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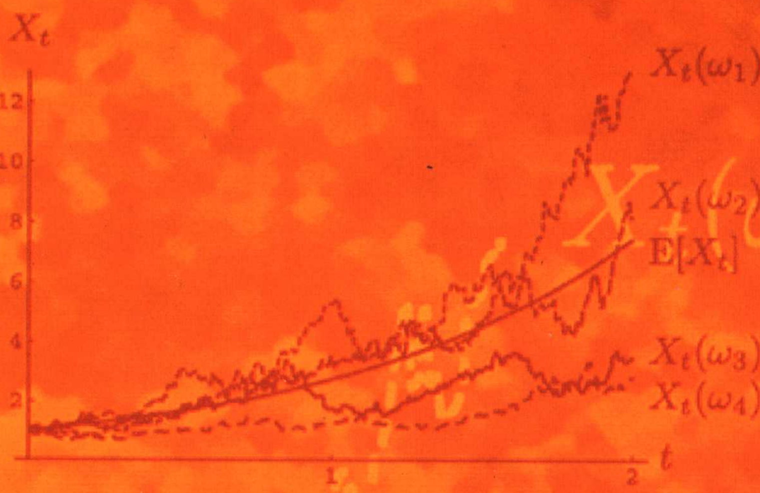

Bernt Øksendal

Stochastic Differential Equations

An Introduction with Applications

Sixth Edition

随机微分方程 第6版

Springer

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To My Family

Eva, Elise, Anders and Karina

We have not succeeded in answering all our problems. The answers we have found only serve to raise a whole set of new questions. In some ways we feel we are as confused as ever, but we believe we are confused on a higher level and about more important things.

Posted outside the mathematics reading room,
Tromsø University

Preface to the Third Corrected Printing of the Sixth Edition

In this Second Corrected Printing some misprints have been corrected and other improvements in the presentation have been carried out. The exercises to which a (partial or complete) solution is provided (in the back of the book) are denoted by an asterix *. I wish to thank the following persons (in alphabetical order) for their helpful comments:

Holger van Bargen, Catriona Byrne, Mark Davis, Per-Ivar Faust, Samson Jinya, Paul Kettler, Alex Krouglov, Mauro Mariani, John O'Hara, Agnès Sulem, Bjørn Thunestvedt and Vegard Trondsen.

My special thanks go to Dina Haraldsson for her careful and skilled typing.

Blindern, August 2005
Bernt Øksendal

Preface to the Sixth Edition

This edition contains detailed solutions of selected exercises. Many readers have requested this, because it makes the book more suitable for self-study. At the same time new exercises (without solutions) have beed added. They have all been placed in the end of each chapter, in order to facilitate the use of this edition together with previous ones.

Several errors have been corrected and formulations have been improved. This has been made possible by the valuable comments from (in alphabetical order) Jon Bohlin, Mark Davis, Helge Holden, Patrick Jaillet, Chen Jing, Natalia Koroleva, Mario Lefebvre, Alexander Matasov, Thilo Meyer-Brandis, Keigo Osawa, Bjørn Thunestvedt, Jan Ubøe and Yngve Williassen. I thank them all for helping to improve the book.

My thanks also go to Dina Haraldsson, who one again has performed the typing and drawn the figures with great skill.

Blindern, September 2002
Bernt Øksendal

Preface to Corrected Printing, Fifth Edition

The main corrections and improvements in this corrected printing are from Chapter 12. I have benefitted from useful comments from a number of people, including (in alphabetical order) Fredrik Dahl, Simone Deparis, Ulrich Haussmann, Yaozhong Hu, Marianne Huebner, Carl Peter Kirkebø, Nikolay Kolev, Takashi Kumagai, Shlomo Levental, Geir Magnussen, Anders Øksendal, Jürgen Potthoff, Colin Rowat, Stig Sandnes, Lones Smith, Setsuo Taniguchi and Bjørn Thunestvedt.

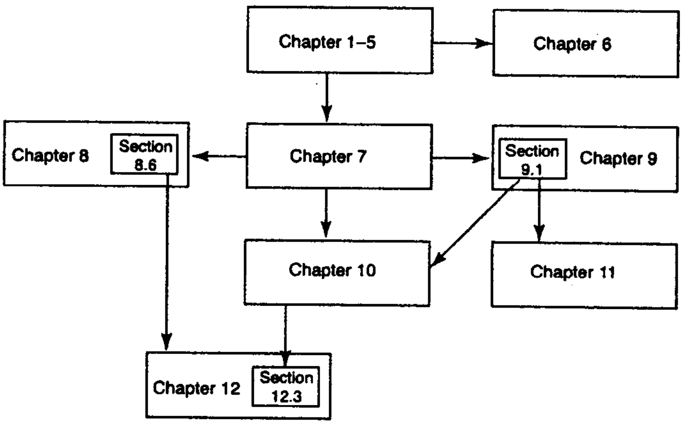
I want to thank them all for helping me making the book better. I also want to thank Dina Haraldsson for proficient typing.

Blindern, May 2000
Bernt Øksendal

Preface to the Fifth Edition

The main new feature of the fifth edition is the addition of a new chapter, Chapter 12, on applications to mathematical finance. I found it natural to include this material as another major application of stochastic analysis, in view of the amazing development in this field during the last 10–20 years. Moreover, the close contact between the theoretical achievements and the applications in this area is striking. For example, today very few firms (if any) trade with options without consulting the Black & Scholes formula!

The first 11 chapters of the book are not much changed from the previous edition, but I have continued my efforts to improve the presentation throughout and correct errors and misprints. Some new exercises have been added. Moreover, to facilitate the use of the book each chapter has been divided into subsections. If one doesn't want (or doesn't have time) to cover all the chapters, then one can compose a course by choosing subsections from the chapters. The chart below indicates what material depends on which sections.



For example, to cover the first two sections of the new chapter 12 it is recommended that one (at least) covers Chapters 1–5, Chapter 7 and Section 8.6. Chapter 10, and hence Section 9.1, are necessary additional background for Section 12.3, in particular for the subsection on American options.

In my work on this edition I have benefitted from useful suggestions from many people, including (in alphabetical order) Knut Aase, Luis Alvarez, Peter Christensen, Kian Esteghamat, Nils Christian Framstad, Helge Holden, Christian Irgens, Saul Jacka, Naoto Kunitomo and his group, Sure Mataramvura, Trond Myhre, Anders Øksendal, Nils Øvrelid, Walter Schachermayer, Bjarne Schielderop, Atle Seierstad, Jan Ubøe, Gjermund Våge and Dan Zes. I thank them all for their contributions to the improvement of the book.

Again Dina Haraldsson demonstrated her impressive skills in typing the manuscript – and in finding her way in the LATEX jungle! I am very grateful for her help and for her patience with me and all my revisions, new versions and revised revisions ...

Blindern, January 1998
Bernt Øksendal

Preface to the Fourth Edition

In this edition I have added some material which is particularly useful for the applications, namely the martingale representation theorem (Chapter IV), the variational inequalities associated to optimal stopping problems (Chapter X) and stochastic control with terminal conditions (Chapter XI). In addition solutions and extra hints to some of the exercises are now included. Moreover, the proof and the discussion of the Girsanov theorem have been changed in order to make it more easy to apply, e.g. in economics. And the presentation in general has been corrected and revised throughout the text, in order to make the book better and more useful.

During this work I have benefitted from valuable comments from several persons, including Knut Aase, Sigmund Berntsen, Mark H. A. Davis, Helge Holden, Yaozhong Hu, Tom Lindstrøm, Trygve Nilsen, Paulo Ruffino, Isaac Saias, Clint Scovel, Jan Ubøe, Suleyman Ustunel, Qinghua Zhang, Tusheng Zhang and Victor Daniel Zurkowski. I am grateful to them all for their help.

My special thanks go to Håkon Nyhus, who carefully read large portions of the manuscript and gave me a long list of improvements, as well as many other useful suggestions.

Finally I wish to express my gratitude to Tove Møller and Dina Haraldsson, who typed the manuscript with impressive proficiency.

Oslo, June 1995

Bernt Øksendal

Preface to the Third Edition

The main new feature of the third edition is that exercises have been included to each of the chapters II–XI. The purpose of these exercises is to help the reader to get a better understanding of the text. Some of the exercises are quite routine, intended to illustrate the results, while other exercises are harder and more challenging and some serve to extend the theory.

I have also continued the effort to correct misprints and errors and to improve the presentation. I have benefitted from valuable comments and suggestions from Mark H. A. Davis, Håkon Gjessing, Torgny Lindvall and Håkon Nyhus, My best thanks to them all.

A quite noticeable non-mathematical improvement is that the book is now typed in T_EX. Tove Lieberg did a great typing job (as usual) and I am very grateful to her for her effort and infinite patience.

Oslo, June 1991 *Bernt Øksendal*

Preface to the Second Edition

In the second edition I have split the chapter on diffusion processes in two, the new Chapters VII and VIII: Chapter VII treats only those basic properties of diffusions that are needed for the applications in the last 3 chapters. The readers that are anxious to get to the applications as soon as possible can therefore jump directly from Chapter VII to Chapters IX, X and XI.

In Chapter VIII other important properties of diffusions are discussed. While not strictly necessary for the rest of the book, these properties are central in today's theory of stochastic analysis and crucial for many other applications.

Hopefully this change will make the book more flexible for the different purposes. I have also made an effort to improve the presentation at some points and I have corrected the misprints and errors that I knew about, hopefully without introducing new ones. I am grateful for the responses that I have received on the book and in particular I wish to thank Henrik Martens for his helpful comments.

Tove Lieberg has impressed me with her unique combination of typing accuracy and speed. I wish to thank her for her help and patience, together with Dina Haraldsson and Tone Rasmussen who sometimes assisted on the typing.

Oslo, August 1989 *Bernt Øksendal*

Preface to the First Edition

These notes are based on a postgraduate course I gave on stochastic differential equations at Edinburgh University in the spring 1982. No previous knowledge about the subject was assumed, but the presentation is based on some background in measure theory.

There are several reasons why one should learn more about stochastic differential equations: They have a wide range of applications outside mathematics, there are many fruitful connections to other mathematical disciplines and the subject has a rapidly developing life of its own as a fascinating research field with many interesting unanswered questions.

Unfortunately most of the literature about stochastic differential equations seems to place so much emphasis on rigor and completeness that it scares many nonexperts away. These notes are an attempt to approach the subject from the nonexpert point of view: Not knowing anything (except rumours, maybe) about a subject to start with, what would I like to know first of all? My answer would be:

1) In what situations does the subject arise?
2) What are its essential features?
3) What are the applications and the connections to other fields?

I would not be so interested in the proof of the most general case, but rather in an easier proof of a special case, which may give just as much of the basic idea in the argument. And I would be willing to believe some basic results without proof (at first stage, anyway) in order to have time for some more basic applications.

These notes reflect this point of view. Such an approach enables us to reach the highlights of the theory quicker and easier. Thus it is hoped that these notes may contribute to fill a gap in the existing literature. The course is meant to be an appetizer. If it succeeds in awaking further interest, the reader will have a large selection of excellent literature available for the study of the whole story. Some of this literature is listed at the back.

In the introduction we state 6 problems where stochastic differential equations play an essential role in the solution. In Chapter II we introduce the basic mathematical notions needed for the mathematical model of some of these problems, leading to the concept of Ito integrals in Chapter III. In Chapter IV we develop the stochastic calculus (the Ito formula) and in Chapter V we use this to solve some stochastic differential equations, including the first two problems in the introduction. In Chapter VI we present a solution of *the linear filtering problem* (of which problem 3 is an example), using the stochastic calculus. Problem 4 is *the Dirichlet problem.* Although this is purely deterministic we outline in Chapters VII and VIII how the introduction of an associated Ito diffusion (i.e. solution of a stochastic differential equation) leads to a simple, intuitive and useful stochastic solution, which is the cornerstone of stochastic potential theory. Problem 5 is an *optimal stopping problem.* In Chapter IX we represent the state of a game at time t by an Ito diffusion and solve the corresponding optimal stopping problem. The solution involves potential theoretic notions, such as the generalized harmonic extension provided by the solution of the Dirichlet problem in Chapter VIII. Problem 6 is a stochastic version of F.P. Ramsey's classical control problem from 1928. In Chapter X we formulate the general *stochastic control problem* in terms of stochastic differential equations, and we apply the results of Chapters VII and VIII to show that the problem can be reduced to solving the (deterministic) Hamilton-Jacobi-Bellman equation. As an illustration we solve a problem about optimal portfolio selection.

After the course was first given in Edinburgh in 1982, revised and expanded versions were presented at Agder College, Kristiansand and University of Oslo. Every time about half of the audience have come from the applied section, the others being so-called "pure" mathematicians. This fruitful combination has created a broad variety of valuable comments, for which I am very grateful. I particularly wish to express my gratitude to K.K. Aase, L. Csink and A.M. Davie for many useful discussions.

I wish to thank the Science and Engineering Research Council, U.K. and Norges Almenvitenskapelige Forskningsråd (NAVF), Norway for their financial support. And I am greatly indebted to Ingrid Skram, Agder College and Inger Prestbakken, University of Oslo for their excellent typing – and their patience with the innumerable changes in the manuscript during these two years.

Oslo, June 1985

Bernt Øksendal

Note: Chapters VIII, IX, X of the First Edition have become Chapters IX, X, XI of the Second Edition.

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