Mastering Elementary Algebra

$< 4 + 2 > 4$

$-3x = 9xy$

$4 \pm \sqrt{6x} - 2$

$y^2 + 7 = 3x$

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MASTERING ELEMENTARY ALGEBRA

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PREFACE

Mastering Elementary Algebra is a basic text for college students who need to study or review elementary algebra. The book will prepare students for technical courses in business, chemistry, or physics as well as for intermediate algebra courses. Its format is suitable for diverse teaching modes—from traditional lecture classes to individualized math programs.

ORGANIZATION

The following approach is used throughout the text:

1. Presentation of the mathematical concept
2. Definitions of new terminology
3. Detailed step-by-step examples
4. Steps summarized in a rule
5. Progress Checks to monitor the student's mastery
6. Numerous Practice Problems—progressing from easy to difficult—after each section
7. Chapter Review and Sample Test at the end of each chapter, with problems referenced by section number
8. Cumulative Review following every three chapters

IMPORTANT FEATURES

1. The reading level of the book is appropriate for first-time algebra students. It is written in an informal style with clear explanations using sound mathematical pedagogy.
2. Rules, properties, steps, and definitions are highlighted in boxes throughout the text. New terminology is introduced in capital letters.
3. Examples are abundant, comprehensive, and detailed. They are written in a clear, step-by-step manner to improve students' problem-solving skills.
4. A special feature in this text is the treatment of word problems. We encourage students to follow these steps:
   a. Identify the type of word problem.
   b. Underline the key words.

c. Construct a table or diagram when helpful.
d. Translate the English sentence into an algebraic equation.
e. Solve the equation.
f. Check the solution in the original problem.

5. Application problems at the end of some chapters allow students to apply their theoretical knowledge to real-life situations.
6. Common student errors are flagged with caution signs.
7. Introductions to chapters are often enhanced by historical notes.
8. A glossary with section reference numbers is in the back of the book.
9. Appendixes contain square root and metric conversion tables and material on scientific notation.
10. Answers to odd-numbered Practice Problems and to all Review Problems, Sample Tests, and Cumulative Review Problems appear in the back of the book.

SUPPLEMENTARY MATERIALS

1. *Instructor's Manual*, which includes:
   a. Objectives for each chapter
   b. Four forms of each chapter test with answers
   c. Four forms of cumulative tests with answers
   d. Four forms of final exam with answers
   e. Answers to the even-numbered Practice Problems
2. Videotaped lectures to accompany each chapter
3. *Study Guide*, which includes:
   a. TV notes to accompany the videotaped lectures
   b. Sample test for each chapter

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CHAPTER 1
INTRODUCTION TO ALGEBRA

1.1
REAL NUMBERS

The first known systematic treatment on algebra was written by an Arabian mathematician, al-Khowarizmi (*c.* 825 A.D.). The title of his book, *Al-jebr w'al muqabalah*, translated from the Arabic means "the art of bringing together unknown to match a known quantity." The key word in the title, *al-jebr*, or "bringing together," gave use to our word algebra. Algebra can be thought of as a tool used to generalize arithmetic.

We begin our text with an overview of algebra, starting with the real numbers and their properties.

THE NUMBER LINE

The numbers used to count objects, 1, 2, 3, . . . , are called COUNTING NUMBERS or NATURAL NUMBERS. (The three dots mean that the numbers continue in the same manner indefinitely. They are read "and so on.") When the number 0 is included with the list of natural numbers, we have a SET of numbers called the WHOLE NUMBERS.

A SET is any collection of objects. The objects belonging to a set are called MEMBERS or ELEMENTS of the set. We will use a pair of braces, { }, to represent objects that are in a set.

We can represent the set of whole numbers, therefore, as {0, 1, 2, 3, 4, 5, . . .} and the set of natural numbers as {1, 2, 3, 4, . . .}.

We can also represent the whole numbers using a NUMBER LINE. First, we will illustrate how to construct a number line.

STEPS TO CONSTRUCT A NUMBER LINE

☐ ***Step 1:*** Draw a straight line.
Choose a point on the line to represent 0. This point is called the ORIGIN on the number line.

Note: The arrowheads at the ends of the number line indicate that the line extends indefinitely in both directions.

☐ ***Step 2:*** Starting from 0, mark off to the right equally spaced points.

☐ ***Step 3:*** Label below the line enough of these points with the whole numbers 1, 2, 3, 4, . . ., to establish the scale.

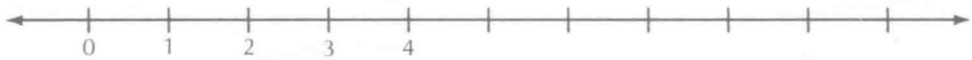
Example

1. Locate the whole numbers 0, 1, 3, 5, and 7 on the number line.

☐ **Step 1:** Draw a straight line.
Choose a point on the line to represent the origin, 0.

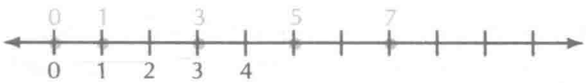
☐ **Step 2:** Starting from 0, mark off to the right equally spaced points.

☐ **Step 3:** Label below the line enough of these points to establish the scale.



☐ **Step 4:** Represent the numbers 0, 1, 3, 5, and 7 using heavy dots on the number line. We call this GRAPHING the numbers. Above the line, label those points that we graphed.

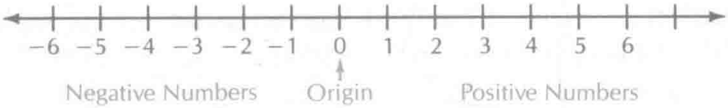
Answer:



REAL NUMBERS

All numbers other than zero are either positive or negative. It is interesting to note that the signs "+" and "−" were not deliberate mathematical inventions. It is probable that they were originally warehouse signs, used to indicate which packages were overweight and which were underweight. There are indications that the Chinese used negative numbers by 200 B.C. They indicated negative numbers by writing them in red. Hindus represented them by putting a dot or a circle above the number.

All numbers to the right of 0 on the number line are POSITIVE NUMBERS and can be indicated by a "+" sign in front of the numbers. In this text, when a number is written without a sign, the number is understood to be positive, that is, 2 means $+2$ and 1.5 means $+1.5$. Numbers to the left of 0 on the number line are NEGATIVE NUMBERS and are indicated by a "−" sign in front of the numbers. -2 (read "negative 2") means two units to the left of 0 and -5 (read "negative 5") means five units to the left of 0.

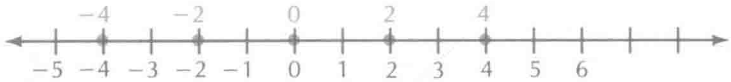


Positive numbers and negative numbers are called SIGNED NUMBERS. The number 0 is neither positive nor negative. Therefore 0 is not a signed number. The set comprising the signed numbers and 0, $\{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$, is called the set of INTEGERS.

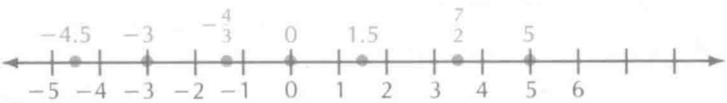
Example

2. Graph the integers -4, -2, 0, 2, and 4 on the number line.

Answer:



In addition to integers, there are other signed numbers on the number line. They are RATIONAL NUMBERS and IRRATIONAL NUMBERS. RATIONAL NUMBERS are numbers that can be written as quotients of two integers (where the divisor is not equal to zero). Some examples of rational numbers are graphed on the number line below.



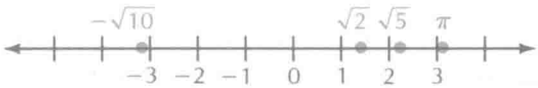
Notice that 1.5 is a rational number because 1.5 can be written as $1\frac{5}{10}$ or $\frac{3}{2}$.

Integers are rational numbers since they can be expressed as quotients of two integers in which the divisor is 1. For example, 0, 2, and -3 are all rational numbers because

$$0 = \frac{0}{1},\ 2 = \frac{2}{1},\ \text{and } -3 = \frac{-3}{1}.$$

All fractions and integers are rational numbers. And, since any repeating or terminating decimal can be expressed as a fraction, those decimals are also rational numbers. For example, the decimal .333 is a rational number because it can be expressed as the fraction $\frac{333}{1000}$.

IRRATIONAL NUMBERS are numbers that can be graphed on the number line but cannot be expressed as quotients of two integers. In decimal form, the irrational numbers do not terminate nor do they repeat a sequence of digits. π is an irrational number because $\pi = 3.1415926536\ldots$ (the sequence does not terminate or repeat) and it cannot be expressed as a quotient of two integers. Some examples of irrational numbers are graphed on the number line below. We will discuss the irrational numbers more extensively in Chapter 8.



$-\sqrt{10}$ is about -3.162; $\sqrt{2}$ is about 1.414; $\sqrt{5}$ is about 2.236.

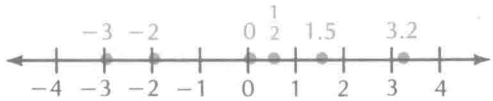
The rational and irrational numbers together make up the set of REAL NUMBERS. All real numbers can be represented by points on the number line. Also, any point on the number line represents a real number.

Examples

3. Graph the following real numbers on the number line:

   $-3, \frac{1}{2}, 1.5, -2, 0, 3.2.$

   Answer:



4. Answer each statement using the following set of real numbers:

   $\{5, 2, 0, -2, -3, -2.6, \frac{4}{3}, \pi\}.$

   a. List the set of integers.
   Answer: $\{5, 2, 0, -2, -3\}$

   b. List the set of natural numbers.
   Answer: $\{5, 2\}$

   c. List the set of whole numbers.
   Answer: $\{5, 2, 0\}$

   d. List the set of rational numbers.
   Answer: $\left\{5, 2, 0, -2, -3, -2.6, \frac{4}{3}\right\}$

   e. List the set of real numbers.
   Answer: $\left\{5, 2, 0, -2, -3, -2.6, \frac{4}{3}, \pi\right\}$

Figure 1 shows the relationships between the various types of real numbers.

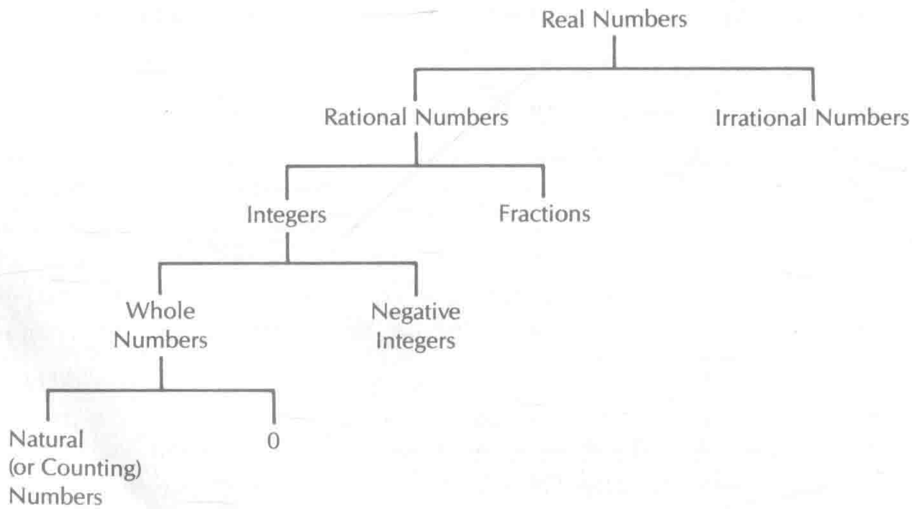


Figure 1

1.1A PROGRESS CHECK

1. Graph the whole numbers 0, 2, 5, and 6 on the number line.

2. Graph the real numbers -3, $-\frac{3}{4}$, 0, and $5\frac{1}{2}$ on the number line.

Tell whether each statement is true or false.

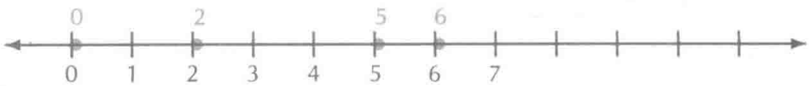
3. The set of integers contains only positive numbers.

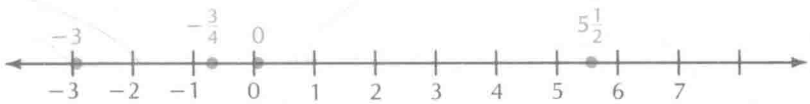
4. 0 is not an integer.

5. All rational numbers are integers.

6. .333 is not a rational number.

Answers:

1. 

2. 

3. False 4. False 5. False 6. False

INEQUALITY SYMBOLS

The number line is often useful in visualizing the relative order of two numbers. For example, to determine which is greater, -2 or -3, graph -2 and -3 on a number line. The number located to the right of the other on the number line is the greater of the two numbers.

-2 is located to the right of -3. Therefore, -2 is greater than -3.

Four INEQUALITY symbols are used to represent the relative order of two numbers. They are "$<$" (is less than), "$\leq$" (is less than or equal to), "$>$" (is greater than), and "$\geq$" (is greater than or equal to). Using the inequality symbol, the statement "-3 is less than -2" is written as $-3 < -2$. The statement "-2 is greater than -3" is written as $-2 > -3$.

Examples

5. Replace the question mark with the appropriate inequality symbol.

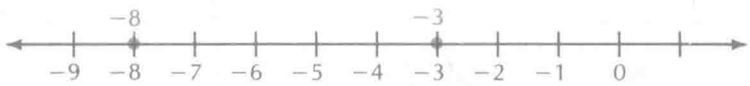
$0 \ ? -1$

To visualize the relative order of the two numbers, 0 and -1, we graph the numbers on a number line.

Since 0 is to the right of -1 on the number line, 0 is greater than -1. So, we replace the question mark with the inequality symbol "$>$".
Answer: $0 > -1$

6. Determine whether each statement is true or false.

   a. $-8 > -3$
   Graph -8 and -3 on the number line.



   Since -8 is to the left of -3, -8 is less than -3.
   Answer: $-8 > -3$ is a false statement.

   b. $1.75 > 1.7$
   Answer: Since 1.75 is greater than 1.7, the statement is true.

1.1B PROGRESS CHECK

Determine whether each statement is true or false.

1. $-7 > -6$ 2. $-100 < 50$ 3. $1.5 > \frac{3}{4}$

Rewrite each question by replacing the question mark with the appropriate inequality symbol.

4. $0 \ ? -\frac{1}{2}$ 5. $-4 \ ? -5$ 6. $-10 \ ? \ 3$

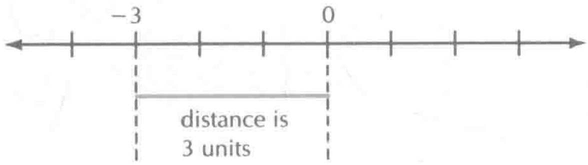
Answers: 1. False 2. True 3. True 4. $0 > -\frac{1}{2}$
5. $-4 > -5$ 6. $-10 < 3$

ABSOLUTE VALUE

Sometimes we may wish to state the distance that a given number is from the origin without regard to its direction from the origin.

> ***Definition:*** *The distance between a number and 0 on a number line—without regard to its direction from 0—is defined as the ABSOLUTE VALUE of the number.*

Since distance is a physical measurement, the absolute value of a number is never negative. For example, the absolute value of -3, written $|-3|$, is 3. The absolute value of a number is represented by a pair of vertical lines, $|\ \ |$.



Examples

7. Give the absolute value of each number.

Number	Absolute Value
10	$\|10\| = 10$
-5	$\|-5\| = 5$
-2.5	$\|-2.5\| = 2.5$
$-\frac{3}{2}$	$\left\|-\frac{3}{2}\right\| = \frac{3}{2}$
0	$\|0\| = 0$

($|10|$ is read "the absolute value of 10")
($|-5|$ is read "the absolute value of negative 5")

8. Indicate whether each statement is true or false.

   a. $|-5| < 5$
   Answer: False because $|-5| = 5$. The statement should be $|-5| = 5$ not $|-5| < 5$.

   b. $|-3| > |-10|$
   Answer: False because $|-3| = 3$, $|-10| = 10$, and $3 < 10$. The statement should be $|-3| < |-10|$.

ADDITIVE INVERSES

One property of real numbers states that every real number has an opposite. For example, the opposite of 2 is -2. The opposite of a number may also be called the negative of that number.

> ***Definition:*** *Two numbers that are the same distance from 0 on the number line but on opposite sides of 0 are called ADDITIVE INVERSES, or OPPOSITES, of each other.*

In Figure 2, the numbers 2 and -2 are called the additive inverses (opposites) of each other because 2 and -2 are 2 units from 0 but on opposite sides of 0.

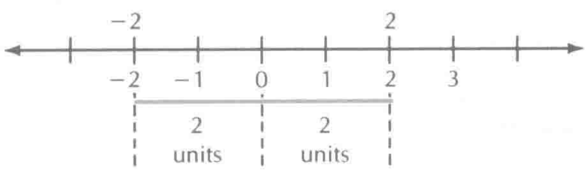


Figure 2

A number and its additive inverse have the same absolute value. For example, 2 and -2 are additive inverses of each other and they both have the same absolute value; that is $|2| = 2$ and $|-2| = 2$. The additive inverses occur in pairs except for the number 0. The additive inverse of 0 is 0 itself.

Example

9. Give the additive inverse of each number.

Number	Additive Inverse
3	-3
-2	2
$-\frac{3}{5}$	$\frac{3}{5}$
2.9	-2.9
0	0

STOP

1.1C PROGRESS CHECK

Give the additive inverse of each number.

1. -3 2. 1.5 3. $\frac{3}{4}$

Give the absolute value of each number.

4. 3 5. -2 6. -7.5

Answers: 1. 3 2. -1.5 3. $-\frac{3}{4}$ 4. 3 5. 2 6. 7.5

NAME ______________________ CLASS ______________ DATE ______________

1.1 PRACTICE PROBLEMS

1. Graph the following set of real numbers on the number line: $\left\{-4, 3\frac{1}{4}, 0, -1\frac{1}{2}, 1.5\right\}$.

2. Graph the following set of real numbers on the number line: $\left\{3, -1.5, 3.2, 0, -3\frac{3}{4}\right\}$.

Determine whether each statement is true or false.

3. Every whole number is a natural number.
4. .333 is not a real number.
5. The number 0 is considered to be positive.
6. Every real number is an integer.
7. The set of real numbers is composed of both the rational and irrational numbers.
8. The set of integers does not contain negative numbers.
9. 0 is an integer.

Answer problems 10–16 using the given set of numbers.

$\left\{6, -3, 0, \pi, \frac{3}{4}, -\frac{5}{2}, 1.8, \sqrt{2}, -4, -8\right\}$

10. List the set of integers less than 6.
11. List the set of whole numbers.
12. List the set of natural numbers.
13. List the set of negative numbers greater than -8.
14. List the set of irrational numbers.
15. List the set of nonnegative numbers.

ANSWERS

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______
12. ______
13. ______
14. ______
15. ______