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SYSTEMS AND CONTROL

Nonlinear Systems

Analysis, Stability, and Control

非线性系统

Shankar Sastry

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Preface

There has been a great deal of excitement in the last ten years over the emergence of new mathematical techniques for the analysis and control of nonlinear systems: Witness the emergence of a set of simplified tools for the analysis of bifurcations, chaos, and other complicated dynamical behavior and the development of a comprehensive theory of geometric nonlinear control. Coupled with this set of analytic advances has been the vast increase in computational power available for both the simulation and visualization of nonlinear systems as well as for the implementation in real time of sophisticated, real-time nonlinear control laws. Thus, technological advances have bolstered the impact of analytic advances and produced a tremendous variety of new problems and applications that are nonlinear in an essential way. Nonlinear control laws have been implemented for sophisticated flight control systems on board helicopters, and vertical take off and landing aircraft; adaptive, nonlinear control laws have been implemented for robot manipulators operating either singly, or in cooperation on a multi-fingered robot hand; adaptive control laws have been implemented for jet engines and automotive fuel injection systems, as well as for automated highway systems and air traffic management systems, to mention a few examples. Bifurcation theory has been used to explain and understand the onset of flutter in the dynamics of aircraft wing structures, the onset of oscillations in nonlinear circuits, surge and stall in aircraft engines, voltage collapse in a power transmission network. Chaos theory has been used to predict the onset of noise in Josephson junction circuits and thresholding phenomena in phase-locked loops. More recently, analog computation on nonlinear circuits reminiscent of some simple models of neural networks hold out the possibility of rethinking parallel computation, adaptation, and learning.

It should be clear from the preceding discussion that there is a tremendous breadth of applications. It is my feeling, however, that it is possible at the current time to lay out in a concise, mathematical framework the tools and methods of analysis that underly this diversity of applications. This, then, is the aim of this book: I present the most recent results in the analysis, stability, and control of nonlinear systems. The treatment is of necessity both mathematically rigorous and abstract, so as to cover several applications simultaneously; but applications are sketched in some detail in the exercises.

The material that is presented in this book is culled from different versions of a one-semester course of the same title as the book that I have taught once at MIT and several times at Berkeley from 1980 to 1997. The prerequisites for the first year graduate course are:

- An introduction to mathematical analysis at the undergraduate level.
- An introduction to the theory of linear systems at the graduate level.

I will assume these prerequisites for the book as well. The analysis prerequisite is easily met by Chapters 1–7 of Marsden's *Elementary Classical Analysis*, (W. H. Freeman, 1974) or similar books. The linear systems prerequisite is met by Callier and Desoer's *Linear Systems Theory*, (Springer Verlag, 1991) or Rugh's *Linear System Theory*, (Prentice Hall, 1993); Chen's *Linear System Theory and Design*, (Holt Reinhart and Winston, 1984); or Kailath's *Linear Systems*, (Prentice Hall, 1980) or the recent *Linear Systems* by Antsaklis and Michel, (McGraw Hill, 1998).

I have never succeeded in covering all of the material in this book in one semester (45 classroom hours), but here are some packages that I have covered, along with a description of the style of the course

- *Analysis, Stability and some Nonlinear Control*
  Chapters 1–7 and part of Chapter 9.
- *Analysis, Some Stability and Nonlinear Control*
  Chapters 1–3, 5–6 followed by Chapters 9, 10 with supplementary material from Chapter 8.
- *Mathematically Sophisticated Nonlinear Control Course*
  Chapters 1, 2, 4, 5–7, with supplementary material from Chapter 3, and Chapters 9–11 with supplementary material from Chapter 8.

Alternatively, it is possible to use all the material in this book for a two-semester course (90 classroom hours) on nonlinear systems as follows:

- (45 hours Semester 1) Chapters 1–7.
- (45 hours Semester 2) Chapters 8–12.

For schools on the quarter system, 80 classroom hours spread over two quarters can be used to cover roughly the same material, with selective omission of some topics from Chapters 3, 6, and 7 in the first quarter and the omission of some topics from Chapters 8, 11, and 12 in the second quarter. A shorter 60 classroom hour long two quarter sequence can also be devised to cover

1. (30 hours) Introductory course on Nonlinear Systems. Chapters 1, 2, 3 (Sections 3.1—3.5), Chapter 4 (Sections 4.1—4.6), and Chapter 5.
2. (30 hours) Intermediate course on Nonlinear Control. Chapter 3 (Section 3.9), Chapter 8, Chapter 9, 10, and parts of Chapter 11.

The structuring of courses at Berkeley favors the two semester structure, with the first course for second-semester graduate students (taught in the spring semester), and the second course called "Advanced Topics in Nonlinear Control" for second-year graduate students (taught in the fall). However, I wish to emphasize that we frequently see undergraduate students taking this course and enjoying it.

Access to a simulation package for simulating the dynamics of the nonlinear systems adds a great deal to the course, and at Berkeley I have made available Matlab, Simnon and Matrix-X at various times to the students as simulation toolkits to use to help stimulate the imagination and help in the process of "numerical experimentation." While I have usually had take home final examinations for the students, I think that it is useful to have "project-based" final examinations with numerical examples drawn from a set of particularly topical applications. A word about the problem sets in this book; they are often not procedural, and frequently need thought and sometimes further reference to the literature. I have found that this is a nice way to draw oneself into what is a very exciting, dynamic and rapidly evolving area of research. I have included these also because over the years, it has been a pleasant surprise to me to see students solve problem sets based on the archival literature with ease, when they are given adequate background. I have chosen applications from a wide variety of domains: mechatronic systems, classical mechanical systems, power systems, nonlinear circuits, neural networks, adaptive and learning systems, flight control of aircraft, robotics, and mathematical biology, to name some of the areas covered. I invite the reader to enjoy and relate to these applications and feel the same sense of scientific excitement that I have felt for the last twenty odd years at the marvels and mysteries of nonlinearity.

The author would be grateful for reports of typographic and other errors electronically through the WWW page for the book:

`robotics.eecs.berkeley.edu/~sastry/nl.book`

where an up-to-date errata list will be maintained along with possible additional exercises.

Shankar Sastry
Berkeley, California
March 1999

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In any large undertaking there are a number of people on whose shoulders we stand. This is certainly the case for me in this book, and the particular shoulders on which I stand are those of my teachers and my students. I owe an immense debt of gratitude to Charles Desoer, Jerrold Marsden, and Roger Brockett for having given me the love for and curiosity about nonlinear systems. Pravin Varaiya, Sanjoy Mitter, and Petar Kokotović have all been my early (and continuing mentors) in this nonlinear endeavor as well. My students have provided me with some of the most exciting moments of discovery over the years that I have worked in nonlinear control and I wish to acknowledge, especially and gratefully, those that have worked with me on subject matter that is represented in this book in chronological order of completion of their academic careers at Berkeley: Brad Paden, Stephen Boyd, Marc Bodson, Li Chen Fu, Er Wei Bai, Andrew Packard, Zexiang Li, Ping Hsu, Saman Behtash, Arlene Cole, John Hauser, Arlene Cole, Richard Murray, Andrew Teel, Raja Kadiyala, A. K. Pradeep, Linda Bushnell, Augusto Sarti, Gregory Walsh, Dawn Tilbury, Datta Godbole, John Lygeros, Jeff Wendlandt, Lara Crawford, Claire Tomlin, and George Pappas. Indeed, these folks will find much in this book that is rather familiar to them since their research work (both with me and after they left Berkeley and set up their own research programs) is prominently featured in this book. In Chapters 8, 10, 11, and 12 I have explicitly pointed out the contributions of Claire Tomlin, Yi Ma, John Hauser, Richard Murray, Dawn Tilbury, George Pappas, and John Lygeros in writing parts of these chapters.

This book has been classroom tested in its different formative phases by John Hauser at the University of Southern California, Richard Murray and Jerrold Marsden at Caltech, Hsia Dong Chiang at Cornell, and Ken Mease at University of

California, Irvine, Claire Tomlin at Berkeley and Stanford, and Shahram Shahruz and George Pappas at Berkeley. I am grateful to them for their painstaking comments. I would also like to thank Claire Tomlin, Nanayaa Twum Danso, George Pappas, Yi Ma, John Koo, Claudio Pinello, Jin Kim, Jana Kosecka, and Joao Hespanha for their help with proofreading the manuscript. I thank Christine Colbert for her superb drafting of the figures and thank Achi Dosanjh of Springer-Verlag, for her friendly management of the writing, and her patience with getting the manuscript reviewed.

Colleagues who have worked with me and inspired me to learn about new areas and new directions abound. Especially fresh in my mind are the set of lectures on converse Lyapunov theorems given by M. Vidyasagar in Spring 1979 and the short courses on nonlinear control taught by Arthur Krener in Fall 1984 and by Alberto Isidori in the Fall 1989 at Berkeley that persuaded me of the richness of nonlinear control. I have fond memories of joint research projects in power systems and nonlinear circuits with with Aristotle Arapostathis, Andre Tits, Fathi Salam, Eyad Abed, Felix Wu, John Wyatt, Omar Hijab, Alan Willsky, and George Verghese. I owe a debt of gratitude to Dorotheé Normand Cyrot, Arthur Krener, Alberto Isidori, Jessy Grizzle, L. Robert Hunt, and Marica di Benedetto for sharing their passion in nonlinear control with me. Richard Montgomery and Jerry Marsden painstakingly taught me the amazing subtleties of nonholonomic mechanics. I thank Georges Giralt, Jean-Paul Laumond, Ole Sordalen, John Morten Godhavn, Andrea Balluchi, and Antonio Bicchi for their wonderful insights about non-holonomic motion planning. Robert Hermann, Clyde Martin, Hector Sussmann, Christopher Byrnes, and the early Warwick lecture notes of Peter Crouch played a big role in shaping my interests in algebraic and geometric aspects of nonlinear control theory. P. S. Krishnaprasad, John Baillieul, Mark Spong, N. Harris Mc Clamroch, Gerardo Lafferriere, T. J. Tarn, Dan Koditschek were co-conspirators into unlocking the mysteries of nonlinear problems in robotics. Stephen Morse, Brian Anderson, Karl Astrom, and Bob Narendra played a considerable role in my understanding of adaptive control.

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Finally, on a personal note, I would like to thank my mother and late father for the courage of their convictions, selfless devotion, and commitment to excellence.

Shankar Sastry
Berkeley, California
March 1999

Standard Notation

The following notation is standard and is used throughout the text. Other non-standard notation is defined when introduced in the text and is referenced in the index. A word about the numbering scheme: Not all equations are numbered, but those that are frequently referenced are. Theorems, Claims, Propositions, Corollaries, Lemmas, Definitions, Examples are numbered consecutively in the order in which they appear and they are *all numbered.* Their text is presented in an *emphasized font.* If the theorems, claims, propositions, etc. are specifically noteworthy they are named in **bold font** before the statement. Exercises are at the end of each chapter and are all numbered consecutively, and if especially noteworthy are named like the theorems, claims, propositions, etc. Proofs in the text end with the symbol □ to demarcate the proof from the following text.

Sets

$a \in A$	a is an element of the set A
$A \subset B$	set A is contained in set B
$A \cup B$	union of set A with set B
$A \cap B$	Intersection of set A with set B
$\ni$	such that
$p \Rightarrow q$	p implies q
$p \Leftarrow q$	q implies p
$p \Leftrightarrow q$	p is equivalent to q
M°	interior of a set M

$\overline{M}$	closure of M
$]a, b[$	open subset of the real line
$[a, b]$	closed subset of the real line
$[a, b[$	subset of the real line closed at a, and open at b
$a \to b$	a tends to b
$a \downarrow b$	a *decreases* towards b
$a \uparrow b$	a *increases* towards b
$\oplus$	direct sum of *subspaces*

Algebra

$\mathbb{N}$	set of non-negative integers, namely, $(0, 1, 2, \ldots)$
$\mathbb{R}$	field of real numbers
$\mathbb{Z}$	ring of integers, namely, $(\ldots, -1, 0, 1, \ldots)$
j	square root of -1
$\mathbb{C}$	field of complex numbers
$\mathbb{R}_+(\mathbb{R}_-)$	set of non-negative (non-positive) reals
$\mathbb{C}_+(\mathbb{C}_-)$	set of complex numbers in the right (left) half plane, including the imaginary axis
$j\omega$ axis	set of purely imaginary complex numbers
$\mathbb{C}_-^\circ$	$\{s \in \mathbb{C} : \text{Re}\, s < 0\}$ = interior of $\mathbb{C}_-$
$\mathbb{C}_+^\circ$	$\{s \in \mathbb{C} : \text{Re}\, s > 0\}$ = interior of $\mathbb{C}_+$
A^n	set of n-tuples of elements belonging to the set A (e.g., $\mathbb{R}^n$, $\mathbb{R}[s]^n$)
$A^{m\times n}$	set of $m \times n$ arrays with entries in A.
$\sigma(A)$	set of eigenvalues (spectrum) of a square matrix A
$(x_k)_{k\in K}$	family of elements with K, an index set.
$F[x]$	ring of polynomials in one variable x with coefficients in a field F
$F(x)$	field of rational functions in one variable x with coefficients in a field F

Analysis

$f : A \mapsto B$	f maps the domain A into the codomain B
$f(A)$	range of $f := \{y \in B : y = f(x) \text{ for some } x \in A\}$
A°	interior of A
$\bar{A}$	closure of A
∂A	boundary of A

$C([t_0, t_1], \mathbb{R})$	vector space of continuous functions $[t_0, t_1] \mapsto \mathbb{R}$
$C([t_0, t_1])$	vector space of continuous functions $[t_0, t_1] \mapsto \mathbb{R}$
$C^k([t_0, t_1], \mathbb{R})$	vector space of continuous functions $[t_0, t_1] \mapsto \mathbb{R}$ with k continuous derivatives
$C^k([t_0, t_1], \mathbb{R}^n)$	vector space of continuous functions $[t_0, t_1] \mapsto \mathbb{R}^n$ whose first k derivatives are continuous
$\|x\|$	norm of an element x in a vector space
$\langle x, y \rangle$	inner-product of two vectors x, y in a Hilbert space
$\hat{f}, \hat{G}$	Laplace transform of scalar (or vector) function f or matrix function G both defined on $\mathbb{R}_+$
$\dot{f}, \dot{G}$	time derivative of scalar (or vector) function f or matrix function G both defined on $\mathbb{R}_+$
$Df(x)$	derivative of a function $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ a matrix $\in \mathbb{R}^{m \times n}$
$D_i f(x_1, x_2, \ldots, x_p)$	Derivative of $f : \mathbb{R}^{n_1} \times \cdots \times \mathbb{R}^{n_p}$ with respect to the i-th argument
$D^2 f(x)$	second derivative of $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ with respect to its argument, a bi-linear map from $\mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}^m$
$D^k_{i_1,\ldots,i_k} f(x_1, x_2, \ldots, x_p)$	k-th partial derivative of $f(x_1, \ldots, x_p)$ with respect to $x_{i_1}, \ldots, x_{i_k}$ a k-linear map from $\mathbb{R}^{n_{i_1}} \times \cdots \times \mathbb{R}^{n_{i_k}} \mapsto \mathbb{R}^m$
$L_p[t_0, t_1]$	vector space of $\mathbb{R}$ valued functions with p-th power integrable over $[t_0, t_1]$
$L^k_p[t_0, t_1]$	vector space of $\mathbb{R}^k$ valued functions with p-th power integrable over $[t_0, t_1]$
$o(x)$	little "o" of x, that is a function $g(x)$, such that $\lim_{\|x\| \to 0} \|g(x)\|/\|x\| = 0$
$O(x)$	capital "O" of x, that is a function $h(x)$, such that $\lim_{\|x\| \to 0} \|h(x)\|$ is well-defined and $\neq 0$

Contents