritz 1909[12]
Weighted residuals
Gauss 1795[18]
Galerkin 1915[19]
Biczeno–Koch 1923[20]
Structural analogue substitution
Hrenikoff 1941[6]
McHenry 1943[5]
Newmark 1949[7]
Piecewise continuous trial functions
Courant 1943[13]
Prager–Synge 1947[14]
Zienkiewicz 1964[21]
Direct continuum elements
Argyris 1955[8]
Turner et al. 1956[9]
Variational finite differences
Varga 1962[17]
PRESENT-DAY FINITE ELEMENT METHOD

1.2 The structural element and the structural system

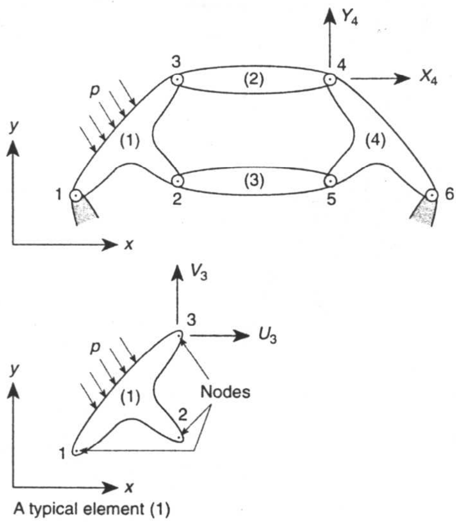


Fig. 1.1 A typical structure built up from interconnected elements.

To introduce the reader to the general concept of discrete systems we shall first consider a structural engineering example of linear elasticity.

Figure 1.1 represents a two-dimensional structure assembled from individual components and interconnected at the nodes numbered 1 to 6. The joints at the nodes, in this case, are pinned so that moments cannot be transmitted.

As a starting point it will be assumed that by separate calculation, or for that matter from the results of an experiment, the characteristics of each element are precisely known. Thus, if a typical element labelled (1) and associated with nodes 1, 2, 3 is examined, the forces acting at the nodes are uniquely defined by the displacements of these nodes, the distributed loading acting on the element (p), and its initial strain. The last may be due to temperature, shrinkage, or simply an initial 'lack of fit'. The forces and the corresponding displacements are defined by appropriate components (U, V and u, v) in a common coordinate system.

Listing the forces acting on all the nodes (three in the case illustrated) of the element (1) as a matrix† we have

$$\mathbf{q}^1 = \begin{Bmatrix} \mathbf{q}_1^1 \\ \mathbf{q}_2^1 \\ \mathbf{q}_3^1 \end{Bmatrix} \qquad \mathbf{q}_1^1 = \begin{Bmatrix} U_1 \\ V_1 \end{Bmatrix}, \qquad \text{etc.} \tag{1.1}$$

† A limited knowledge of matrix algebra will be assumed throughout this book. This is necessary for reasonable conciseness and forms a convenient book-keeping form. For readers not familiar with the subject a brief appendix (Appendix A) is included in which sufficient principles of matrix algebra are given to follow the development intelligently. Matrices (and vectors) will be distinguished by bold print throughout.