

**Phase Transition Approach to  
High Temperature Superconductivity  
Universal Properties  
of Cuprate Superconductors**  
**高温超导中的相变方法**

**T. Schneider & J. M. Singer**

Imperial College Press  
世界图书出版公司

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**T. Schneider & J. M. Singer**

University of Zurich, Switzerland

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SUPERCONDUCTIVITY**

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# Preface

The discovery of superconductivity at 30K by Bednorz and Müller in 1986 ignited an explosion of interest in high temperature superconductivity. The initial development rapidly evolved into an intense worldwide research effort that still persists after more than a decade to understand the phenomenon of cuprate superconductivity, to search for ways to raise the transition temperature and to produce materials which have the potential for technological applications. During the past decade of research on this subject significant progress has been made on both the fundamental science and technological application fronts. A great number of experimental data is now available on the cuprates, and various properties have been well characterized using high quality single crystals and thin films. Despite this enormous research effort, the underlying mechanisms responsible for superconductivity in the cuprates are still open to question. Nevertheless, we believe that the overall picture of the experimental situation is clear enough to warrant the writing of a textbook that presents our understanding from the phase transition point of view, surveys and identifies thermal and quantum fluctuation effects, identifies material independent universal properties and provides constraints for the microscopic description of the various phenomena. The aim is to present this material in a format suitable for the use in a graduate level course.

A survey of phase transitions in cuprate superconductors, including the normal conductor to superconductor, normal conductor to insulator and superconductor to insulator transitions, tuned by a variation of temperature, doping, magnetic field, disorder etc., must be based on a background of theory of thermal and quantum critical phenomena, found in textbooks and review articles. Brief introductory chapters and appendices covering special topics provide a sketch of this background.

Topics discussed include:

- Experimental evidence for classical critical behavior: critical behavior close to optimum doping; specific heat, penetration depth, magnetization, crossing point phenomenon, magnetic torque, magnetic field tuned phase transitions, doping dependence of finite temperature critical properties, universal scaling functions, universal relation between specific heat, transition temperature, correlation length and penetration depths at criticality, finite size scaling, corrections to scaling, dimensional crossover, Kosterlitz-Thouless transitions in thin films.
- Evidence for quantum critical properties: superconductor to insulator transition in two dimensions; universal relation between transition temperature and zero temperature penetration depth; universal conductance at criticality; nature of the insulating phase.
- Implications: failure of the interlayer tunneling model, suppression of the transition temperature due to dimensional crossover and quantum fluctuations, pseudogap features, doping and pressure dependence of critical amplitudes, Bose gas approach, effective pair mass and emerging phase diagrams.

The immense scope of this subject dictated a selective choice of the experimental data cited to illustrate the progress made in this field over the past decade. For comprehensive accounts of specific topics in high temperature superconductivity the reader is referred to various review articles.

Finally, we are pleased to acknowledge the assistance of many colleagues in encouraging our efforts, in helping to guide the focus of several revisions, and on providing generous assistance in improving the quality of the presentation. We have benefited from discussions with (or encouragement by) J. Hofer, M. Willemin, H. Keller, K. A. Müller, C. Meingast, H. Beck, P. Martinoli, J. Perret, Ø. Fischer, J.-M. Triscone, G. Triscone, A. Junod, M. Roulin, C. Rossel, P. F. Meier, E. Stoll, I. Herbut, Z. Tesanovic, F. Nogueira, A. Sudbø, V. Loktev and J. Engelbrecht (we apologize in advance to all those inadvertently omitted). Moreover, we appreciate the support given by the Physics Department of the University of Zurich, the Swiss National Science Foundation and Imperial College Press, London.

*Zurich, December 1999*

*T. Schneider and J. M. Singer*

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# Chapter 1

## Introduction

### 1.1 Cuprate superconductors

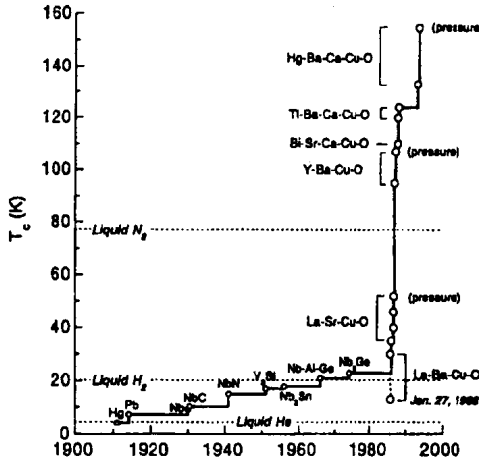


FIGURE 1.1: Maximum superconducting transition temperature  $T_c$  versus time. Taken from [189].

The dramatic increases in the transition temperature  $T_c$  that has been observed since 1986 are illustrated in Fig. 1.1 where the maximum value of  $T_c$  is plotted versus date. Prior to 1986, the A15 compound  $\text{Nb}_3\text{Ge}$  with  $T_c \approx 23\text{K}$  held the record for the highest value of  $T_c$  [85]. After the discovery of superconductivity at 30K in the cuprate system  $\text{La-Ba-Cu-O}$  [24], the maximum value of  $T_c$  has increased steadily to its current maximum value in  $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_8$

[260]. When this compound is subjected to pressure [43], the critical temperature  $T_c$  increases to 166K.

### 1.1.1 Structure

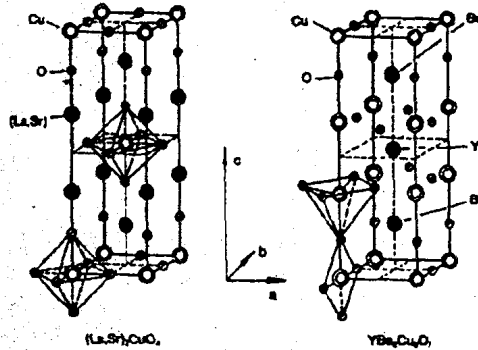


FIGURE 1.2: Schematic crystal structure of  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ . Taken from [266].

Approximately 100 different cuprate materials, many of which are superconducting, have been discovered since 1986. These materials are layered perovskite-type structures which consist of  $\text{CuO}_2$  planes separated by layers comprised of other elements A and oxygen,  $\text{A}_m\text{O}_n$ , and, in some cases, layers of Ln ions. The  $\text{A}_m\text{O}_n$  layers act as charge reservoirs and control the doping of the  $\text{CuO}_2$  planes with charge carriers. The mobile charge carriers are believed to reside primarily within the  $\text{CuO}_2$  planes. In several of the compounds containing Ln layers, the Ln ions with partially-filled  $4f$  electron shells and magnetic moments have been found to order antiferromagnetically at low temperatures [190]. Fig. 1.2 shows the structure of two of the most extensively studied materials,  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ . Their so-called *parent compounds* are  $\text{La}_2\text{CuO}_4$  and  $\text{YBa}_2\text{Cu}_3\text{O}_6$ , referring to the undoped, antiferromagnetic, non-superconducting structure. The  $\text{La}_2\text{CuO}_4$  layering scheme consists of equally spaced,  $\text{CuO}_2$  sheets, with the oxygen atoms stacked one atop the other, the copper ions alternating between two sites in adjacent layers. Half of the oxygen are in the planes, and the other half between them. In  $\text{La}_2\text{CuO}_4$  the unit cell contains two  $\text{CuO}_2$  planes, with a spacing of  $\approx 6\text{\AA}$ . We see from Fig. 1.2 that in  $\text{YBa}_2\text{Cu}_3\text{O}_7$  three planes containing Cu and O are sandwiched between two planes containing Ba and O and one plane containing Y. There are two  $\text{CuO}_2$  planes that have the Y plane between them. A third

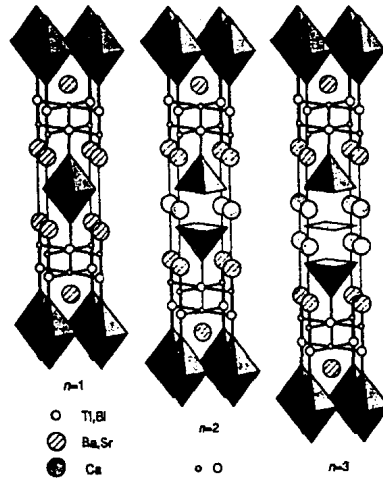


FIGURE 1.3: Multilayer materials: Structures of 2201, 2212 and 2223 Bi - Sr - Ca - Cu - O (BSCCO) and Tl - Ba - Ca - Cu - O (TBCCO) systems with  $n = 1, 2, 3$  directly adjacent  $\text{CuO}_2$ -layers. Taken from [5].

$\text{CuO}$ -plane, often referred to as chains, consists of  $-\text{Cu} - \text{O} - \text{Cu} - \text{O}-$  chains along the  $b$ -axis. Thus, there are two  $\text{CuO}_2$  planes per unit cell. Even three or more closely spaced  $\text{CuO}_2$  planes are realized in some of the Bi, Tl and Hg based cuprates (see Fig. 1.3).

### 1.1.2 Doping

Metallic behavior is derived from the insulating and antiferromagnetic parent compounds by partial substitution of out-of-plane ions or by adding or removing oxygen. In  $\text{YBa}_2\text{Cu}_3\text{O}_7$  hole doping is achieved by changing the oxygen content from  $\text{O}_{6.5}$  to  $\text{O}_{7-x}$ , while  $\text{La}_2\text{CuO}_4$  can be doped either by alkaline earth ions or oxygen to exhibit superconducting properties. The phase diagram of  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  depicted in Fig. 1.4 shows that holes are very efficient in destroying the antiferromagnetic long-range ordered state. Indeed, for  $x = 0$ ,  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  is a highly anisotropic antiferromagnetic insulator ( $T_N \approx 300\text{K}$ ). It is well described by a  $2D$   $S = 1/2$  Heisenberg antiferromagnet. On adding Sr,  $T_N$  drops very sharply and vanishes at  $x \approx 0.01$ . Then, there is the spin glass phase, and magnetic short range order does persist up to fairly high doping levels, as elucidated by inelastic neutron scattering. Electrical resistivity measurements reveal, that at a certain doping level – the so

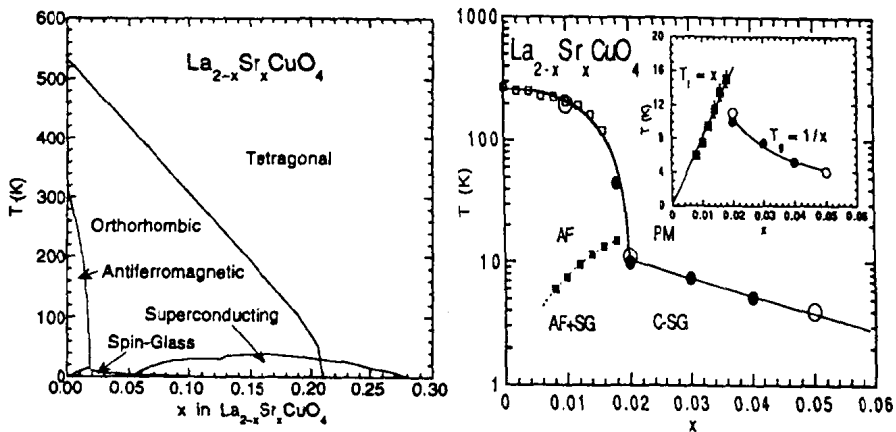


FIGURE 1.4: Left panel: structural, magnetic and superconducting phase diagram of  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ . Taken from [141]. Right panel:  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  in the doping regime  $x < 0.06$ . Abbreviations: PM, paramagnetic; AF, antiferromagnetic; SG, spin-glass; CSG, cluster spin-glass;  $T_f$  and  $T_g$ , spin-glass freezing temperatures for  $x < 0.02$  and  $x > 0.02$ , respectively. Taken from [142].

called underdoped limit  $x_u$  – the system undergoes at  $T > 0$  an insulator to anomalous conductor transition and at  $T = 0$  an insulator to superconductor transition [299]. In  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  this occurs close to  $x_u \approx 0.05$ , where  $x$  is the hole concentration per Cu atom in the  $\text{CuO}_2$  plane. Many inelastic neutron scattering measurements on the  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  system have been carried out. They established that AF correlations survive into the normal conducting as well as the superconducting regime, which may be important with regard to the mechanism for high temperature superconductivity of the layered cuprates. As the hole concentration is increased, the superconducting transition temperature  $T_c$  rises, reaches a maximum value  $T_c^m$  at  $x_m$ . In the so called underdoped regime  $x_u \leq x \leq x_m$  the material exhibits anomalous normal state properties. With further increase of  $x$  the compound becomes more metallic, but now  $T_c$  decreases and finally vanishes in the overdoped limit  $x_o$ . At  $x_o$  a superconductor to normal conductor transition is expected to occur at  $T = 0$ . Such a  $(T, x)$  phase diagram appears to be generic for cuprate superconductors. Examples include  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ ,  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ ,  $\text{Y}_2\text{Ba}_4\text{Cu}_7\text{O}_{15+x}$  [87],  $\text{Bi}_2\text{Sr}_2\text{CuO}_{6+x}$ ,  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  [102],  $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+x}$  [223] and  $\text{HgBa}_2\text{CuO}_{4+x}$  [82]. Experimental data for the doping dependence of the superconductor transition temperature are shown in Fig. 1.5.

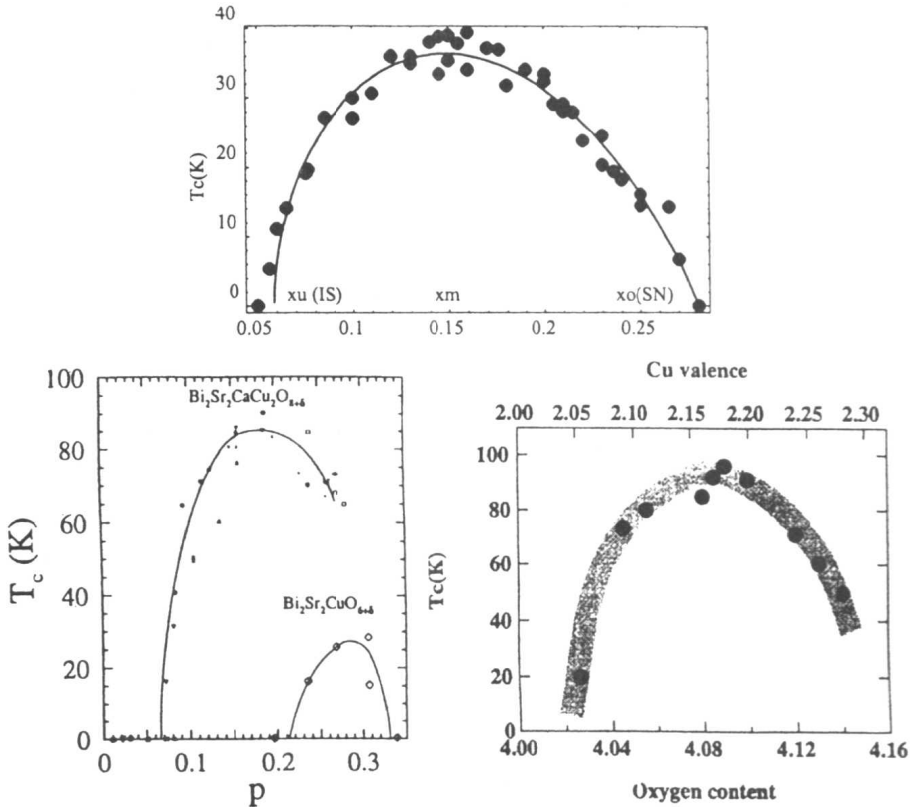


FIGURE 1.5: Upper Panel: Superconducting transition temperature  $T_c$  versus  $x$  for bulk  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ . Data taken from [299], [307], [208] and [83].  $x_u$  and  $x_o$  denote the under- and overdoped limits, respectively, where  $T_c$  vanishes, while at  $x_m$ , corresponding to optimal doping, the transition temperature reaches its maximum value. Lower panel, left:  $T_c$  versus hole concentration  $p$  for  $\text{Bi}_2\text{Sr}_2\text{CuO}_{6+p}$  and  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+p}$ , taken from [102]. Lower panel, right:  $T_c$  versus hole concentration  $\delta$  for  $\text{HgBa}_2\text{CuO}_x$ , taken from [82].

Apparently, a unique property of cuprate superconductors is the occurrence of a phase transition line  $T(x)$  in the temperature-doping plane, which separates the superconducting from the normal conducting phase. It should be recognized that this feature is absent in conventional superconductors. In  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ ,  $\text{Bi}_2\text{Sr}_2\text{CuO}_{6+x}$  and  $\text{HgBa}_2\text{CuO}_{4+x}$  both, the underdoped and overdoped limits are experimentally accessible, while in other compounds, including  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ , only the underdoped and optimally doped regimes appear to be accessible. Another prominent example is  $\text{Y}_{1-x}\text{Pr}_x\text{Ba}_2\text{Cu}_3\text{O}_{7-\delta}$ , which becomes more and more underdoped as  $x$  increases and  $T_c$  vanishes at  $x \approx 0.55$ .

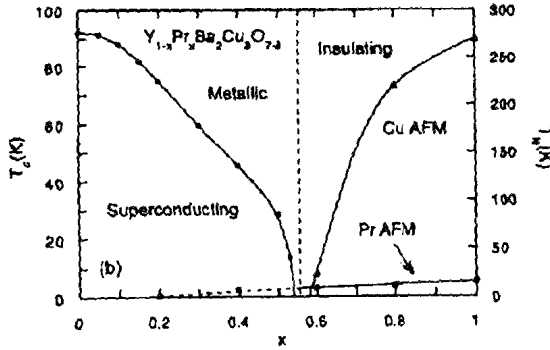


FIGURE 1.6: Phase diagram in the  $(T, x)$  - plane of  $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$ , showing metallic, superconducting, insulating and antiferromagnetically ordered regions. Taken from [214].

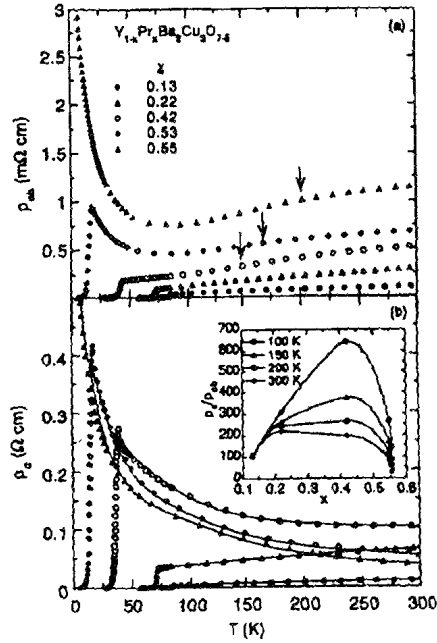


FIGURE 1.7: In-plane resistivity  $\rho_{ab}(T)$  (a) and out-of-plane resistivity  $\rho_c(T)$  (b) for a  $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$  single crystal. Inset to panel (b): anisotropy  $\rho_{ab}/\rho_c$  versus  $x$  at different temperatures. Taken from [139].



Displayed in Fig. 1.6 is the  $(T, x)$  phase diagram of this material. In any case, the occurrence of this phase transition line and the associated two critical endpoints, occurring in the underdoped and overdoped limits where  $T_c$  vanishes, leads to the important question, how the ground state of the cuprates evolves as a function of doping.

This evolution is reflected in the temperature dependence of the *ab*-plane and *c*-axis resistivities  $\rho_{ab}(T)$  and  $\rho_c(T)$  (see, e.g., [48]). Both  $\rho_{ab}(T)$  and  $\rho_c(T)$  exhibit in the underdoped region a crossover from metallic conducting (i.e.,  $d\rho/dT > 0$ ) to semiconducting behavior (i.e.,  $d\rho/dT < 0$ ) with decreasing temperature,  $\rho_c(T)$  is semiconducting or metallic close to optimal doping, depending on the material, and  $\rho_{ab}(T)$  and  $\rho_c(T)$  are both metallic in the overdoped region. As an example of the evolution of  $\rho_{ab}(T)$  and  $\rho_c(T)$  with doping we refer to the data shown in Fig. 1.7 for a  $Y_{1-x}Pr_xBa_2Cu_3O_{7-\delta}$  single crystal. It should be noted, however, that insulating features of  $\rho_c(T)$  close to optimal doping are most likely due to incoherent tunneling [139]. The generic phase diagram of the cuprates as a function of dopant concentration appears to be a critical line  $T_c(x)$ . At the critical endpoints, corresponding to the underdoped ( $x = x_u$ ) and overdoped ( $x = x_o$ ) limits,  $T_c$  vanishes, so that at  $T = 0$  and  $x = x_u$  a doping tuned insulator to superconductor (IS) transition, and at  $x = x_o$  a superconductor to normal metallic conductor (SN) transition occurs. Resistivity measurements in 60T pulsed magnetic fields to quench superconductivity also uncovered the corresponding insulator to normal-conductor transition in underdoped  $La_{2-x}Sr_xCuO_4$  [28]. Noting that at  $T = 0$  no thermal fluctuations are left, quantum fluctuations are expected to play an essential role in the zero temperature phase transitions driven by the dopand concentration.

### 1.1.3 Effective mass anisotropy and spatial dimensionality

Critical properties of a phase transition depend crucially on the spatial dimensionality of the system. Useful information on the dimensionality of cuprate superconductors is obtained from the effective mass anisotropy entering the Ginzburg-Landau Hamiltonian (for a detailed discussion we refer the reader to Sec. 2),

$$\mathcal{H} = \int d^3R \left( \sum_i \frac{\hbar^2}{2M_i} \left| \frac{\partial}{\partial x_i} \Psi \right|^2 - r|\Psi|^2 + \frac{u}{2}|\Psi|^4 \right), \quad i = x, y, z. \quad (1.1)$$